Eight strategies for dealing with differences in student readiness to learn mathematics

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Abstract

All classes are mixed in student ability. This session will present eight strategies for teaching mathematics that cater for the diversity in student readiness to learn. Examples of classroom lessons, relevant to the years 4 to 10, of each of the strategies will be presented and discussed.

What makes dealing with the diversity of readiness difficult?

The Australian school context

- The long "tail"
- Decline in specialist mathematics and science study in senior years
- Widespread dissatisfaction (from teachers, students and parents) with the way mathematics and science is taught
  - Emphasis on "telling" followed by practice
  - Overuse of texts designed for practice rather than learning (and certainly not for fostering creativity and student decision making)
  - Disconnected from meaning and relevance
- Belief by teachers (and parents and students) that some (many?) cannot learn mathematics

Yet teachers commonly experience …

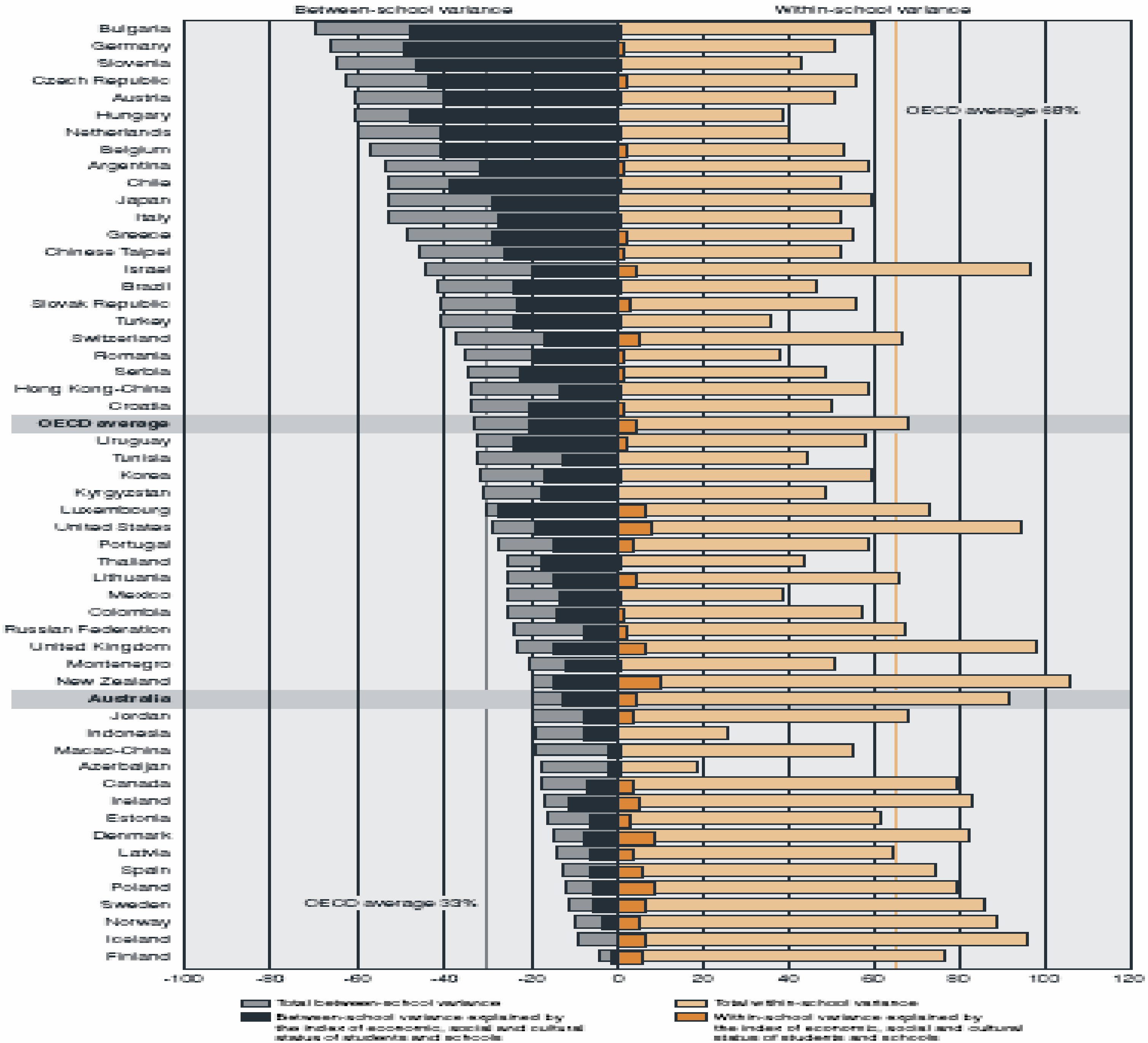
- … fast learners who shout answers and criticise others who are still thinking, and who complain to their parents about being under-extended
- … some learners who have more or less given up believing that they cannot learn, and who prefer to interrupt others
- … (school derived) pressure on teachers to skim from topic to topic
- … routines in schools that leave teachers with limited time for collaboration, sharing ideas, innovating, …

What are you hoping for?

- Creativity, imagination, adaptability, willingness to think and make decisions, persist, …
- … and life long learners (note concern about following Asian models) …
- … or correct answers, compliance, acceptance of place in life, …

What is the issue with difference?

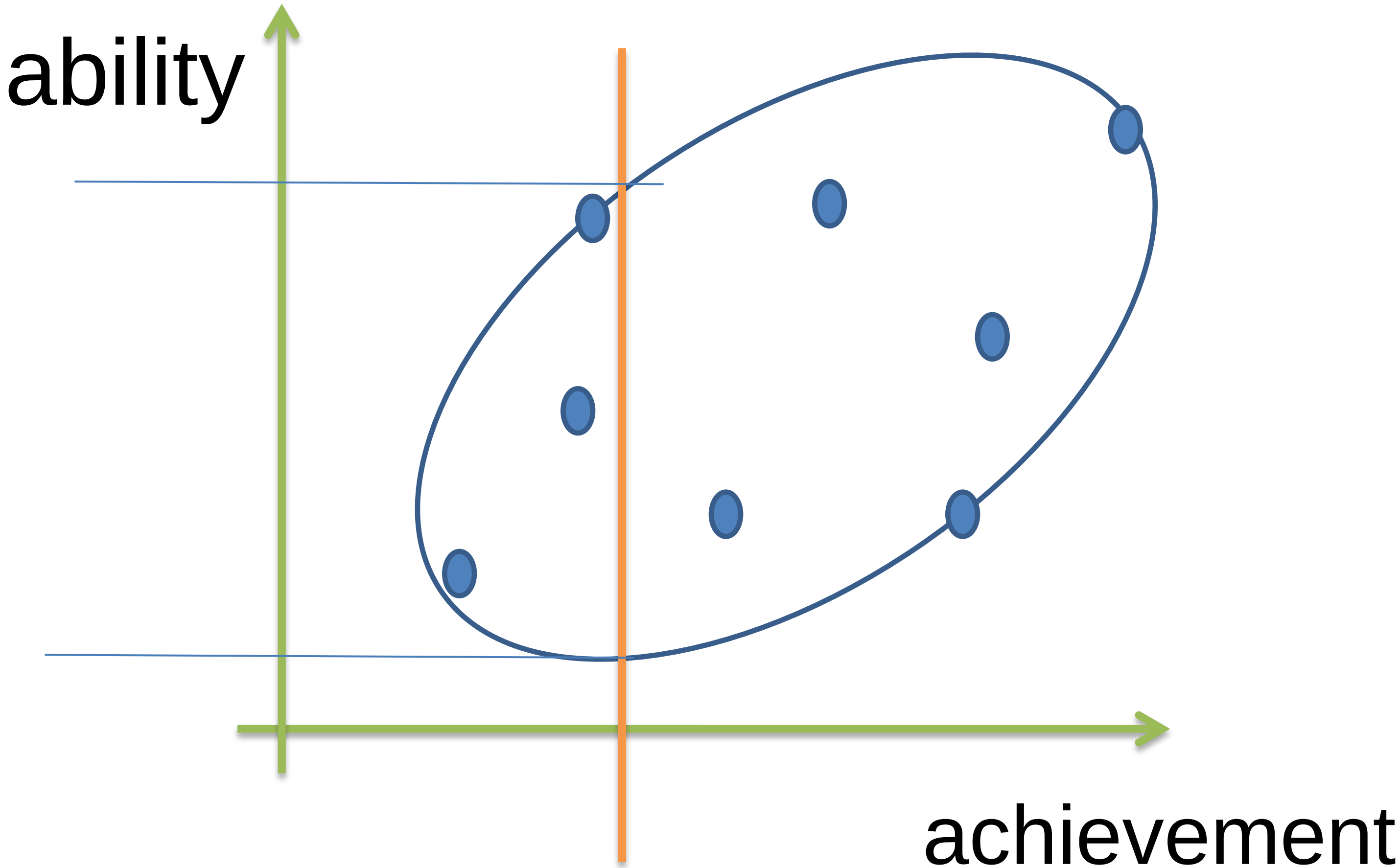
- ACARA says there is a 5 year gap: Cockcroft reported a 7 year gap
- While we know that there are factors contributing to differences in readiness (Indigenous, SES, rural, gender), even within these subgroups there is the same degree of diversity
- Differences in readiness refer not only to achievement, but aspirations, expectations, resilience, mindsets, confidence, satisfaction, etc



MAV difference 2013

Figure 7.1 Variance in student performance between schools and within schools in scientific literacy

But every group is mixed so lesson planning needs to anticipate differences in readiness


ability
achievement

Eight strategies for dealing with diversity, while offering experiences covering at least some common content

Common to all 8 approaches

- The intention is to foster the sense of a classroom community to which all students contribute
- The experience of engaging with the task happens before instruction
- The classroom pedagogies are explicit
  - The set of groups
  - Expectations for individual work
  - Modes of assessment

- Few rather than many tasks
- All students are given time to engage sufficiently to participate in the review
- Asking students to solve problems in more than one way

What these approaches are NOT!!

- Asking questions that are so easy that everyone can do them
- Setting up groups that allow some students to hide

Strategy 1
Posing questions with multiple entry and exit points

- Such tasks :
  - are simply posed drawing on content with which the students are familiar (although the connections may be new)
  - can be responded to in both simple and complex ways, with responses of all students being potentially of interest
  - are easily accessible (so everyone can start) but have potential for sophisticated responses (and so can challenge those who can to go further)

Some lower primary examples

- What is something in this room that there is exactly 4 of?
- I have three coins in my hand. How much money might I have?
- Three numbers add to 11. What might be the numbers?

Write your name using 50 match sticks.

Some secondary examples

- Describe some events that have a probability of 1/4?
- What are some equations whose graph goes through the point (2,3)
- Draw some rectangles with a perimeter of 20 cm. Work out the area of each of your rectangles.
- Three numbers add to 1. What might be the numbers? (at least one is a fraction and at least one is a decimal)

- What might be f(x)?

$$\int_2^4 f(x)dx = 10$$

Strategy 2
Posing challenging tasks supplemented with enabling and extending prompts

- Such tasks:
  - are set at levels that provide reasonable challenge for all students
  - include an expectation that students will communicate their responses clearly in writing and verbally
- The prompts:
  - connect directly to the original task
  - allow all student to contribute to task reviews

What are enabling prompts?

- Enabling prompts can involve slightly varying an aspect of the task demand, such as
  - the form of representation,
  - the size of the numbers, or
  - the number of steps,

  so that a student experiencing difficulty, if successful, can proceed with the original task.
- This approach can be contrasted with the more common requirement that such students
  - listen to additional explanations; or
  - pursue goals substantially different from the rest of the class.

- I did a division question correctly for homework, but the printer ran out of ink. I remember it looked like

$$\_\_ \ \_\_ \ 4 \div \_\_ = \_\_ \ 4$$

- What might be the digits that did not get printed?
- (Give as many answers as you can)

An enabling prompt

- What might be the missing digits

$$\_\_\,2 \div \_\_ = 4$$

An extending prompt

- How many different answers are there?
- What patterns can you see?

Strategy 3
“Learning” tasks followed by “Consolidating” tasks

- The “learning” task is one that is harder than we would expect most students to do
- The learning occurs from working on this task, even if unsuccessful, and listening to the solutions of students who are successful
- The “consolidating” task has a similar structure and is designed to allow students to implement the strategies learned from the learning task and discussion

An example of a lesson “suggestion”

A example of a junior primary lesson

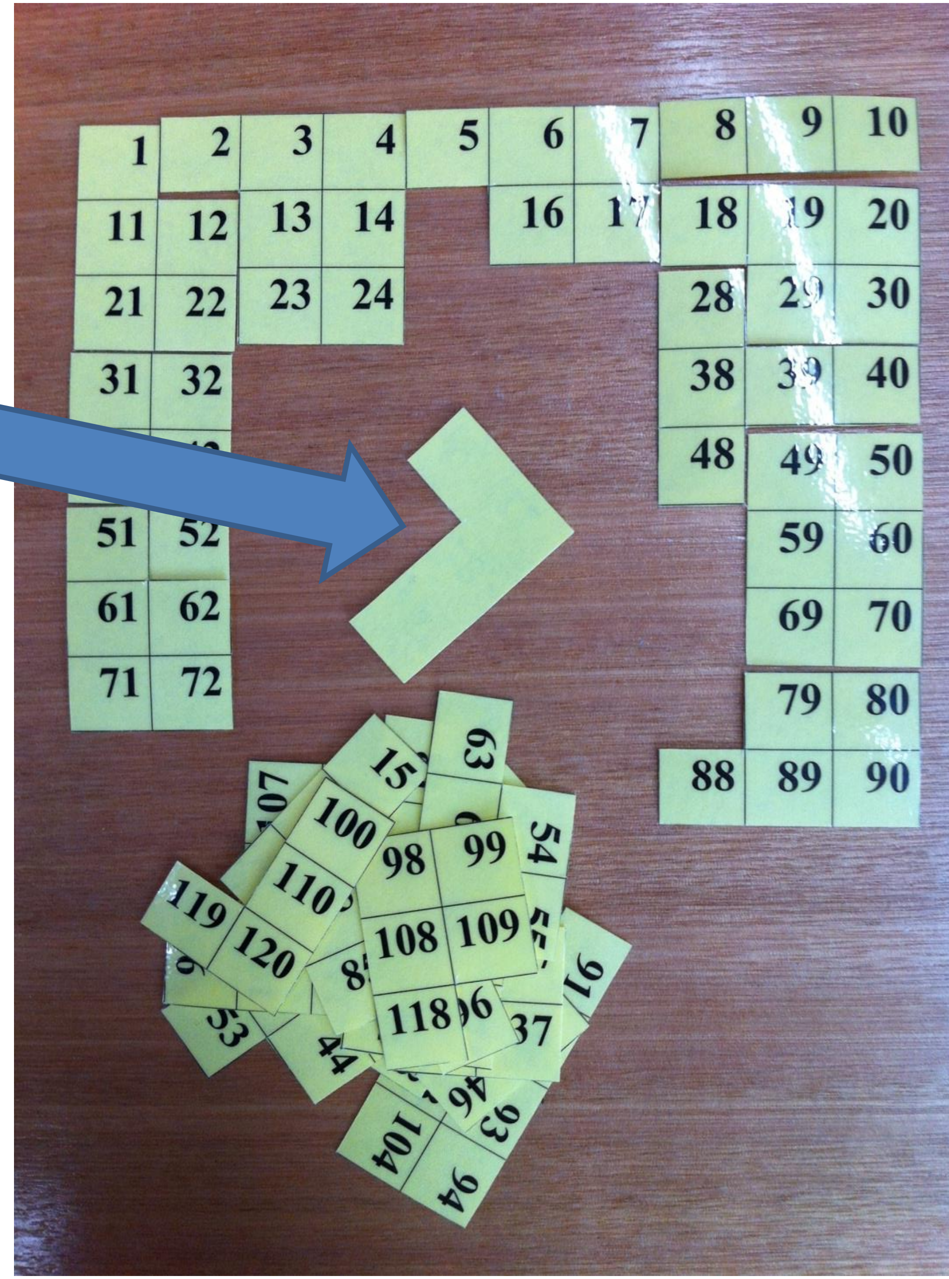
- As an introduction students would complete a number jigsaw and maybe follow up tasks like the following …

Which ones are wrong?

1	2	3	4	5	6	7	8	9	10
11	12	14	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	37	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	53	55	56	57	58	58	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80

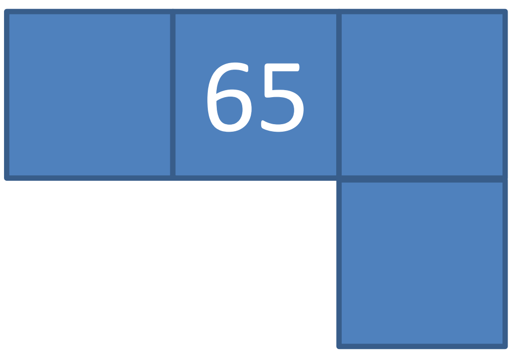
The learning task

What might be the numbers on the L Shaped piece? (I know that one of the numbers is 65)



An enabling prompt

- What might be the missing numbers on this piece?



As a consolidating task

- The numbers 62 and 84 are on the same jigsaw piece.
- Draw what might that piece look like?

Surface area = 22

- A rectangular prism is made from cubes.
- It has a surface area of 22 square units.
- Draw what the rectangular prism might look like?

Enabling prompt:

- Arrange a small number of cubes into a rectangular prism, then calculate the volume and surface area.

Extending prompt:

- The surface area of a closed rectangular prism is 94 cm^2.
- What might be the dimensions of the prism?

A consolidating task

- The surface area of a closed rectangular prism is 46 cm^2.
- What might be the dimensions of the prism?

Strategy 4
Where the prerequisite experience is part of the task

- Sometimes the barrier to engagement is the lack of information needed to start
- It is possible to present interesting activities that provide the necessary preliminary experience, with the intention that the learning tasks are subsequent to this experience

Strategy 5
Using textbooks (and worksheets) in ways that create opportunities for decision making

- The usual way that teachers use texts and worksheets is to explain the process and then have the students work through the page in order
- In fact the texts are designed to be used that way
- This strategy is to change the ways that students think about the purpose of text examples

Some examples

- In what ways are the questions in this exercise similar?
- In what ways are the questions different from each other?
- Which questions match the example at the start and which do not?
- In what ways is question 2 harder than question 1?
- In what ways is question 10 harder than question 1?
- In what ways does question 1 help you answer question 2?

Some more examples

- Which questions can you do in your head?
- Which is the first question you cannot do? Start working at the question before that one.
- Read the last question first. What do you need to learn to be able to do that questions? Which of the earlier questions look like they might help?
- Work in pairs. One of you does the odd questions. The other does the even ones. Then each of you can explain your working to the other.

Strategy 6
“Realistic” investigations

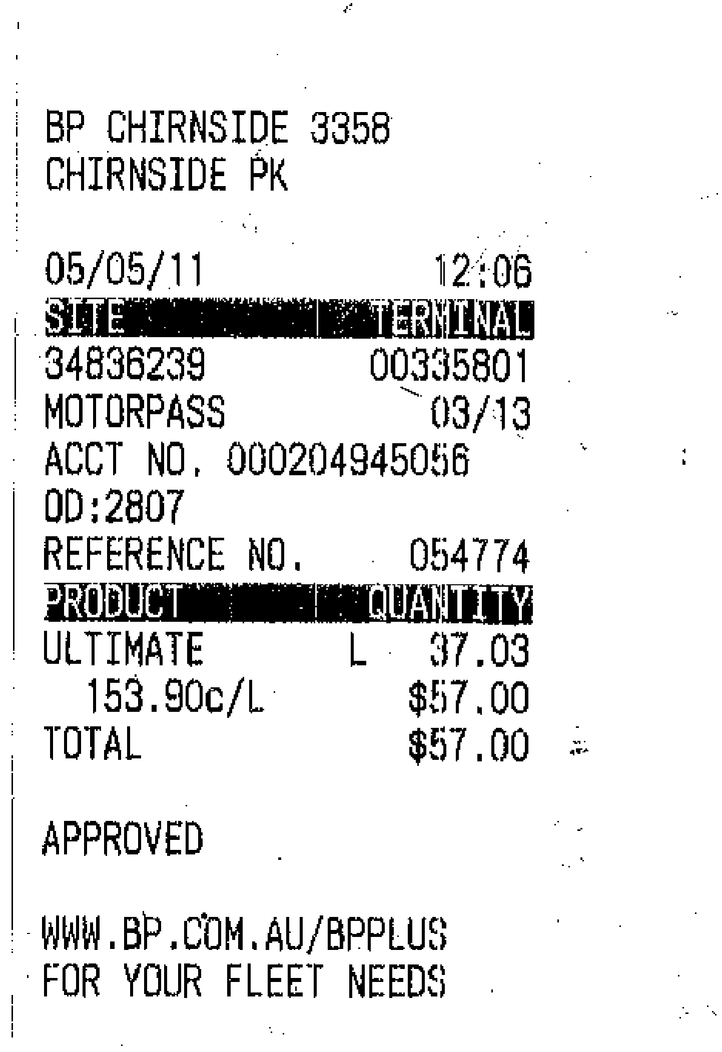
- Such investigations should:
  - be sufficiently complex for groups to engage
  - require some planning, data collection, analysis, and interpretation
  - have results that are worth reporting on
- There are many resources that have collections of these, such as MCTP, RIME, Maths 300

Calculating the fuel consumption

I am concerned that my car is not getting the best fuel economy.

I filled up my car on 27th April, noting the odometer as being 2345 km. When I filled the car next, I got this print out.

What is the fuel economy of my car in L per 100 km?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153.90c/L		$57.00
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FOR YOUR FLEET NEEDS

- I noticed that my Citroën C3 uses 20 L of fuel to travel 250 km. The advertising information for the car says the following.

FUEL CONSUMPTION (L/100KM)	88KW PETROL
urbain	9.4
cycle extra-urbain	7.0
combiné	7.6

Should I be happy?
Explain your thinking clearly.

HOW MUCH MONEY?

What is the maximum number of \$1 coins that can fit, lying flat in a single layer without overlapping, in a shape with an area of 4 m^2?

Explain how you worked out your answer

Nor will we focus on realistic and socially relevant experiences

Teacher led experiments

- Out of 10 rolls, how many times do you expect to get a 2?
- What do you imagine might be the largest and smallest estimates of people in this group?
- If each person here rolled a die 10 times, what would be the smallest and largest number of “2”s that any person would get?
- Out of 100 rolls how many times do you expect to get a 2?
- What do you image might be the largest and smallest estimates of people in this group?
- If each person here rolled a die 100 times, what would be the smallest and largest number of “2”s that any person would get?

1 mm of rain on 1 sq m of roof is 1 L of water.

Design a tank for this building that captures all of the rain that usually falls this month.

- The Shell Centre Materials
  – http://www.mathshell.com/
- Formative Assessment Lessons and Tasks
  – http://map.mathshell.org/materials
- Australian Curriculum
  – www.australiancurriculum.edu.au/crosscurriculumpriorities/sustainability
- NSW DEC Environmental and Sustainability Education
  – www.curriculumsupport.education.nsw.gov.au/env_ed/
- USA Population Education
  – www.populationeducation.org

- USA World of 7 Billion
  - www.worldof7billion.org
- Gapminder
  - www.gapminder.org
- If the world were a village of 100 people
  - http://geography.about.com/od/obtainpopulationdata/a/worldvillage.htm
- ABS Website Survey Data
  - http://www.abs.gov.au/AUSTATS/abs@.nsf/Lookup/1318.3Feature%20Article14Aug%202009

- Living Planet Index
  - http://wwf.panda.org/about_our_earth/all_publications/living_planet_report/
- Happy Planet Index
  - http://www.happyplanetindex.org/countries/australia/
- Ecological Footprint Interactive Graph
  - http://www.epa.vic.gov.au/ecologicalfootprint/calculators/
- Earth Overshoot Day
  - http://www.footprintnetwork.org/en/index.php/gfn/page/earth_overshoot_day/

- nrich
- http://nrich.maths.org/frontpage
-
- transum
- http://www.transum.org/
-
- hotmaths
- http://www.hotmaths.com.au/
- tarsia - there is not actually a website with this name, but a number that offer software (example below) http://www.tes.co.uk/article.aspx?storyCode=6107407&s_cid=RESads_MathsTarsia
-

Strategy 7: Mathematical games that are a mix of skill and luck

- There are many resources of games, the best ones are those that address mathematically important ideas
- Actually most games will address different learning needs, but if they are totally skill based then winners and losers are too obvious, and if totally luck based the learning is less obvious

An example of such a game

In turn, players roll a 10 sided die (numbered 0 to 9) and, after each roll, write the number rolled in one of the rectangles on a board that looks like

$$\square \; \square \; \square \div \square$$

The winner has the answer closest to 100 (for example).

Extending the game

How could you place 3, 4, 5 and 6 on a board like this, to make the answer closest to 100

$\square \ \square \ \square \div \square$

Directions

Strategy 8
Task sequences that develop complexity progressively

- The basic task should be “meaningful”
- The sequence starts with elements that are unrealistically simple, and the complexity develops progressively
- It is not critical if all students do not get to the destination

Buying pizza

- The cost of a small pizza is $\$x$
- What would be the cost of y small pizzas?
- If I have $\$z$, how many small pizzas can I buy?

Buying pizza

- The cost of a small pizza is $7
- What would be the cost of 4 small pizzas?
- If I have $35, how many small pizzas can I buy?

Buying pizza

- The cost of a small pizza is $11.70
- What would be the cost of 7 small pizzas?
- If I have $100, how many small pizzas can I buy?

Buying pizza

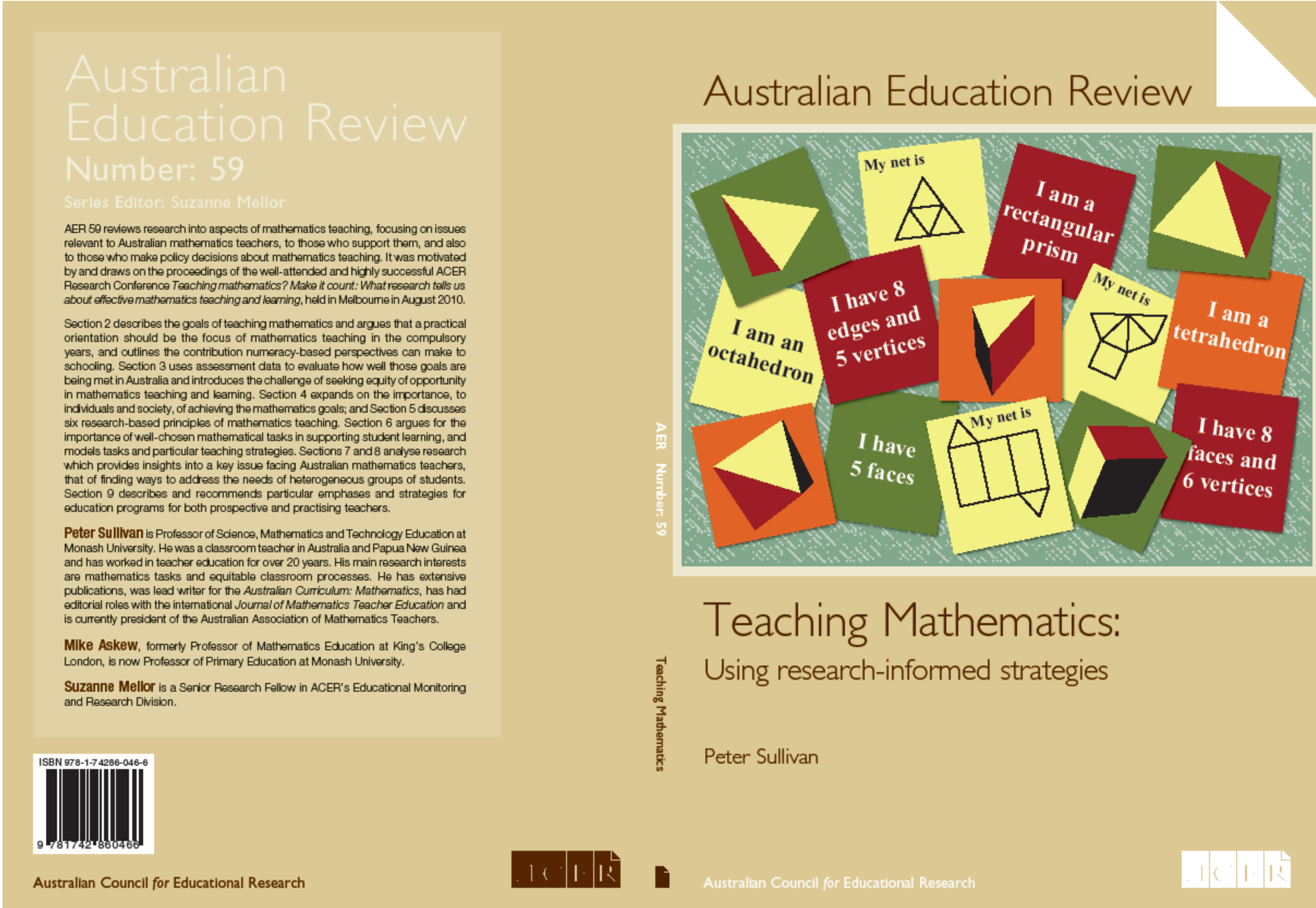
- The cost of a small pizza is $11.70
- What would be the cost of 37 small pizzas?
- If I have $975, how many small pizzas can I buy?

Buying pizza

- The cost of a small pizza is $$x$
- What would be the cost of y small pizzas?
- If I have $$z$, how many small pizzas can I buy?

Conclusion

- All classes are mixed
- The classroom community is an asset (not a liability)
- Telling students what to do and how to do it is disengaging for them
- Try something out



Australian Education Review

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AER 59 reviews research into aspects of mathematics teaching, focusing on issues relevant to Australian mathematics teachers, to those who support them, and also to those who make policy decisions about mathematics teaching. It was motivated by and draws on the proceedings of the well-attended and highly successful ACER Research Conference *Teaching mathematics? Make it count: What research tells us about effective mathematics teaching and learning*, held in Melbourne in August 2010.

Section 2 describes the goals of teaching mathematics and argues that a practical orientation should be the focus of mathematics teaching in the compulsory years, and outlines the contribution numeracy-based perspectives can make to schooling. Section 3 uses assessment data to evaluate how well those goals are being met in Australia and introduces the challenge of seeking equity of opportunity in mathematics teaching and learning. Section 4 expands on the importance, to individuals and society, of achieving the mathematics goals; and Section 5 discusses six research-based principles of mathematics teaching. Section 6 argues for the importance of well-chosen mathematical tasks in supporting student learning, and models tasks and particular teaching strategies. Sections 7 and 8 analyse research which provides insights into a key issue facing Australian mathematics teachers, that of finding ways to address the needs of heterogeneous groups of students. Section 9 describes and recommends particular emphases and strategies for education programs for both prospective and practising teachers.

Peter Sullivan is Professor of Science, Mathematics and Technology Education at Monash University. He was a classroom teacher in Australia and Papua New Guinea and has worked in teacher education for over 20 years. His main research interests are mathematics tasks and equitable classroom processes. He has extensive publications, was lead writer for the *Australian Curriculum: Mathematics*, has had editorial roles with the international *Journal of Mathematics Teacher Education* and is currently president of the Australian Association of Mathematics Teachers.

Mike Askew, formerly Professor of Mathematics Education at King's College London, is now Professor of Primary Education at Monash University.

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Teaching Mathematics

Australian Education Review

Teaching Mathematics:

Using research-informed strategies

Peter Sullivan

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