Making Mathematics Meaningful, Realistic and Personalised: Changing the Direction of Relevance and Applicability

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Increasingly, there is a view that school mathematics should be aligned to the kind of problem-solving situations that individuals regularly encounter in their lives in order to ensure that mathematical thinking can be embedded into activities that are both useful and relevant to their personal contexts. However, connections between school and out-of-school mathematics can become quite problematic when we attempt to consider the diverse range of interests and perspectives an individual is likely to bring to a problem-solving situation. In order to develop learning environments that are meaningful (and often realistic) teachers have been encouraged to create problem-solving activities that are open-ended in nature or establish frameworks that challenge students to pose their own problems. Teachers are also challenged to demonstrate how school mathematics is both applicable and relevant to 'life after school'. Interestingly, these approaches attempt to legitimise school mathematics by demonstrating its usefulness to out-of-school contexts rather than bringing the mathematics students naturally and enthusiastically embrace outside school into regular classroom contexts. The type of mathematics students' use outside school is radically different—both in content and approach—to the mathematics they encounter in school. In this presentation I will argue that there needs to be a directional shift in the way in which we establish relevance and applicability in mathematical engagement in order to ensure that student learning remains meaningful, realistic and personalised.

In-School and Out-of-School Contexts

It could be argued that learning and doing mathematics is an act of sense making and consequently involves cultural, social, and cognitive phenomena that cannot be separated (Schoenfeld, 1989). If designed in a manner that considers these "learning dimensions," mathematical problems can be personalised in ways that provide opportunities for enhanced mathematical meaning. Unfortunately, school mathematics rarely considers the social and cultural contexts of learning (Lowrie, 2004) because planning in-school mathematics experiences to build on students' mathematical understandings from out-of-school experiences are both difficult and problematic (Masingila & de Silva, 2001).

The separation of cultural, social and cognitive aspects of mathematical engagement is perpetuated when problem solving is confined to students solving traditional word problems. Generally, word problems do not provide opportunities for students to access knowledge or consider experiences associated with their everyday lives. In fact, Lave (1998) argued that children's intuitions about the world are constantly violated in situations where they are asked to solve word problems. Other researchers (including Greer, 1997; Lowrie, 2004) have documented that students tend to ignore relevant and authentic aspects of reality and exclude real-world knowledge when solving word problems in classroom contexts. Moreover, the connections between students' everyday and classroom mathematics are not always attainable because of the disheartening fact that the contexts differ significantly (Lave, 1988; Lowrie & Clancy, 2003).

There is a view that children construct a set of beliefs and assumptions about problem solving (often developed by solving routine word problems that simply require the execution of one or more of the four arithmetic operations) that actually reduces the likelihood of connections to realistic contexts. Bonotto (2002) indicated that this only can change if there is a transformation in the teacher conceptions, beliefs and attitudes towards mathematics in ways that alter the culture of the classroom. Unfortunately, the disconnection between realistic and traditional problem solving establishes a situation where children begin to assume that what they know about the real world is not useful or valid. On the other hand, authentic problem-solving contexts provide opportunities for children to acquire knowledge and skills in situations that are not only meaningful but also relevant to their personal experiences which are established both in school and out-of-school contexts.

Lesh and Harel (2003), for example, maintained that the kind of problem-solving situations that should be emphasised in school are simulations of real-life experiences where mathematical thinking is useful in the everyday lives of the student or their family and friends. In recent years there has been a view that conditions that make out-of-school learning so effective must be recreated in classroom activities by "creating classroom situations that promote learning processes closer to those arising from out-of-school mathematics processes" (Bonotto, 2002, p. 3). As Boaler (1993) noted:

The reasons offered for learning in context seem to fall into two broad categories, one concerning motivation and interest of students through an enriched and vivid curriculum, the other concerning the enhanced transfer of learning through a demonstration of links between school mathematics and real world problems. (p. 14)

Such a focus creates a directional shift in terms of the mathematics content and processes that could be presented in classroom situations. Irrespective of whether or not the mathematical understandings are applied to out-of-school contexts or adapted from these realistic contexts, it is imperative that researchers and teachers: 1) develop a more comprehensive understanding of the processes students employ to solve problems in these out-of-school contexts; and 2) appreciate how mathematics studied at school is accessed and utilised in the student's world beyond school.

Connections Between Contexts

Connections between in-school and out-of-school mathematics are most commonly formed when teachers encourage students to recognise the mathematics learning and engagement they participate in outside school. In such situations, students are encouraged to reflect upon the content knowledge they accessed and the strategies they selected to solve tasks in everyday activities. In addition, teachers encourage students to reflect on how in-school learning and practice can be used and applied to situations out of school. As Masingila, Davidenko and Prus-Wisniowska (1996) asserted:

We believe that while some differences may be inherent in mathematics learning and practice in and out of school, the differences can be narrowed so that instead of being disjoint activities that do not influence each other, mathematics learning and practice in and out of school can build on and complement each other. In this way, students can bring to bear their mathematical knowledge gained in out-of-school experiences on their school mathematics. Likewise, students can use their school mathematics in solving problems that occur in everyday situations. (p. 177)

De Corte, Verschaffel and Greer (2000) maintained that in order for students to make meaningful connections between problem solving and real-life contexts they need to be immersed in innovative learning environments that are radically different from traditional classroom practices. They proposed that tasks should be well structured, diverse and authentic. Authentic tasks reflect the nature of real problems because they are complex, ill-structured, contain multiple perspectives and offer multiple pathways or solutions (Young, 1993). Since skills and knowledge are best acquired within realistic contexts (Grabinger, 1996), authentic tasks should be aligned between the context in which learning is represented and the real-life setting in which that knowledge will be called upon (Bennett, Harper & Hedberg, 2002). Consequently, the problem solver should be able to engage within the problem context from both sense making and process perspectives. Ideally, students should be encouraged to extend, adapt, revise and adopt mathematical ideas to a context that they can place themselves within.

It seems to be the case, however, that students do not intuitively use real-life knowledge or evoke realistic scenarios to contextualise problems—irrespective of how "authentic" they may be. There is a view that children construct a set of beliefs and assumption about problem solving (often developed by solving routine word problems that simply require the execution of one or more of the four arithmetic operations) that actually reduces the likelihood of connections to realistic contexts. Bonotto (2002) indicated that this only can change if there is a transformation in the teacher conceptions, beliefs and attitudes towards mathematics in ways that alter the culture of the classroom. Bennett et al (2002) commented that the level and nature of authenticity will depend on a) the level of sensory fidelity in task representation so that practical skills may be developed, b) the extent to which critical thinking or problem solving can be enhanced, and c) the potential for social interaction and engagement.

In a study that traced the out-of-school mathematical environments that children are exposed to, de Silva, Masingila, Sellmeyer and King (1997) observed middle school children a) using several mathematical concepts within a single activity, b) making decisions that were based on optimising goals, and c) communicating their ideas in order to make sense of complex relationships. Importantly, they concluded that these children were exposed to potentially rich mathematical contexts. How then, can such rich contexts be reproduced in classroom environments?

Bonotto (2002) proposed that classroom-based activities that aim to create connections between reality and mathematics should be founded on the use of concrete materials. In the present study, these cultural artefacts included brochures, menus, bus timetables and photographs. Such artefacts were relevant to the children because they allowed them to make connections to real-life experiences, offered significant reference to concrete situations, allowed them to keep their reasoning processes meaningful (Bonotto, 2002) and enhanced their capacity to think metacognitively (Lowrie & Clancy, 2003). This study specifically examined the influence the cultural artefacts had on the participants' problematisation of the problem scenario and the way in which they accessed out-of-school experiences to interpret and subsequently apply knowledge to the problem context.

The Approach

This investigation outlines the way in which authentic and cultural artefacts are used to promote mathematical meaning and sense making. In particular, the study describes the influence these artefacts have on students' learning in both in-school and out-of-school contexts.

The In-School Context

Three open-ended tasks were constructed for the investigation. Each task required the problem solver (children from a Grade 5 class) to consider a scenario associated with the planning of a family excursion to a theme park. One task required the construction of a proposed budget that a family would spend at a theme park, the second required the construction of a timetable which identified what a family would do at particular time intervals during the day, while the third task required the students to plan the day by locating rides and attractions and subsequently indicating a sequence of events.

Although a broad range of content knowledge and sense making was required to complete the tasks, the first targeted number sense, the second measurement sense and the third spatial sense. With each task, the problem solver was given a problem scenario, relevant pamphlet (including restaurant menus), and theme park map (see Figure 1 for an example). A range of visual literacies (Clancy & Lowrie, 2002) and information graphics (Harris, 1996) were required to make sense of the respective maps and to problematise the problem scenario. These realistic resources provided more information than was actually required for the problem solver to complete the task but certainly added to the authenticity of the task.

Figure 1. Wet`n`Wild theme park artifact.

The Out-of-School Context

The second context examined the way in which a 7 year old boy accessed a range of literacy and numeracy skills when playing an electronic game in a naturalistic setting. The Pokemon phenomenon—which is used as an example of out-of-school mathematical engagement—consists of a range of different synergistic texts such as movies, videos, books, internet cheat sites, card games, computer games, board games and hand held Game Boys. One of the key aspects of playing the Pokemon Game Boy is the notion of journey. Each game involves having a mission which involves going on a Pokemon journey in order to collect different species of Pokemon. These journeys require the players to move across a range of landscapes. Over time the game continues to evolve and so players constantly need to seek out information from the different forms of text.

The Pokenav (see Figure 2) provides access to important information about the location of cities and pathways (Routes) that are recommended for travel from one city to another. Morgan (the 7 year old case study participant) accessed additional information about specific pathways from the Pokemon books. In these magazines Morgan encountered different graphical representations of cities—including maps with different scale, orientation and perspective.

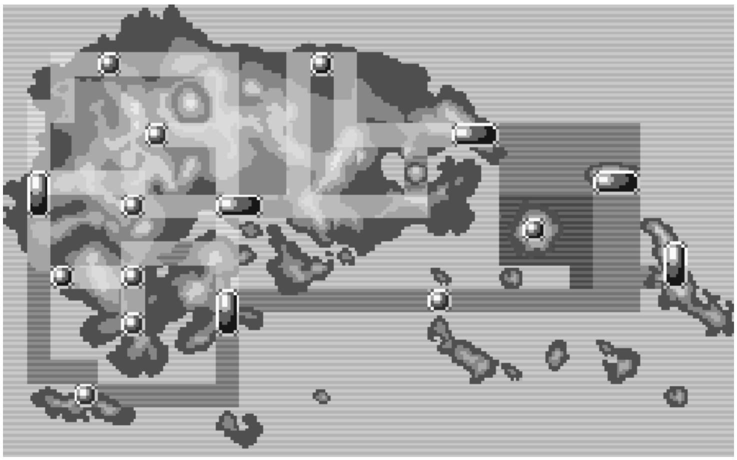

Figure 2: A visual representation of the map illustrated in the Pokenav.

Creating Personalised Scenarios Within In-School Contexts

Realistic scenarios or simulations of real-life experiences were often formulated in the early stages of problem representation when the children began to consider the theme park problem. Most of the participants intuitively used real-life knowledge to contextualise the problem—many of the students "placing" themselves within the actual theme park setting. The mathematical knowledge required to complete the task was dramatically shaped by these initial decisions.

I started at the helicopter. I started here because I could get a view of the whole park. So I could see what rides I could go on. It would let me see which rides are the best and where all the restaurants are…it gives me a bird's eye view to plan my map. [Jake]

I started at the cyclone and I did it to wake me up in the morning. I then looked at the table to find all the other frightening rides. [Harry]

Well I would leave the scariest rides until last…I made sure that I looked at the table that told you which rides were the most thrilling and planned the day from there…with the scariest rides put at the end of the day. [Mary]

Although only six children had actually visited the theme park referenced in their problem most of the participants had previously visited an amusement park at some stage. For example, Jake, Harry and Mary had not attended the park described in their problem but they had visited similar parks before. These three children frequently used the park map (and particularly the tables and other graphical information) to solve the problem. Their mathematics sense making was formulated from a desire to create an authentic setting. Furthermore, their problem-solving approaches were strongly influenced by previous "theme park experiences" with out-of-school knowledge influential from both sense making and process perspectives. In other words the students extended, adapted and revised mathematical ideas within a personalised context. Rather than fashioning situations that allowed them to easily derive solutions, the children engaged in processes with a level of sophistication beyond that usually encountered in the classroom. Thus, sense making was influenced by authentic scenarios and not the ease with which algorithms could be calculated.

The easiest part was choosing the food…because you had so many menus and quite a good menu…like I went to the back one [at one end of the park] twice and the ice-cream store once. [Michael]

I chose cold drinks for everyone because you would come here on a hot day (a water park)…you wouldn't want to be there on a cold day I don't reckon…I made sure that everyone got their favourite soft drink too…they were different prices so I didn't do 4 times the price for each drink. [Jane]

Similarly, the participants were willing to modify the actual question when they felt that it did not represent a "real" scenario. In the following transcripts Stuart and Sam were concerned that their timetables were not realistic because they were required to factor in a ten minute transition between rides (set in the initial problem scenario). Interestingly, sequences of ten minutes (between rides), twenty minutes (morning tea) and forty minutes (lunch) were presented so that the children could sequence itineraries within hourly timeslots.

I didn't like that you had to have 10 minutes between each ride…because the last time I was there it only took a few minutes to walk from one ride to another. [Stuart]

I wanted to change the problem a bit…because when you are at the Ski Challenge it wouldn't really take you 10 minutes just to walk to the Reef Discovery…And it would take longer than 10 minutes to walk right up to the hotel. 10 minutes could be the average but it's better to do it right…some are 2 minute walks while others could be almost 15. [Sam]

Although such reasoning would make the task more difficult to solve, both children modified the problem to ensure that it was realistic. As Boaler (1993) noted:

Activities must be genuinely open and allow students to move in the directions appropriate to their perception of the problem…Thus whilst the start of an activity may have a specific context, the development of the activity must enable students to follow routes which are their own…If the students' social and cultural values are encouraged and supported in the mathematics classroom, through the use of contexts or through an acknowledgement of personal routes and directions, then their learning with have more meaning to them. (p. 19)

It could be argued that this provided evidence of how students modified and adapted the task to make it more personally authentic.

Importantly, children who had never visited theme parks (only two in this class) attempted to personalise the problem context to make the problem more meaningful. Isabella, for example, created her personal scenario by establishing degrees of plausibility. She developed a budget that was based on her visiting a theme park with her older sister who was at boarding school—reasoning that her big sister had promised to take her to the Gold Coast one day. She continually validated the authenticity of her solution by seeking the advice of three of the children who had previously visited the theme park. She did not feel that she was disadvantaged by not having visited a park but commented that she tried to imagine herself at the park when solving the problem. The main difference between her approach to solving the problem and that of other students in the class was her willingness to share her solution and reasoning with her peers. Most other children in the class were content to individually work on the problem.

Accessing and Using Maps in Out-of-School Contexts

In order to play the game more efficiently, Morgan accessed and utilised various artifacts that involved interpreting maps. In fact, Morgan demonstrated the capacity to access important information from the Pokemon world by analyzing maps in different representations and scaled forms—including graphical information from magazines. These maps included "full" maps that represented the entire Poke-world (see Figure 2) and more detailed "zoom" maps that allow the player to navigate his way through towns (see Figure 3), cities, and various natural environments (including caves, mud slides and waterfalls) between these cities and towns. In addition, less detailed positional maps (see Figure 4) were regularly analysed in order for Morgan to determine where he was positioned in relation to significant landmarks.

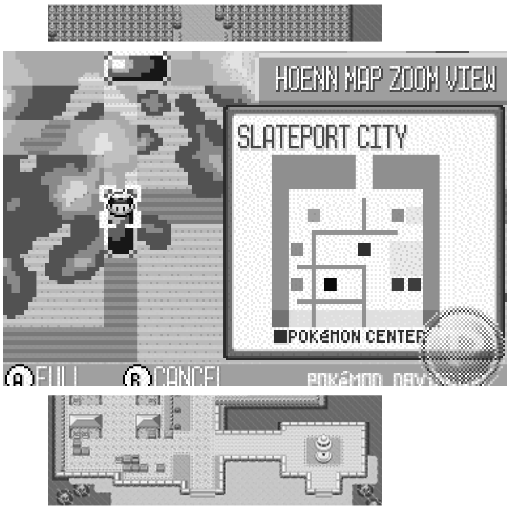

Figure 4. A positional map within the Gameboy.

Figure 3. A town (Slateport City) displayed within the Gameboy.

Essentially, the maps were utilised to locate information that was necessary to find (or catch) Pokemon. Morgan's capacity to reason visually and locate information in a relatively sophisticated manner was required in order to solve both routine and open-ended problems within the game context. As Morgan commented:

The Mountain Falls [is] the closest city you can go to. Once you go from Mauville City, that's where I am [showing the location on the screen], you go up there to there [pointing to another location on the map], then you go across here and follow that thing [a pathway], you end up in Mountain Falls. That's where the Magna Team are. You need to battle the leader two times…And this is Everyday City right over here [pointing]. That's the whole thing. I need to go over there, that's the Pokemon Centre right over there (Morgan is referring to the Pokenav that shows the whole Houen area map and the individual cities that are colour coded to represent different buildings).

The Pokenav (see Figure 2) provides access to important information about the location of cities and pathways (routes) that are recommended for travel from one city to another. Morgan accessed additional information about specific pathways from the Pokemon books. In these magazines Morgan encountered different graphical representations of cities—including maps with different scale, orientation and perspective. Although the magazine maps were more detailed (and in a larger scale) than the corresponding graphical representations in the Gameboy, Morgan found it advantageous to cross reference information whilst playing the game. The magazines became an important reference point for travel between cities because these maps provided more information within a single frame—not only was the scale easier to interpret, more information was represented within the given space. The Gameboy screen is relatively small (7 cm x 4 cm) and as a consequence the player would need to use scroll buttons (across eight compass-point directions) to view the information that could be represented in the magazine maps. Within the game-play context the player is able to navigate through space in both "full" and "zoom" modes (represented in Figures 1 and 3). The zoom mode displays information in a more detailed manner (possibly magnified ten fold) than the map that represents the Houen City. Morgan simultaneously moved between these two perspectives while regularly referring to the maps in the magazine.

[There are] two maps…The little map that shows you half of the town. You have to move over and it shows you the other half…You flip backwards and forwards between the cities to get more information [and] to see where you are going.

Morgan had developed an awareness of scale and proportion. Moreover, he appreciated the fact that you could only see part of the map in the zoom function mode and realised that one part of the map was connected to the other even though both parts of the map were not visible on the single screen.

Transferring Skills Within and Across Different Contexts

Impressively, Morgan not only remembered the directional sequences when revisiting cities, he was able to explain why it was important to go back to these destinations. His conversations referenced the Gameboy, the magazines and the Pokedex within the Gameboy simultaneously. Moreover, Morgan was able to effortlessly move between several graphical representations when describing his movements "inside" and "outside" the game context.

…the big [map] shows you just where the towns are. Like if they are going longways, tallways, diagonals or circles. It shows you the world, like here the world and here are the little towns and here's the other one and there are little ones here (referring to the maps on his Pokenav).

[With the zoom function] There's actually three maps… Because when you go on like if you click anywhere and go small, so you just see one more part and if you click say again, it just shows the whole world and then the little one. So it shows you one little square, it can show you the whole world and it can show you how cities are.

He was able to appreciate that his (the trainer's) location within a city or town could be represented in different ways on the same screen (as in Figure 3). Although he had not encountered notions of scale, proportion or perspective (in a school context) he was able to conceptualise the relationship between landmarks in different spaces as a series of routes. Furthermore, he was able to integrate these routes into networks of landmarks in ways that allowed him to make approximations of relative distances, and thus constitute a form of scale (Lehrer & Pritchard, 2002).

It was certainly the case that Morgan had not encountered notions of scale, proportion or perspective at school and yet he was interpreting maps and applying knowledge to situations with a level of sophistication that would probably not be expected of him at school for another three or four years. This form of mathematics was so realistic because it was contextualised within a social framework that was both meaningful and personally authentic. This type of mathematics was certainly useful to Morgan and his friends.

Conclusion Comments and Implications

The mathematics that the participants "naturally" used in both in-school and out-of-school contexts was challenging and sophisticated. Irrespective of the context, the students were able to utilise a range of reasoning skills to complete tasks in an environment that was empowering and intrinsically motivating. Moreover, the activities encouraged the students to make connections between mathematical ideas and use multiple artefacts to increase sense making. Almost all participants indicated that these activities bore no resemblance to the type of mathematical activities they were accustomed to solving in their "regular" classroom environment. De Corte, Verchaffel and Greer (2000) suggested that meaningful connections between problem solving and real-life contexts were more likely to happen in radically-different situations. It could be argued that these activities were not innovative—nevertheless they are certainly radically different that those experienced by these participants at school.

The following implications arise from the study:

- The open-ended nature of the tasks allowed the participants to pose problems within the original structure of the problem. Such rich tasks provided opportunities for the problem solver to make connections to real-life situations and encourage deep learning;
- It is important for teachers to encourage flexibility within problem contexts in order to allow students to make connections to real-life situations;
- The shared knowledge of the participants (and the opportunity access a range of authentic artefacts) helps establish powerful and rich learning environments;
- Such activities transform students' beliefs about problem solving and alter the culture of mathematical engagement; and
- If students' cultural and social values are encouraged in the classroom (including recognition of the mathematics they access in out-of-school settings) their learning will be meaningful.

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