RICH TASKS FOR MIDDLE YEARS

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This paper will explore the development of rich mathematical tasks for the middle school. The authors' experiences suggest that rich tasks resonate with sound pedagogical practices that are based on inclusive teaching and learning strategies. For the past four years the presenters have been part of a group of educators developing one such task. Multo, which is currently part of Maths300 (HREF1)is a simple game based on multiplication facts and the game of bingo that has grown, through teacher reflection and practice, into a potentially rich mathematical learning experience.

This paper will first detail the Multo lesson notes, (written in first person). Then in the second part, inclusive teaching strategies used in the lesson will be discussed.

Multo: A rich task

Multo is an excellent investigation based on the Multiplication Facts from $[0 \times 0]$ to $[9 \times 9]$ and Bingo. It starts out as a simple, good fun game but it has deeper layers of learning. It connects number skills and probability concepts, then turns into an investigation that contains all the elements of *Working Mathematically*, making a rich powerful learning journey that helps students

develop important mathematical understandings. The lesson also suits a range of ability levels, enabling every student to experience success.

Introducing the game

I tell the students we are going to play a game called Multo, which is like Bingo only we use basic multiplication facts.

We talk about Bingo and what we need to play, such as a grid, and how we need these things to play Multo as well. Then I start off by modelling a game.

Modelling the game

I model the game first so students can get a clear picture of how the game works. This also lets me show them some useful strategies in a non-threatening way. I show a card and ask them what the answer is, how many different ways can they see to make the same answer, find all the prime numbers, find all the cards whose product is 16 and so on. Lots of language practice as well. Usually students get the hang of this very quickly and start playing along.

I show the pack of 100 Multo Flashcards to the students. I like to gather the students around and spread them out on the floor or a table so they can see them all. We talk about the multiplication facts, giving students an opportunity to discuss and share their knowledge of factors. It is also a wonderful, spontaneous time to review multiplication and have some fun practising our mental arithmetic before we start. I explain to the students that for the Multo game we need a 4×4 grid and I draw a large grid onto a piece of butcher's paper. I explain that in each square on the grid I will need to write a number. This number is an answer to the multiplication problems written on the flash cards that might be drawn from the pack.

Then I explain how to win the game.

There are four ways to win Multo. You need to get 4 correct answers in either:

a row = 4 across
a column = 4 down
a diagonal = 4 diagonally
or 4 corners = one number in each corner

Next I fill out the numbers in the grid. I usually ask the students to help me do this. There can be a problem if they put in too many numbers that cannot be answers (for example: 31, 100, 53 etc). Some students often start to work out there are better numbers even at this early stage. I try not to give

too many clues, however I often have to 'rig it a bit' to make sure that we can get several Multos in the sample game. I always like to model the strategy that I want the students to use. I find a really useful strategy is: LOOK, THINK, DISCUSS, CHECK, RECORD

Now it's time to play – I ask the students to gather and shuffle the flashcards. I draw a flashcard from the pile, show it to the students and say the multiplication fact out loud. I make sure that I model the strategy I wish the students to use. So I:

1. Look at the flashcard (and say the problem out aloud, e.g. "four times three equals _____"
2. Think about it and say the answer "twelve"
3. Discuss this with the group
4. Check the answer using a calculator
5. Record by crossing out the answer on the grid.

We win (and finish) when we have a Multo.

Class organisation

Working in small groups or pairs helps the less confident students to participate and encourages peer tutoring or learning. It also encourages students to dialogue with each other and develops skills of cooperation within the competitive games context.

Playing the game

I give each group some butchers' paper and a marker. I ask each group to draw a large 4x4 grid onto their butcher's paper. Then to write onto their grid 16 different numbers that are the answers to the flash cards that might be drawn from the pack. I encourage the students to discuss the numbers they write down. The Challenge is to create a grid that gives them a good chance of winning the game. I shuffle the flashcards, draw a flashcard from the pile, show it to the students and say the multiplication fact out loud. I remind each group to use the *look, think, discuss, check, record* strategy by looking at the card and thinking together to work out the answer mentally. I encourage them to say the multiplication problem and talk with each other. When the group has the answer, they can check it using a calculator and record it on their grid by crossing it out. The winning group are the first to get Multo but usually the students like to keep playing until everyone has a Multo.

I usually invite teams that already have a Multo to get keep playing (using another colour marker can be useful). Note: the winning grid must be shown to the class. Then students know it's a fair game and learn incidentally from winning grids.

Developing skills and understandings

By playing the game again, students can begin to design better grids based on what they saw happen in the game and through the reflective questions. It may take many games before students are ready to be challenged to think about strategies. Some teachers introduce Multo by playing with their students at the end of lessons for several weeks or even a term until they feel students are ready. They then focus on it for several lessons, eventually using the computer to work the data.

Using focus questions to reflect and refine

I watch each student carefully so I know just when to ask that focus question that will make the most of the learning moment. Sometimes I ask the whole group and sometimes I challenge individual students who are ready.

While we are playing the game, I always use focus questions to encourage the students to think strategically about what they are doing, i.e. to see the strategies they are using and the patterns that are emerging.

These are some examples of focus questions I use:

What is the lowest number you can use? Why?
What is the highest number you can use? Why?
Are there any numbers you would never use? Why?
What are the best numbers to use? Why?
How often does each number occur? Why?
Where are the best places to put the numbers? Why?

Reflection and recording

After playing several games, I often 'pause' and spend time reflecting and recording as a group on what we have done and learnt so far. I like to use this as an opportunity to model to students how to write in their own mathematics book by constructing a joint piece of writing that everyone contributes to. This piece of writing will become pages in our Class Mathematics Book.

I scaffold the process of writing down and organising data in a meaningful way in our mathematics journals through modelling how to do it and constructing a group-negotiated text. By making the act of writing public, I can make the features and process of writing explicit and show students what goes on inside a writer's mind as they construct written text. I think out aloud – organise my thoughts and ideas, change my mind, wonder about the right word to use and perhaps spell (although I tend to de-emphasise spelling and focus on meaning instead). I always model making mistakes, crossing out, using arrows and being messy.

To construct a group-negotiated text, I use butcher paper which I attach to an easel or the wall. I scribe what the students tell me onto butchers' paper. Sometimes I help them to refine their thoughts and I always make sure that we are all happy with what is written down. When displayed on the wall, a group-negotiated text is a powerful and accessible example (which each person contributed to and therefore 'owns') of how students can construct their own writing. For this reason, I am very careful to set out my writing just as I would like students to eventually record data on their own in their Mathematics books or Mathematics journals.

FOCUS QUESTIONS TO GUIDE REFLECTIVE WRITING

- *What did we do?*

This is asking students to recall what they did. This is a way of focussing them on the exploration or lesson and the order of events. It will produce a kind of writing (or genre) called a recount. As long as they have participated in the experience, most students can produce this writing easily. It is one of the first kinds of writing children 'spontaneously' produce. It is important that recounts are not the *only* kind of writing students produce but it is an *excellent starting point* and can be used as a basis for other more sophisticated forms of writing later on (such as explanations and reports).

- *Why?*

This is asking them to use higher order thinking and analyse the reasons for doing certain things (taking action in certain ways). How much the teacher wants the student to focus on *Why?* will depend on the exploration and the purpose of the writing at this stage.

- *What did we find out?*

This asks students to link findings and consequences to their actions which is part of the working mathematically process.

- *How?*

This focuses students on what they did to find something out, that is, the process.

Don't forget to use this writing to create a language and mathematically rich learning environment by spreading it out and displaying it on the wall for all learners to see.

Use the Class Mathematics book

The Class Mathematics Book is made up of all the modelled and group-negotiated writing you do. Not only is it a good way of storing this work but it is a model of the individual's mathematics books that you and your students keep. There may be other related work you have done that is recorded in your class mathematics book and the individual's mathematics book that you can refer to. This models how to use a mathematics book, and demonstrates the need to write down things in a way that enables you to revisit them and understand what you did at a later date.

Wall glossary

I make sure all new words and special mathematics terms are written onto a whole-class constructed Mathematics Wall Glossary. Usually we stop and use our Mathematics and English dictionaries to check words and explore some of the differences in words when they are used in a mathematical context. For example, as students apply reasoning and strategies to the design of the grid.

Developing strategies

I wait until the students know how to play the game, are developing their multiplication skills and are starting to develop strategies for choosing numbers, and then I pose the challenge: *Who can design the best (winning) Multo grid?*

This time, as each group is designing the winning grid, I ask them to justify each number on their grid by writing around their grid, the reasons they have for putting each number on their grid. This will identify the different strategies students are using. I let them know that each group will be

sharing their grid with the whole class after we test their grids, which begs the question: *How will we test our grids?* The answer: *By playing a game of Multo!*

I keep a watchful eye on students writing as it gives me lots of information about their understandings. Usually they will develop their ideas and correct themselves through playing the game. Sometimes I use focus questions to individually challenge students about their ideas. Other times it will come out during group discussion. Once each group has finished their grid and writing, we play a game. The competition can be fierce and a 'prize' is often a great idea!

Identifying poor strategies

I put myself into the game by designing a grid that includes some problems. When I am discussing strategies I use my grid to highlight problems instead of drawing attention to students' grids, which may cause humiliation. This also makes me an equal participant and reinforces the teacher – student relationship as a partnership.
This is a sample grid that you may like to use.

18	13	6	48
10	12	7	24
15	0	16	4
1	36	9	8

I don't write my reasons around my grid because later, we often write around it as a group.

Guided reporting back

I use reporting back as an opportunity to help students refine and develop their language and understandings, as well as their mathematical reasoning. After we have finished playing, each group shares their grid by reporting back to the whole group. I ask them to explain their grid and their reasons.

If this is the first time the students have come across the idea of strategy I spend time developing our understandings and make sure it is added to the Mathematics Glossary.

With the students I brainstorm, discuss and list the strategies that the class is using onto butchers' paper (again as part of the Class Mathematics Book). To brainstorm I write a question on butchers paper such as: *What strategies are we using to make a winning Multo grid?* To help the discussion and to develop ideas further, I use my grid as an example and invite the students to analyse it.

With all of this extra information about good strategies to use it is time to pose the challenge again: *NOW…Who can design the best (winning) Multo grid?*

This is a wonderful question to set for homework, and for this reason I suggest it is a good idea to copy the list of useful strategies into their mathematics books. I tell them that we will be testing their grids by using the computer.

Linking the game to the computer

The software is designed to replicate the classroom experience as closely as possible so the students can easily move from one mode to another. Looking at this idea the other way around, the classroom experience should prepare the students for, and link to the computer experience as seamlessly (or closely) as possible.

While we are playing Multo with cards and paper, I often challenge students to come up with ideas for measuring the successfulness of a grid; such as the idea of counting the number of cards it took before a Multo was reached. And further, to use this idea to compare grids by comparing the number of drawn cards for each grid. But often it is more valuable for students to discover these ideas themselves as they use the computer to investigate and experiment.

When to introduce the computer

As the game is played, it provides verification of the students thinking and the logic of where they placed the numbers.

This is the ideal moment to introduce the computer program as it has a real purpose. When the students first use the computer program, try the first option, *Play a Game of Multo* a few times so that they become comfortable with the computer simulation of the game. Then I ask the focus question: *How do we test which grid is the best?*

There are many ways of using the software to find the best grid, such as by comparing grids, the average number of cards, percentage wins or individual numbers in grids.

Testing grids on the computer is an ideal opportunity to develop working mathematically strategies and skills. I model this for students, and use a framework for doing and recording our class investigations. I often use the following headings for my circular framework:

Formulate Hypotheses
Run Experiment
Record Results
Analyse Data
Refine Model

Students then use it to frame their own investigations as well as a framework for recording these in their mathematics books or mathematics journals.

Inclusive teaching and learning strategies

Multo was developed by the Maths No Fear team (Charles Lovitt, Juli Cathcart and Steve Flavel) and participating teachers from the Northern Territory, Queensland, New South Wales and Western Australia. Through the development of *Multo* and other Maths No Fear lessons, the team, with Juli Cathcarts' guidance, developed key understandings and ways of working with all mathematics learners. Many of these key understandings belong to the group called *inclusive teaching and learning strategies*.

The structure of first and other languages of indigenous students differ from English in that the language encodes very different systems of organising, relating and naming the environment. The learning of mathematics for indigenous students is about the learning of a different way of making meaning of the world. In this way, most indigenous Australian students' ways of learning differ from ESL students whose first language (and cultural background) is both alphabetical and mathematical.

Scaffolding

A scaffold is a temporary structure that is used to support the construction of a new building. When the building is complete, the scaffolding is removed. In education, the word scaffolding refers to the uses of special strategies, activities and processes that support students to construct new

knowledge and skills. Over time these strategies will move from teacher-centric, temporary structures, to student-centric, more permanent structures, that will form part of the students foundation for further learning.

Rich learning environments

Some aspects of learning are very much like hunting. A good hunter needs to be able to read their environment for clues and messages.

A well-organised classroom provides opportunities for students to hunt for meaningful contexts, ideas and language from their displayed products of previous learning. Students can then use these artefacts to build a richer hunting ground. This ground can be populated with:

- wall stories that build up a picture and record of the students learning as a unit of work progresses;
- wall glossaries of specialised language for mathematics, with examples taken from the student's own production;
- examples of ways to do things; and
- charts with appropriate annotations.

The most successful class environments for students to hunt in will be those in which the students take ownership of the grounds through active participation in their production.

The Mathematics Book

Writing is a record of thoughts, experiences and ways of thinking. The act of writing and reading encourages reflection on learning and to have a dialogue with oneself in very powerful ways. Encouraging students to write regularly about their learning can facilitate this reflective practice. This can be effectively achieved by extending the traditional mathematics exercise book into a journal. Through this extension, students gain a rich and personal record of their mathematics learning. Asking students to write about their feelings, to re-read their books and to dialogue with previous entries, can further enrich The Mathematics Book.

Ideally, the teacher also keeps a mathematics book, using and recording in it, as they would like their students to. In this way the teacher is modelling how to use a mathematics book.

In a mathematics book students can write notes, pose questions and record their reflections alongside their work. Teachers can pose questions and tasks

to encourage these students' actions within the lesson. Later students can be encouraged to review their work and write reflective summary pieces. At the back of The Mathematics Book, or alongside each page, a section can be set aside for a glossary.

These are some of the ways students have been observed using their mathematics books in the *Multo* lesson.

- To doodle – to explore ways of tackling the problem
- Create simple cases in order to solve the problem
- Record data systematically
- Graph and draw relationships
- Make and test generalisations
- Pose questions
- Reflect on progress so far
- Record connections and to link derived generalisations to different problems
- Explain new terminology and symbols (Glossary)
- Summaries of understandings gained
- Develop and practice algorithms
- Record feelings (AHA, Stuck, Give up) (Mason et al, 1987)
- Explain (in own words) the main ideas and concepts under study

The class mathematics book

To write meaningfully in their own mathematics books, students need to learn the appropriate skills. Teachers scaffolding the process of learning how to use mathematics books can effectively teach these skills.

One way to do this is for students to work as a whole group with the teacher. This enables the teacher to model the process of recording to the students. The class 'big' mathematics book can be constructed with butchers paper and crayons, with the teacher acting as a scribe. By keeping the sheets of paper loose during the unit, they can be displayed onto the wall to create a 'wall story'. In this way they help to create a language-rich and mathematically rich learning environment. Using this rich student generated display, the teacher can model productive ways of interacting with the display in order to support their own mathematics learning. The display is also celebratory of the students learning and at the end of the unit the sheets can be bound into a book so that the ' big' Class Mathematics Book is a continuously growing

record of the class learning. And during this process the students are immersed in language and mathematics.

By making the act of writing public, we can make the features and process of writing explicit and show students what goes on inside a writer's mind as they construct written text. Teachers can think out aloud – organise their thoughts and ideas, change their mind, and wonder about the right word to use. It is important to model making mistakes, crossing out, using arrows and being messy.

Teachers can construct a group-negotiated text, by using butcher paper attached to an easel or the wall. They can scribe what the students say onto butchers' paper and sometimes help students to refine their thoughts. Teachers of *Multo* report that is better to always make sure that everyone is happy with what is written down.

When displayed on the wall, a group-negotiated text is a powerful and accessible example (which each person contributed to and therefore 'owns') of how the students can construct their own writing. For this reason, it has been found that great care needs to be taken in setting out writing just as you would like students to.

Small group work

Teachers reported that effective small group work in *Multo* encourages discussion and dialogue, peer tutoring, and learning through observation and listening.

The Maths No Fear team believes that mathematics is a social construction, where student mathematicians are trying to construct new ways of making meaning. By its social nature, it is not possible for students to construct meaning in isolation from their fellow mathematicians.

A natural extension of small group work is that students can record information in their groups with one student acting as a scribe for the others, and each student contributing to the construction of the text. Writing on butchers' paper works well. This can be de-constructed by the whole group and synthesised into text for the class mathematics book, displayed as text to support individual writing and used to support guided reporting back. For the teacher, it provides a documented assessment opportunity.

Students can be assigned specific roles such as scribe (or recorder), reporter, leader and encourager. It can be useful to spend time helping students to develop an understanding of their roles.

Purposeful, guided reporting back

Purposeful, guided reporting back provides the teacher with a wonderful opportunity to have input into the students' development, by helping them to refine their mathematical logic and language expression.

Students need to practice speaking in English as often as they can, in the appropriate mathematical contexts. While they are doing a mathematical activity, they are using language in context, so their language is context embedded. This means that their words do not need to be specific as the other members of the group share the context and will know what 'it' or 'the thing' means. In other words, the audience shares the context (they are there) and the purpose is to relate with an immediate group of people.

Reporting-back creates a need for students to talk about an activity.

Rather than move straight from doing an activity to writing about the activity in our books, we insert a stepping stone, a piece of scaffolding, an opportunity for the teacher to support the necessary shift in the type of language needed in moving from talking to writing. We call this stepping-stone or piece of scaffolding – guided reporting back.

Students need to know that they will be reporting-back what they have done and found out to others who were not involved in their activity (i.e. the audience shares some of the context, as they are doing similar activities, but not all of it). This means that in their writing or recording and in their reporting-back, students need to provide enough information for those students who did not share in the activity to be able to understand what happened. When they write down what they have done or found out, they will be compelled to use more specialised, specific and structured language.

During the reporting-back time the teacher can input into the students' language development by carefully asking questions or helping them to clarify meaning and develop specific language use, especially of mathematical terms. Often words are added to the Class Mathematics Glossary during this time. Trialing *Multo* teachers found that this reporting back time was an opportunity to celebrate the students learning by saying something positive about the students work.

After reporting back students can then be asked to use the group writing to individually write in their mathematics book or to redraft as a group for the Class Mathematics Book. At this stage it is important that the students understand that the audience will not share what they know. They will need

to make sure their writing contains all the information and language needed. Often it is a good idea to write for a once removed audience such as the class next door or their parents.

Content versus process

Introducing new content using a method or process (ways of doing things) that the students are familiar and comfortable with means they can focus on the content, and see it separately to the process. And, introducing a new way of doing things using familiar content will enable students to focus on the process and see this separately to the content.

Ask the question – Am I trying to introduce too many new things at once? Can the students really see what I am trying to show them or are there other things cluttering and confusing the learning moment?

A concluding thought

Creating a supportive learning environment is a challenge that may require changing attitudes to learning and the very culture of the school. Changing personal practice first and then sharing and reflecting on your practice with your colleagues, is an effective and satisfying way of achieving whole school change.

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