Drawing, Gesturing and Talking: Using What Comes Natural to Young Learners of Mathematics

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“Mummy, I know what four 4’s are...16.”

“...it only works if you go cross-eyed, then the 2’s become 4’s.”
Profound Importance of Early Mathematics

✧ Mathematical skills and knowledge at kindergarten entry are best predictors of overall school achievement (Claessens et al, 2009; Duncan et al, 2007)

✧ “Gains in mathematical knowledge from preschool through first grade are even more predictive of age 15 mathematical achievement” (Siegler et al, 2014)
Prior to school, most children have an intuitive and strong grasp of “everyday mathematics”

“surprisingly broad, complex, and sometimes sophisticated” that may be innate or drawn from daily experiences (Ginsburg et al. 2008, p.3)

Unfortunately ...

1. Not all children have the same experiences;
2. Teachers/curriculum underestimate their capabilities;
3. Intuitive “everyday mathematics” does not guarantee success with formal mathematics
Everyday Maths vs School Maths

What's nine multiplied by six?

Er... dunno

We're nine goals down!

That's fifty-four points
Professional Noticing & Intentional Teaching
1. Hand movements (static & dynamic)
   – Gesturing: Pointing and Tracing
   – Drawing

2. Talk Moves
   Deliberate strategies to engage all students in mathematical conversations
Gesture: Pointing & Tracing

Pointing for attention & interest as early as 12 months of age

Tracing with fingers in soft surfaces 3500 BC
Paleolithic cave finger flutings by 2 year olds
Montessori: Multi-sensory

“I keep pointing at the children; they keep staring at my finger”

Maria Montessori (1912, 1914)

Sandpaper letters
Biological wiring to pay attention

Humans possess *bimodal visuo-tactile neurons* that influence our visual attention in the space near our hands.

We recognise *faster* & pay *greater attention* to objects near our hands.
Intentional Gesturing: 
Pointing & Tracing

English as an Additional Language or Dialect (EALD) Study and Maths Word Problems
- Intervention with four Year 2 & 3 (aged 7-9) EALD students over 3 weeks to improve their word problem solving capacities.
- Pointing & tracing emphasised as part of a multi-sensory approach.

Mathematics Achievement Quiz
Pre-intervention Interview
3 x 30 minute teaching-sessions
Post-intervention Interview
On Saturday, Brian bought 24 lollipops for his birthday party.

On Sunday, he bought another 6 lollipops. How many lollipops did he buy altogether?
Findings after 3 lessons

Two students used pointing and tracing strategies after being *explicitly reminded* to do so. There were no changes to pre/post problem solving.

Two students independently used pointing and tracing strategies with improvements in problem solving understanding & number correct.

Both these students also reported feeling more confident solving word problems. One commented: “*my finger is now a weapon to fight the words*”. 
Order of Operations
(Ginns et al 2016)

54 Grade 4 students: Tracing vs Non-tracing

7 \times (31 - 20) + 56 \div (5 + 3) = ?

Students tracing out the order of operations correctly solved significantly more problems than non-tracing students.
Tracing Worked Examples

Step 1: This is a triangle. [Trace out the triangle]

Step 2: The given angle is 50°. [Trace out the given angle]

Step 3: When two lines cross, vertically opposite angles are equal, so this angle is also 50°. [Trace out the two vertically opposite angles]

Step 4: An exterior angle of a triangle is equal to the sum of the two opposite interior angles, so $x = 50 + 60 = 110$. [Trace out the two opposite interior angles. Then, trace the exterior angle.]

Grade 6 children who traced over key elements of angle relationship worked examples performed better than those just reading.
Gesturing by Teacher: Live vs Video

Speech instruction accompanied by gesture (pointing) resulted in significantly more learning than instruction conveyed through speech only.

Gesture’s effect was stronger for video instruction than live instruction.
Drawing Mathematics

• **Generative Drawing Principle:** Drawing by hand involves cognitive and meta-cognitive processes.

**Students who created drawings while reading had:**
✧ Better recall
✧ Deeper understanding  (Leopold & Leutner, 2012)

The “**pen**” is mightier than the “**keyboard**”.  
(Mueller & Oppenheimer, 2014)
“Drawing can be a window into the mind of a child” (Wolek, 2001)

Drawing considered a natural aspect of children’s expressive development (Machon, 2013)

- often seen as “static” products, but can represent dynamic process reflecting emerging understandings.
  - Young children’s drawings of diagrams during problem solving assisted some students to find appropriate solutions (Diezmann & English, 2001)

- drawing in early years mathematics has not been given wide attention (Woleck, 2001),
- not highly valued in mathematics classrooms (Soundy, 2009)
Drawing: ‘natural’ to ‘mathematical’

Children might use icons and symbols in pre-school, but the function of drawing shifts in school. This shift can be troubling to many children (Machon, 2013)

For instance:

• A correct drawing does not always lead to a correct solution.
• At school, some children do not apply drawing strategies to help solve maths problems (Soundy & Drucker, 2009)
Types of Drawings & Purpose

Six, 6 year olds, first term of Kindergarten, Malaysia. Exploring children’s disposition towards & use of drawings to help solve mathematics problems

Natural or Not ?

• Some children were reluctant to draw
• Some *resisted* drawing in mathematics classes
• Did not view ‘drawing’ as a valid strategy to use in mathematical problem solving.
Pictographic
( realistic depictions of objects – eg. animals’ toes )

“There is an elephant and a tiger. How many legs are there altogether?”
Types of Drawings & Purpose

“There are 2 elephants. How many legs are there altogether?”

Iconic
(simple shapes)
Modeling the problem situation
• “Drawing for solution”
• Produced to obtain answer
Types of Drawings & Purpose

Iconic & Symbols
Representation of objects (quantity) and action (operation using arithmetic symbols)

Modeling the problem situation (purpose)
- Drawing as solution
- Drawn after giving answer

“There are 5 balls. I add 1 more. How many balls are there altogether?”
Teacher: How many of these books will we need to cover this desk?
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Drawing and gesturing to help solve problems in Kinder

Emma: ... I just have to count all the squares now.
Jb: How are you going to count them?
Emma: Mmm... I’m going to skip-count by fives.

Jb: Can you show me where each group of five books is on your drawing ...?
Emma: (running her finger down each column of rectangles) There’s five... five, five, five... all the way to the end...twelve times...there are twelve books in each row.

Emma then wrote “5 rows of 12” next to her diagram.
Drawing and gesturing to help solve problems in Kinder

Emma’s working for 5 rows of 12
Two-thirds of the talk in classrooms is done by teachers (even during class ‘discussions’)

Two-thirds of the talk is about controlling or directing

Child (first day of kinder): I HATE school!

Mother: Why?

I can’t read yet; I can’t write yet; and I’m not allowed to TALK. So how am I supposed to learn?
Why Encourage Talk

- Talk reveals understanding and misunderstanding.
- Talk supports academic language development.
- Talk supports deeper reasoning.
- Talk supports social & emotional engagement and development.
Talk Moves

Pedagogy for building **intellectual** and **emotional** engagement in mathematics


1. Turn and Talk
2. Repeating
3. Revoicing
4. Adding-on
5. Reasoning
6. Wait time
7. Revise

Kazemi & Hintz (2014)
Turn-and-Talk

“Turn and talk to the person next to you ...”

Circulate and listen to partner talk. Use this information to decide who to call upon. Allows students to refine their thinking.

Adapted from Kazemi & Hintz (2014)
Intentional talk, Stenhouse Publishers.
Repeating

“Can you repeat what she/he said in your own words...”

A student repeats or rephrases what another has said. Repeating can slow down the conversation so that an important or complex idea can be highlighted.

Revoicing

“So you’re saying ...”

Repeat some or all of what the student has said, then ask the speaker to verify whether the revoicing is correct (“.... Is that right?”).

Revoicing can be used to clarify or highlight an idea.

Adding-on

“Would someone like to add to this...”

Invite students to participate in the conversation or clarify their own thinking.

Reasoning

“Do you agree or disagree, and why?”
“Why does that make sense?”

Students are encouraged to compare their thinking to the thinking of others.

Students can respond: “I see his point, but I disagree because ....” or “I agree with this because ...”.

Adapted from Kazemi & Hintz (2014)
Intentional talk, Stenhouse Publishers.
Wait Time

“Think about your answer first...”

Wait after asking a question before calling upon a student. Give the student time to collect their thoughts.

Adapted from Kazemi & Hintz (2014)
Intentional talk, Stenhouse Publishers.
Revise

“Has anyone’s thinking changed?”
Would you like to revise your thinking?”

Encourage students to reflect and revise their thinking as they have new insights.

Adapted from Kazemi & Hintz (2014)
Intentional talk, Stenhouse Publishers.
**Gesturing:**
Pointing & Tracing
- biologically wired to ‘notice’ the space near hands

**Drawing** with pen & paper while learning:
- Improve recall
- Enhance deeper understanding

**Talk Moves:**
- Academic language development
- Social/emotional benefits

- Intuitive skills
- Intentional instruction needed to link everyday uses to formal mathematics
The range of what we think and do is limited by what we fail to notice. And because we fail to notice that we fail to notice, there is little we can do to change until we notice how failing to notice shapes our thoughts and deeds.

(Goleman, 1985, p. 24)
DON’T FORGET TO DO YOUR EVALUATION
Thank you

Some References


