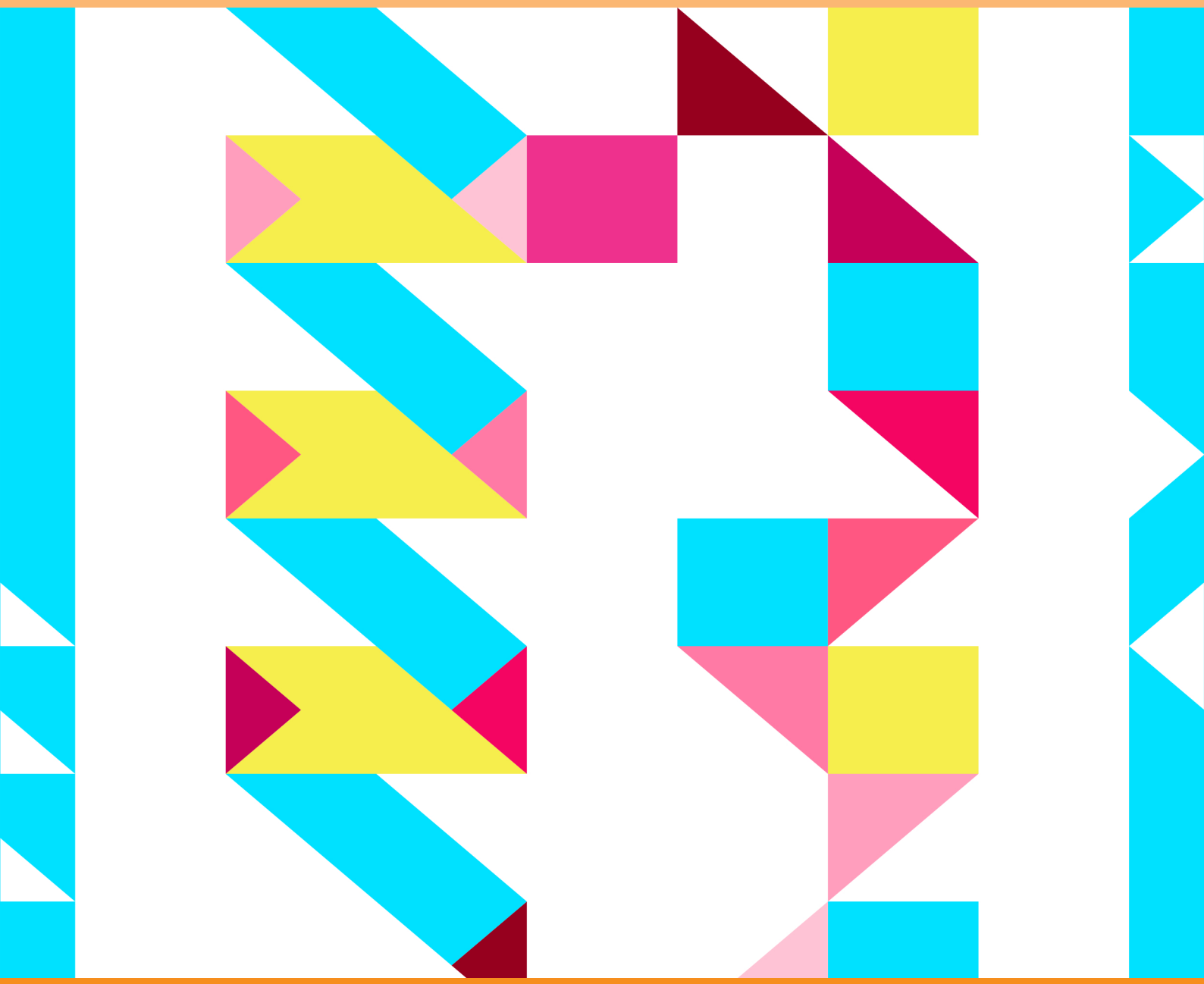
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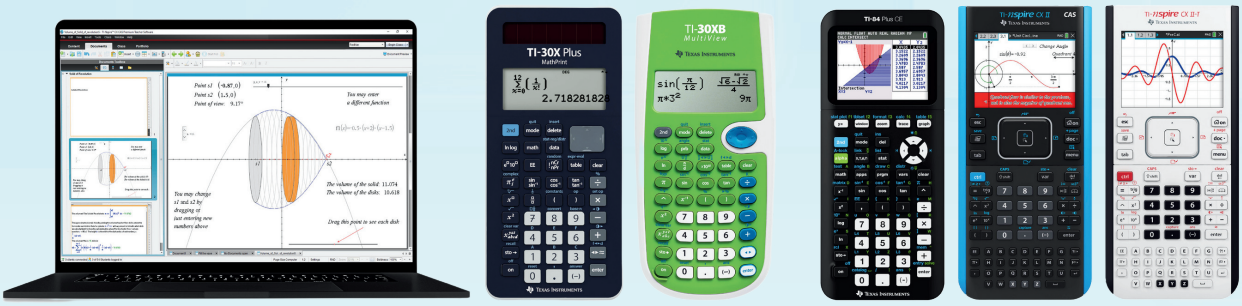
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EDITORIAL

Justine Sakurai

In the evolving landscape of mathematics education, educators must navigate the differences between the fundamental mathematical processes that build disciplinary expertise, and the varied pedagogical approaches used to teach these processes.

There is a clear distinction between mathematical processes and mathematical pedagogies. As education bodies prescribe pedagogical practices in the classroom, for example, explicit teaching in Victoria, it is critical for mathematics teachers to recognise that mathematical processes are underpinning skills for mathematics. These must be covered in the curriculum as stated by VCAA and ACARA.

Mathematical processes can be taught from a wide range of pedagogical perspectives. These include explicit teaching. They may also include tasks that promote reasoning and sense making in students, meaningful discussions, purposeful questioning, and activities that promote productive struggle. The National Council for Teachers of Mathematics (NCTM) has an excellent resource for considering which pedagogical tool to choose for use in the classroom (www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit/Resources/7-EffectiveMathematicsTeachingPractices.)

Understanding that mathematical processes are an independent concept to pedagogy, it is worth considering evidence from the mathematics education literature which argues that some pedagogies support specific processes. For example, when teaching proof (an undeniable mathematical process), the think, record, discuss cycle (Brown, 2008) can be helpful. Likewise, when teaching statistics, the statistical cycle, which is well-accepted in industry and in overseas educational bodies, is a key pedagogical tool. Interestingly, this pedagogical approach has been developed by statisticians and so mirrors the statistical cycle process.

Another example of where mathematics educators have defined a mathematical process that is used extensively in industry, is mathematical modelling. Similarly to the statistical cycle, mathematical modelling pedagogy as described in the literature can support the learning of the mathematical modelling process. Mathematical modelling is a skill that is required by engineers, scientists, financial analysts, manufacturing industry, telecommunications, economists, and healthcare professionals, to name just a few broad categories. Speaking to mathematical modellers in any of these industries, you will find a similar common story: that new workers are not coming through with the modelling skills needed to undertake the work.

This issue has long been recognised, hence the inclusion of these mathematical processes in the curriculum. The teaching of these processes is everyone's responsibility. Issues such as a crowded curriculum, exam focus, etc., have traditionally meant that these skills are sometimes not given priority.



The Victorian Teaching and Learning Model 2.0. which can be found at www.education.vic.gov.au/school/teachers/teachingresources/practice/improve/Pages/Victorianteachingandlearningmodel.aspx has four elements of teaching: planning, enabling learning, explicit teaching and supported applications. Supported applications is intended for learners to apply their learning.

As mathematics educators, we have a collective responsibility to prioritise these essential mathematical processes including, but not limited to, proof, statistical analysis, algorithmic thinking, mathematical modelling, computational thinking. Curricula and assessment constraints should not restrict teachers from including the mathematical processes in the teaching of the mathematics. By intentionally selecting pedagogies that enhance these processes, we can better prepare students with the practical skills that industry demands, ultimately bridging the gap between classroom mathematics and real-world applications.

REFERENCE

Brown, J., (2008) Structuring mathematical thinking in the primary years [Keynote address]. ConnectEd Maths: Mathematical Association of Victoria, Melbourne.

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IMPACT OF INCREASING SOCIAL INTERACTIONS IN THE CLASSROOM

Melinda Kelly, Newhaven College

The learning of mathematics is often impeded by students' affective state, that is students' beliefs and emotions can have a significant impact on their ability to learn mathematics. Negative emotions can reduce a students' motivation to learn and engage with mathematics and stress and anxiety can affect working memory and attention. Research has consistently shown that students who are emotionally involved in their mathematics learning tend to have more positive attitudes towards the subject, enjoy it more, and achieve higher levels of proficiency. That is, by elevating students' mathematical wellbeing their attitudes, achievements and engagement with the subject should increase (Hill et al., 2021). Hannula (2018) emphasised the importance of understanding group level dynamic processes in the study of mathematics-related affect. He argues that while there has been significant research on individual level factors that effect mathematics learning and performance, such as attitudes, beliefs, and emotions, there has been less research on the ways in which these factors interact within a group setting. With the intention of improving students' mathematical wellbeing, I incorporated small group work into each lesson throughout the teaching and learning of a topic in Year 11 Mathematical Methods. The learners in the class ranged from students who really love mathematics to those who do not like it, and included two students with self-identified mathematical anxiety.

I wanted to know if facilitating regular small group work in the mathematics classroom might result in an increase in students' mathematical wellbeing, and therefore have a positive impact on student learning.

The class had 17 students, and the topic that we were studying was *Gallery of Graphs*. I tried this strategy over a three week period.

For part of each lesson during the teaching of the topic students were put into groups of three to four students. The groups were either; student choice, or mixed ability. The students' ability levels were based on knowledge and skills students had shown via 'exit tickets' or 'entry tickets'.

Students were not told how they had to work together but were encouraged to make their learning visible by writing on windows, glass doors, whiteboards or mini whiteboards. I observed my students closely during this intervention, and they were given a survey at the end of the topic so their opinions on the intervention could be gauged.

RESULTS

- Students' felt comfortable working in groups and felt that they learnt well working in this way (Figure 1).
- Students in the class felt safe to try new things and make mistakes as they felt they would be helped by other students if they made errors (Table 1).
- The majority of students (71%) felt that they learnt best by working with others and hearing how others solve problems and then working independently. No students believe they work best entirely independently (Figure 2).
- 65% of students are unhappy in the maths classroom when they are not able to communicate with those around them (Figure 3).

EVALUATION

The students were a little reluctant to share their ideas the first time they were asked to complete work in a group which they were assigned to. However, after the first lesson they were very happy to work in groups and it did not seem to matter which students they were grouped with and every student actively participated every time.

> Sometimes in class I find that I am constantly helping those around me and not getting my work done as well. I realise it is good to help others as it helps you to better understand the topic, it does annoy me somewhat as I would like to get my work done in class. When we worked in groups I worked with different people who didn't need my help all of the time, so I felt I learnt more. (Student A)

The students in this class made a choice to study this subject, as Mathematical Methods is not compulsory. This may indicate that these students value mathematics. Yet, some of the students claim they do not 'like' maths but chose this subject to keep their future options open. Although not all students in the class 'like' mathematics they were all positive about working in small groups in mathematics. They made claims such as, 'it is fun' and 'working in groups is really good because you still have to think for yourself and can get little hints and make sure you're on the right track from fellow students'.

The data collected during my class showed that the strategy of incorporating small group work into each lesson in the Year 11 Mathematical Methods class had a positive impact on student affect and learning. Students felt that the classroom was a safe space to try new things and make mistakes and felt they learnt when working in small groups. Overwhelmingly the students in my class indicated that they learn best by firstly working with others to hear ideas, and then working independently, with 71% of my students believing this best reflected the way they like to learn. None of the students in my class thought that they learn best by working entirely independently. 65% of the class felt unhappy when talk was not allowed during class, which may indicate the benefit to talking in small groups for students' learning of maths.

During this topic where small group work was implemented in every lesson, students demonstrated engagement, perseverance in the face of challenges, support for one another and enjoyment of the class, all of which suggest a positive state of mathematical wellbeing.

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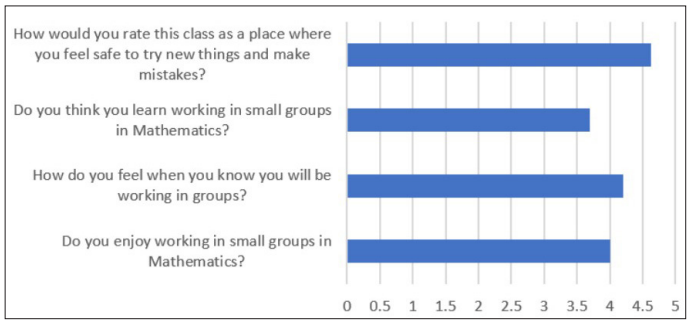


Figure 1. How students felt about group work at the end of the program (Likert scale, 5 being the most positive).

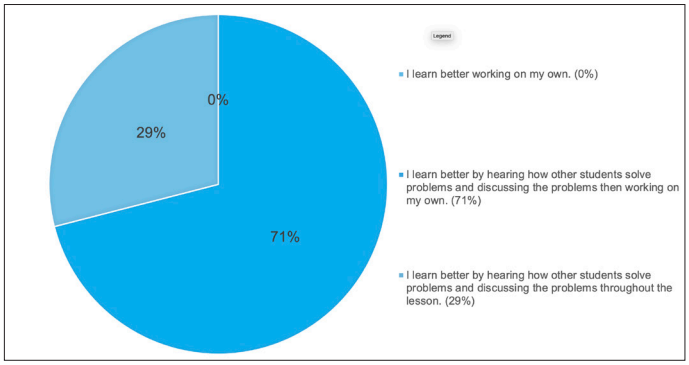


Figure 2. Which option most reflects your learning style.

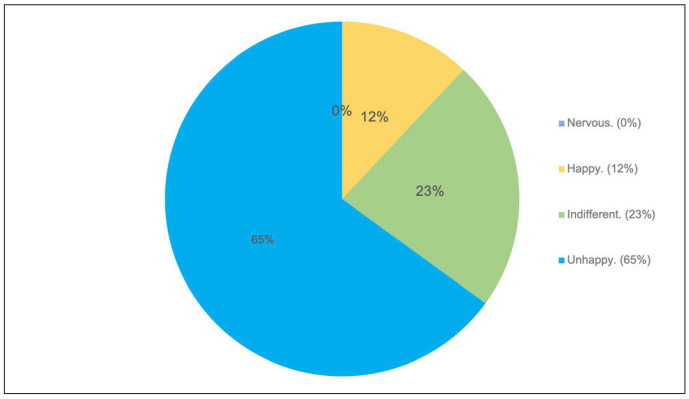


Figure 3. How do you feel when you have to work in silence during maths?

LIMITATIONS

The sample size for this strategy was small (n = 17) and was trialled in one class from one school. To better measure impact on learning in mathematics this strategy would need to be trialled with different ages, classes and schools. The impact of this strategy in a class where mathematics is still compulsory would be particularly beneficial. To better measure the impact of the strategy of group work a student survey undertaken before as well as after the program would allow changes in students' perceptions about group work to be measured.

If a question is answered incorrectly the class tries to help understand why you got it wrong rather than just telling you you're wrong.
I know that if I get a question wrong I won't get laughed at etc. We also have a nice group of kids.
I think our class is a comfortable place to share answers and learn from each other.
Most people in the class just want to learn so it makes it easier to make mistakes.
When I get things wrong I get help to fix my mistakes.
No one judges or makes negative comments when a question is answered incorrectly, therefore I feel comfortable answering questions in class.

Table 1. Student comments on their rating for the class as a place where they feel safe to try new things and make mistakes.



Figure 4. Students making their learning visible.

REFERENCES

Hannula, M. S. (2018). From anxiety to engagement: History and future of research on mathematics-related affect. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education: Umeå, Sweden July 3-8, 2018* (Vol. 1, pp. 19-34). PME International Group for the Psychology of Mathematics Education.

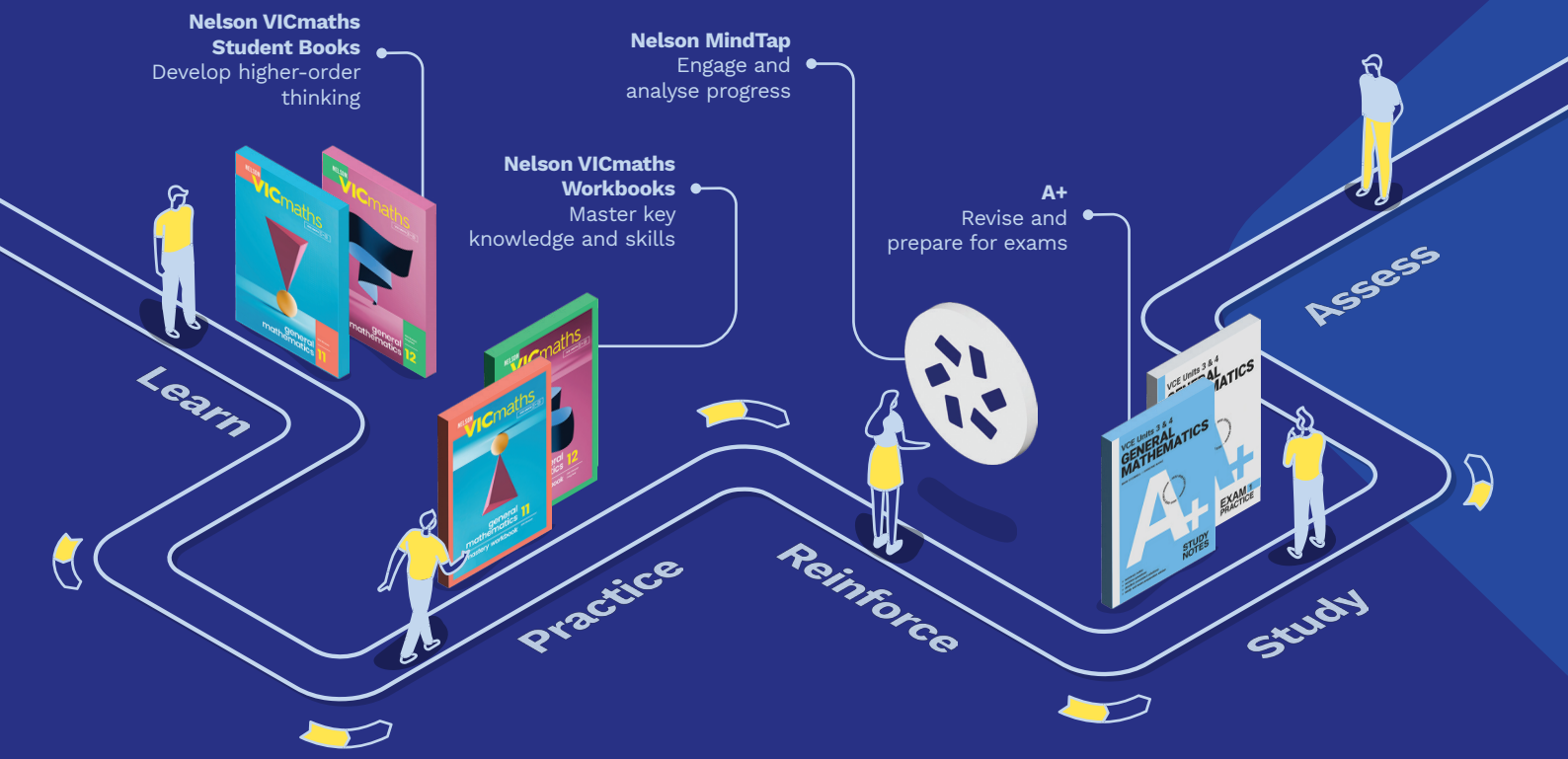
Hill, J. L., Kern, M. L., Seah, W. T., & Van Driel, J. H. (2020). Feeling good and functioning well in mathematics education: Exploring students' conceptions of mathematical wellbeing and values. *ECNU Review of Education*, *4*(2), 349–375. https://doi.org/10.1177/2096531120928084

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MATHEMATICS RESEARCH IN SCHOOLS

Dr Paul Fijn, School of Mathematics and Statistics, The University of Melbourne

Mathematics within a classroom can often be a solitary task, where finding the 'correct' answer is the only goal. This is a valuable way to learn techniques, and the certainty/formulaic nature suits some students. Beyond school, however, mathematicians are creative, collaborative and often exploring objects and theories just because they are interesting: even (sometimes especially!) if they are unsure an answer exists. Using open-ended mathematics research questions can be a method to engage students who otherwise do not see maths as 'for them'.

GETTING STARTED

Creating good research questions is time-consuming, requires broad expertise, and is difficult to do on your own. Instead, it's better to find groups who already produce these and make use of their work! The Maths and Stats Research Challenge (https://ms.unimelb.edu.au/engage/outreach/mathematics-and-statistics-research-challenge) is one source for many problems suitable for a range of year levels and interests. It can be an opportunity for students to practise a deliberate exploration of a topic outside of a closed book, formal assessment type of context.

The second challenge is in supervising and guiding students: it can be daunting to not know how to approach the problem yourself; and it is also possible good approaches use mathematics you have never experienced (or at least never felt comfortable with). However, this isn't a limitation: it is easier to guide students in their own thinking if you don't already have a method in mind. Good questions will always allow multiple approaches to make progress, and they are valuable in different ways.

WHAT IF I DON'T KNOW HOW TO START?

In many cases, starting with small examples will help with understanding the question/checking that you understand the 'rules' defined in it. Some questions might be scaffolded to already do this, but it is always useful to start with some small examples. Students who enjoy using technology can also use this to work out some larger examples which, otherwise, might be too tedious to check by hand. See Figure 1.

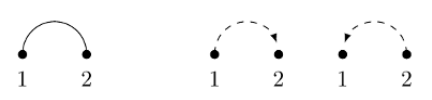
GUIDING STUDENTS

What distinguishes maths research from other fields is usually the abstract nature, and the concept of 'proof'. In other sciences, proof is typically an indication that it is the best theory currently fitting all known information. However, in mathematics it can be possible (but sometimes very difficult) to prove that a formula always works. While a proof might be the aim, it is usually better to think about justifying reasoning: progress is the aim, not simply finding the solution (if it exists!).

HOW MANY WAYS CAN YOU CONNECT PAIRS OF POINTS WITH NON-INTERSECTING UPPER HALF-CIRCLES?

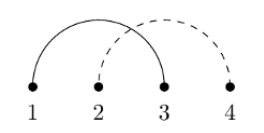
Start by working out some small examples/checking rules:

$N = 2$

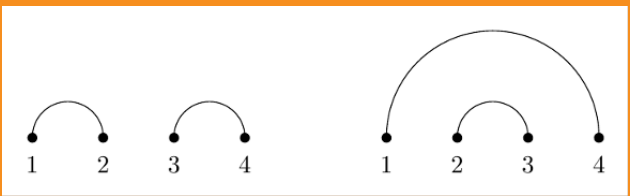


We just want to connect the points, so it doesn't matter if we go from 1 → 2 or from 2 → 1.

$N = 4$



We can't connect 1 → 3 because then 2 can't be connected to 4 without intersecting.



There are two ways to connect four points.

An important idea: we can see that to pair up the points we need an **even** number of them...sometimes considering special types of numbers can help.

Figure 1. An example of working with small examples to check understanding of the 'rules' relevant to a problem.

To help students learn to think in this way, here are some prompts:

- Does there seem to be any patterns in what you've found so far?
- Would presenting the information another way reveal more interesting aspects of the problem?
- Do you think that would work if the [problem] was [harder, larger, more complicated, different]?
- That's a fantastic formula/theory you've come up with, is there a way to find out if it always works?
- Is there a formula you can find, rather than just calculating lots of possible answers?

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MATHEMATICS RESEARCH IN SCHOOLS (CONT.)

Dr Paul Fijn, School of Mathematics and Statistics, The University of Melbourne

It can also be difficult to help students find resources: some might be inaccessible to an audience with their background in mathematics (e.g. Khan Academy, depending on the topic), some are only known by those who use them frequently (e.g. the Online Encyclopedia of Integer Sequences, oeis.org), and others might be unreliable (eg any AI, particularly large language models, can give incorrect information).

However, this is the key to *re*-search: you need to actually look at a range of sources, and work out which ones might be helpful to you. Learning how to search: which words/phrases should I look up? Which results should I look at? Colleagues who trained as teachers in a field other than mathematics have an advantage here, where looking more broadly is encouraged. This can make it a collaborative supervision/guidance process, too, not just for students to collaborate!

Finally, it is important to think of research as a process and trying to make progress: there will always be things they could have thought about, things they chose not to investigate, problems they didn't solve. This is both normal and expected! You also do not need to know if everything is 'correct': the best research projects are those that explain the story of the research well.

COMMUNICATING MATHEMATICS

As a student who was good at maths in school, I undervalued the communication of maths: if you got the right answer, why were there such arcane rules about exactly which steps you needed to show? Fortunately, I learned the error of my ways and came to think of maths as not just the solution, but communicating the thought process and reasoning as well.

START WITH THE STORY

Students should think about how they might explain what they did to someone who hasn't read the prompt: they should start with explaining the problem, then talk about what they did with it. Sometimes they might want to discuss what they tried, other times they can focus on what they found was the 'best' approach. It really depends on what best shows their story while approaching the research topic.

USING NOTATION

Mathematics notation can be very efficient ($a^2 + b^2 = c^2$ is much simpler than 'the sum of the squares of the two lesser sides equals the square of the hypotenuse'). Some problems, with the right notation, become much simpler to understand and see the pattern. On the other hand, too much notation can sometimes obscure the point and make it difficult to follow. Ultimately, there aren't any rules, but a good guide is 'use notation if it makes things easier to understand'.

VISUALISING MATHEMATICS

Often the best ways to show mathematics can be visual: it's excellent for patterns, can help clarify by showing examples, and are familiar to most students (e.g. graphs, number lines). A few well-selected diagrams (or photographs of experiments) can improve the readability and clarity immensely.

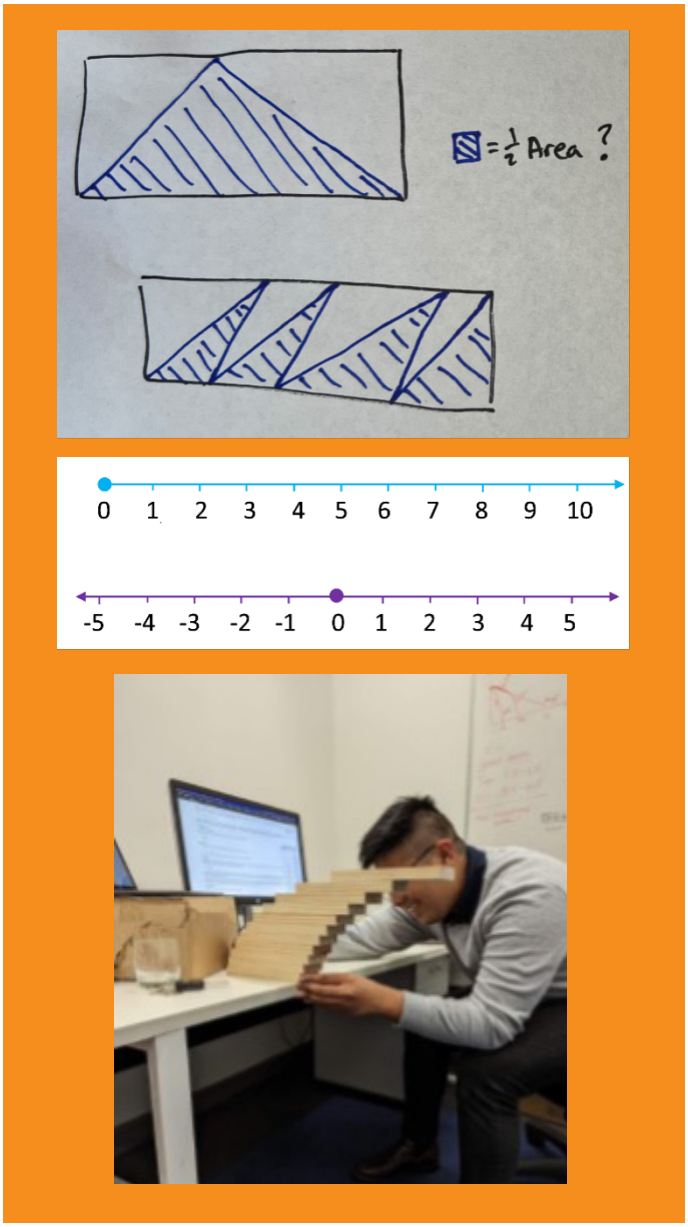


Figure 2. A variety of ways to visualise mathematics: drawing diagrams/examples (top); using number lines (middle); creating a physical model (bottom).

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CRITICAL EVALUATION

My favourite strategy when writing anything is to both start and finish by writing the introduction. Writing the introduction at first helps to clarify the problem, think about what story you want to tell, and explain how you chose what to do. (Re-)writing it last means you can make sure it matches the story you actually told, even if your plans changed from the beginning – this is a normal part of research, we don't always end where we expected to!

Getting feedback (e.g. from family, other students) can also help: people who don't know the problem or anything that they have done will quickly pick up when something is unclear or when something 'obvious' has not been mentioned.

CELEBRATE!

Mathematics research (like all research) can be fun, interesting and you can learn a lot from it. It's important to recognise that making progress (sometimes even just understanding the problem can take a lot of work!) is the goal, not 'finishing' a project. Research is rarely 'finished': try to remind students that presenting progress is just as much of an accomplishment.

The Maths and Stats Research Challenge is free and open to students in Australia and New Zealand from Year 5 to Year 12. Students are encouraged to work in groups of up to three. Details available at: https://ms.unimelb.edu.au/engage/outreach/mathematics-and-statistics-research-challenge.

AUTHOR NOTE

Thank you to my collaborators, the Maths and Stats Outreach Team at the University of Melbourne: Dr Susan James, Dr Cindy Huang, Dominic Maderazo and Cait Pryse.



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PLAN TO THROW ONE AWAY: THE NEWTON-RAPHSON METHOD

Dr George Galanis

Have you ever had a mathematics or science problem that appeared so intractable that you could not see any solution? Or you did not even know where to start? Such problems are sometimes referred to as 'wicked problems'. Since there is a name for it, it must be common occurrence in mathematics and science. So what do we do about wicked problems when we encounter them? There is probably no single method for tackling wicked problems, but we can study examples and perhaps glean some ideas from them. I was confronted by a wicked problem in my first job just after I graduated with my science degree. It was a three month work-experience placement at the Telecom Research Laboratories.

On my first day I went through all the things you go through starting a new job, I was introduced to the staff in the section and my direct supervisor, and I was shown around the labs. It was around then I was told that they do not normally employ science graduates in that group, rather they employed electronics engineers. But in my case, the head of group (the boss), who interviewed me, was impressed with my 'hobby'. I held the highest level of amateur radio license. The job they had for me was essentially a computer programming job but with an electronics-engineering application. They wanted me to convert a program written in FORTRAN, which ran on the Telecom mainframe computer, to run on the Hewlett Packard desktop computers that they had in their laboratory. The Hewlett Packard computers were actually controllers for lab equipment and were more like advanced calculators than full blown computers. The boss believed that my combination of knowledge of electronics and computer programming would increase the chance of a successful outcome. As I discovered later, they had given this job to two other new graduates before and they had not succeeded in converting the program. It was then I realised what professional job pressure was. What if I did not succeed?

The boss gave me the program listing, it was a thick ream of perforated computer paper with FORTRAN code printed on it bound with cardboard covers. He then took me to the teletype in the back room, a small storage room with no windows, and shelves of supplies on the walls, so working in there was full of interruptions by staff picking up their office supplies. We sat down at the teletype that was connected to Telecom's mainframe computer and he showed me how to navigate through the menu system. He walked me through a couple of simple examples, and I took copious notes, because unlike the way we interact with such computer programs nowadays, the menus and data entry was arcane. In the first couple of weeks I sat at my desk, and pored over pages and pages of the program listing that looked like unintelligible gibberish. The main part of the gibberish was around five thousand lines of code that contained the algorithm for designing electronic filters. Essentially the filters were for separating bands of frequencies, mostly designed for audio up to the low end of the high-frequency spectrum.



Figure 1. The IBM mainframe.



Figure 2. A Hewlett Packard HP9825A.

My task was presented to me as 'simply' translating FORTRAN into the Hewlett Packard Language (HPL). The thinking was you take a line of FORTRAN and convert it into the equivalent statement in HPL. But it is not that simple. FORTRAN had different ways of branching than HPL. The input output and filing systems were different. The FORTRAN worked in a different operating system than the HPL. And the FORTRAN program I had to start with was complex and convoluted.

During my computer science course, I had learned that the term for the type of program I was to convert was 'spaghetti code', because the threads of code would be like threads of spaghetti in a bowl. It was quite common to find software written like that back in those days. And here I was, in my first job enmeshed in this spaghetti. By the time I got to the end of the second week I realised that I was making very little, if any, progress towards the stated aim of converting the code from FORTRAN to HPL.

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PLAN TO THROW ONE AWAY: THE NEWTON-RAPHSON METHOD (CONT.)

Dr George Galanis

It was on the Friday afternoon of the second week when I went to my supervisor, a PhD level electronics engineer, to express my concern. After a few minutes of conversation I said to my supervisor something along the lines of, 'This program must be based on an algorithm that is written up somewhere?' There was very little in the way of documentation for the program, as once the program had been written and working the engineers in the section just used the program. My supervisor took my concern seriously and said to me, that there was a paper that described the core of what the program was doing, and it was really that core they needed to run on the desktop computers. He took me to the library and there the librarian helped us locate the journal the paper was in. I took a photocopy of the paper and I took it home for some reading over the weekend.

To my surprise the paper was well written, and it seemed easy to follow. I read through the paper on Friday evening and discovered that the essence of the problem I was attempting to solve was finding the roots of polynomials using the Newton-Raphson method, all the roots, including the complex roots (but that's another matter for maybe another article). Readers may recall that the Newton-Raphson method is an iterative algorithm for finding the roots of polynomials. You start with a guess for the root, this is x_0 and then use the iterative process:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

The reason the Newton-Raphson method was written up in this paper was that the typical electronics design problem has polynomials of a high order, and the polynomials end up in the form of something like:

$$f(x) = (x-a)(x-b)(x-c)\ldots-(x-k)(x-l)(x-m)\ldots$$

The functions are in the form of the difference between two polynomials in their factor form. These polynomials come from the mathematics used to describe the behaviour of the filters, and the mathematics behind them is from Laplace transforms (a generalisation of the Fourier transform, in fact the 'x's are actually 's's, but I will use x in this article). I did not know much about Laplace transforms, but it did not matter, because for the electronics engineers in the lab, the 'poles and zeroes' (the values in the factored form) of the polynomials were their starting point. So in reality the problem I had to solve was finding the roots of polynomials of the form above.

One way to approach finding the root could be to expand out the factors and produce a polynomial in the traditional form:

$$f(x) = x^m + p_{m-1}x^{m-1} + p_{m-2}x^{m-2} + p_{m-3}x^{m-3}\ldots$$

But the problem with expanding the polynomial is the accumulation of errors. When you compute the coefficients and then plug in values of x and compute the value of the polynomial and its derivative and apply those values to the Newton-Raphson algorithm the errors accumulate to such an extent that the results become unusable, at least for the size of the floating point numbers available on most computers back in the late 1970s.

To get around this problem the paper described how to work with the Newton-Raphson method without expanding the polynomial. The key was to find the derivative of $f(x)$ without expanding the function. This requires the use of the product rule. So taking the first part of $f(x)$ and for, say, a polynomial with only 3 terms we have:

$$\begin{aligned}
&\frac{d}{dx}[(x-a)(x-b)(x-c)] \\
&= \left[\frac{d}{dx}(x-a)\right][(x-b)(x-c)] + (x-a)\left[\frac{d}{dx}[(x-b)(x-c)]\right] \\
&= [(x-b)(x-c)] \\
&\quad + (x-a)\left[\left[\frac{d}{dx}(x-b)\right](x-c) + (x-b)\left[\frac{d}{dx}(x-c)\right]\right] \\
&= [(x-b)(x-c)] + (x-a)[(x-c)+(x-b)] \\
&= (x-b)(x-c) + (x-a)(x-c) + (x-a)(x-b)
\end{aligned}$$

If we now apply this idea to all the terms, including the second part of the function $f(x)$ we get a 2nd order polynomial of the form of:

$$\begin{aligned}
f_3'(x) = {} & (x-b)(x-c) + (x-a)(x-c) + (x-a)(x-b) \\
& -(x-l)(x-m) - (x-k)(x-m) - (x-k)(x-l)
\end{aligned}$$

It is a relatively simple matter to produce an algorithm for an arbitrary number of terms for the derivative of $f(x)$ given the factored form of a polynomial. I had used the Newton-Raphson method a few times in my university course, so it was not new to me. So, on Saturday morning I wrote out the pseudo code, just a generic form of code that is not machine specific to get a feel for what the algorithm will look like before I wrote the actual HPL code. Sunday morning I opened up the HPL manual, HPL was Hewlett Packard's version of BASIC, I was familiar with BASIC so I found it straight forward to write the HPL needed to make the algorithm work.

On Monday I went in to work and typed the HPL I had written on Sunday into one of the HP9825As that were in the laboratory. There were a few bugs to iron out, and by Tuesday morning I had the program running reliably. I had added a section of code to enter the factors for each part of a function and the computer did the rest in a loop and produced the roots of the polynomial. It was only a hundred or so lines of HPL. It was just before lunchtime and my supervisor came around to see how I had gone with the paper. I showed him the root finding program on the desktop computer. He sat down at the

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computer and typed in some numbers that meant something for him in terms of an actual design problem. He copied down the roots the program produced and scratched away on some scrap paper, and he looked at me and said ‘wait here’. He went into the next office and got the boss. They both sat at the computer and typed in several functions. Over the space of about a quarter of an hour they tapped away, wrote numbers on a scrap of paper. Finally my boss turned to me and said, ‘It works, well done, you can go home now.’ I was surprised, was I out of a job? But going home was too generous to pass up. So I went home and relaxed for the afternoon.

The next morning I went in to work. The boss had a jolly personality and when he saw me he joked, ‘What are you doing here? You’ve created a problem for us. We actually weren’t expecting you to finish this so quickly, in fact, we weren’t really expecting an outcome given we haven’t had one before.’ Knowing I was a keen windsurfer he said to me, ‘It’s a beautiful day today, why don’t you go windsurfing. Come back tomorrow morning, I need to develop a work program for you.’ It was indeed a beautiful sunny day with a light breeze, so I went home, put the windsurfer on the car and went sailing for the afternoon.

As it turned out, my work program became extending the root finder I had written to become part of a larger system that Telecom would use for designing electronic filters. That also included extending the system to work with microwave frequencies and for components for AusSat, the first Australian produced communications satellite. The boss offered me a permanent position and I spent the next two years at the labs working on various software projects for solving engineering design problems.

The reason I did not drop out of my placement at Telecom was not because I was a genius, there were many friends I had at university that had better academic records than me and many of my colleagues at the laboratory were better qualified than me. Rather, I believe it was that I had the courage to own up that I was not making any progress. Being open about that led to my supervisor taking action to help me redefine my job into something achievable. Much later in my career I discovered Frederick Brooks’ book *The Mythical Man Month* (1969), described the exact problem I had confronted at Telecom. Brooks described ‘The Tarpit of software engineering…In the mind’s eye one sees dinosaurs, mammoths and sabertoothed tigers struggling against the grip of the tar. The fiercer the struggle, the more entangling the tar, and no beast is so strong or so skillful but that he ultimately sinks’. Brooks’ describes possible solutions to the problem of the software ‘tarpit’. One of the chapters in his book is called, ‘Plan to Throw One Away’, because, ‘you will anyway’.

Why do we expect complex systems to work first time? Rather, Brooks suggests that we make an attempt, and then use the expertise gained from the first attempt to design and build a better system the second time. The wisdom of my supervisor and my boss at Telecom was to see the program I had written as a better starting point than hanging onto the original FORTRAN program. Over the span of my career I had seen some well-known organisations and experienced scientists and engineers not heed Brook’s advice and become stuck and large projects failed because they did not want to let go of their legacy systems.

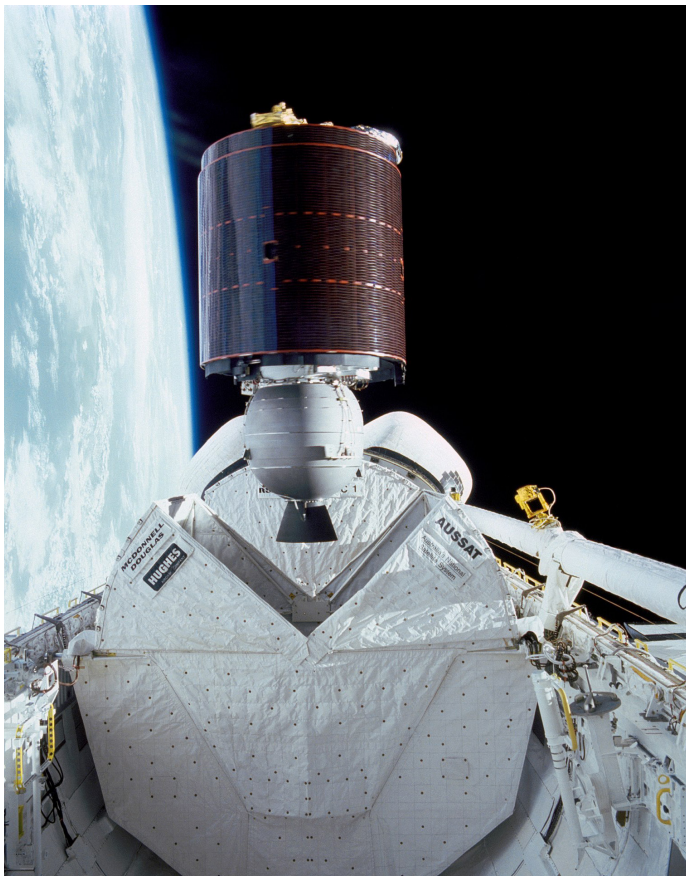


Figure 3. AUSSAT 1 being released from STS-51 (Space Shuttle).

I sometimes wonder where my career would have gone if I had not taken that courageous leap of owning up to not making any progress. What if the boss had not accepted that his program could be implemented in a better way? I have worked with many scientists and engineers during my career and it is not uncommon for people to see their work as their ‘baby’. If my boss at Telecom had been that way inclined then I would have really been stuck and probably would have failed by the end of three months (or any period). Working as an engineer and scientist can be daunting. There is an element of luck and seizing opportunities as they arise, but luck and seizing opportunities requires mastery of the subject matter and trust in one’s mastery. If you have truly mastered your field my advice to students is: own up if you are not making progress, it is better than the alternative.

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TENNIS: MATHEMATICAL MODELLING IN THE SCHOOL CLASSROOM

Jill Brown, Deakin University

In this article, a real-world task is presented and discussed. The aim is to illustrate how a mathematical modelling task might be introduced. This article illustrates the idea that it is important to discuss the context separately (before if possible) from the actual problem needed to be solved. It is also critical for the teacher(s) to attempt any task themselves, prior to implementation.

Each January, Tennis Australia holds the Australia Open. The Australian Open (AO) is one of the four tournaments that constitute the tennis grand slam.

During one media interview, it was reported that thousands of large tennis balls were sold. What do you know about tennis and the AO?

The real-world context for this task is the AO Tennis event. If implementing this task, share/use the information to-date to begin a conversation about the context with your class. Thus, you are providing some details, and adding to this as students contribute additional insights. This discussion may include problem finding and problem posing. It will certainly provide all students some understanding of the game of tennis and a tennis competition.

It is important to keep in mind that students will be working in groups on the task and there will be multiple discussion (whole class interactions and teacher group interactions) as students progress on the task. All discussions, students to student, and students to teacher will further develop student understanding of the context and hence the task.

This raises the question, how many tennis balls are needed in preparation for the AO?

THE MATHEMATICAL MODELLING TASK: THE TENNIS BALL TASK

> During 2024, Chris was doing work experience at Melbourne Park. Chris's supervisor Lee needed to organise the order for tennis balls for the 2025 AO. On the same day, both a power outage and a data hack occurred. Lee asked Chris to work out how many balls should be ordered and to provide a written report explaining how this was determined, including any assumptions made by Chris.
>
> Solve the task for Chris and write a report to Lee clearly explaining how the solution was determined.

This is the question to be explored and solved.

This question should be presented to the class for discussion. Small groups will likely generate more ideas, but individual teachers should make such decisions including based on their



knowledge of the class, the time of year, and themselves. This discussion should generate details as to what is known and what needs to be known. What can be ignored and what is irrelevant should also be discussed. This discussion might include:

- How often are the balls changed?
- How long is a game of tennis? [irrelevant]
- Connections between large balls for signing and small balls for play. [irrelevant]
- How many players?
- How many different competitions make up the AO?
- How many games in a set? (Maximum, minimum, average: which metric should be used?) How many competitions constitute the AO? (it is not just for men).
- What is a seeded (wildcard) players and how many are in the competition?

NOTES

Note that whilst some of these statements and questions are irrelevant, all are helpful in supporting students progress their understanding of the mathematical modelling task and how they might solve it.

1. It is important to let students arrive at the idea that they should consider the maximum number of balls required. This may not be recognised until after the task is initially solved and students are reflecting on their solutions. In early modelling, some students won't consider this until they are finished the mathematical modelling task and considering alternative solutions and approaches by other groups.

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2. For mathematical modelling beyond this task, it is recommended to wait until students make this connection, that is, find the maximum number of balls first, by themselves. It is an important aspect of mathematical modelling to consider such ideas before beginning to solve future tasks. In some other cases, task solvers would use minimum values to solve a task, but in this case the maximum should be considered for a more realistic solution (e.g. the worst case scenario).

3. However, it is important also for students to recognise that following a first solution, it is generally a simple step to modify their model to adapt for changed thinking.

4. In this particular task, using the minimum number of games initially and adjusting following recognition that the maximum should be used is not a major task. This is true of many tasks.

5. Reflecting on one's first solution is important.

SOME USEFUL FACTS

You can allow students to source this information (in class or for homework after the introductory discussion and posing of the problem to be solved) or provide it for them post discussion about what needs to be known. The AO uses Dunlop AP balls.

Fact 1: Rules and scoring. See www.tennis.com.au - includes rules of the game and scoring, including a two minute video on the site or directly at https://youtu.be/zVdWbWi7kI8

Fact 2: Most championships are played with the best of three sets. Some have five sets. The AO is actually a collection of championships. The men play best of five sets and others are best of three.

Fact 3: In the AO, balls are changed every nine games, except at the start of the match where the balls are changed after seven games to account for using the balls in the warm up.

Fact 4: Six balls are used at one time.

Fact 5: The AO uses Dunlop AO balls. The Dunlop Australian Open (AO) Balls can be bought by the box, 18 cans with four balls per can or 24 cans with three balls in each at approximately $190. This is not necessarily the price the AO pays, nor does it include delivery, but could be useful for students wanting to account for the cost of the balls.

Fact 6: The men's and women's AO 2025 opening round has 128 players, including 32 seeds and eight wild cards.

Fact 7: See https://ausopen.com/draws for the draw for each of the various competitions that make up the AO (i.e., women, men, girls, boys, singles, doubles, wheelchair, quad wheelchair, legends).

The prior experiences of your students with mathematical modelling may impact the number of ideas generated by students. It is important to keep in mind that it is mathematically sensible to both simplify the problem and to solve the problem using the simplest methods available. Whatever approach is taken, it is important for student groups to reflect on their solution.

Mathematical modellers (students and employed) can always suggest what they may have done differently, and what they might do if they had more time (noting time is money in the real world). Mathematical modellers should always reflect on their assumption, model(s), and results as part of their solution.

Looking at the solutions of other groups can be productive for future modelling. Often students (and humans in general) are easily convinced by something they agree with. Questions such as: Does the process make sense? Is the mathematical work (including calculations) clear, and correct? Is it clear that the mathematical solution can be used to answer the real-world problem?

Your students might search online for the answer and discover 71 352 tennis balls were actually used in 2024. However, that is not the solution to this task. This number might be used as part of validating their mathematical solution. Mathematical modellers need to be able to explain all aspects of the modelling process taken to determine their solution. The complete solution process should be clearly communicated to others.

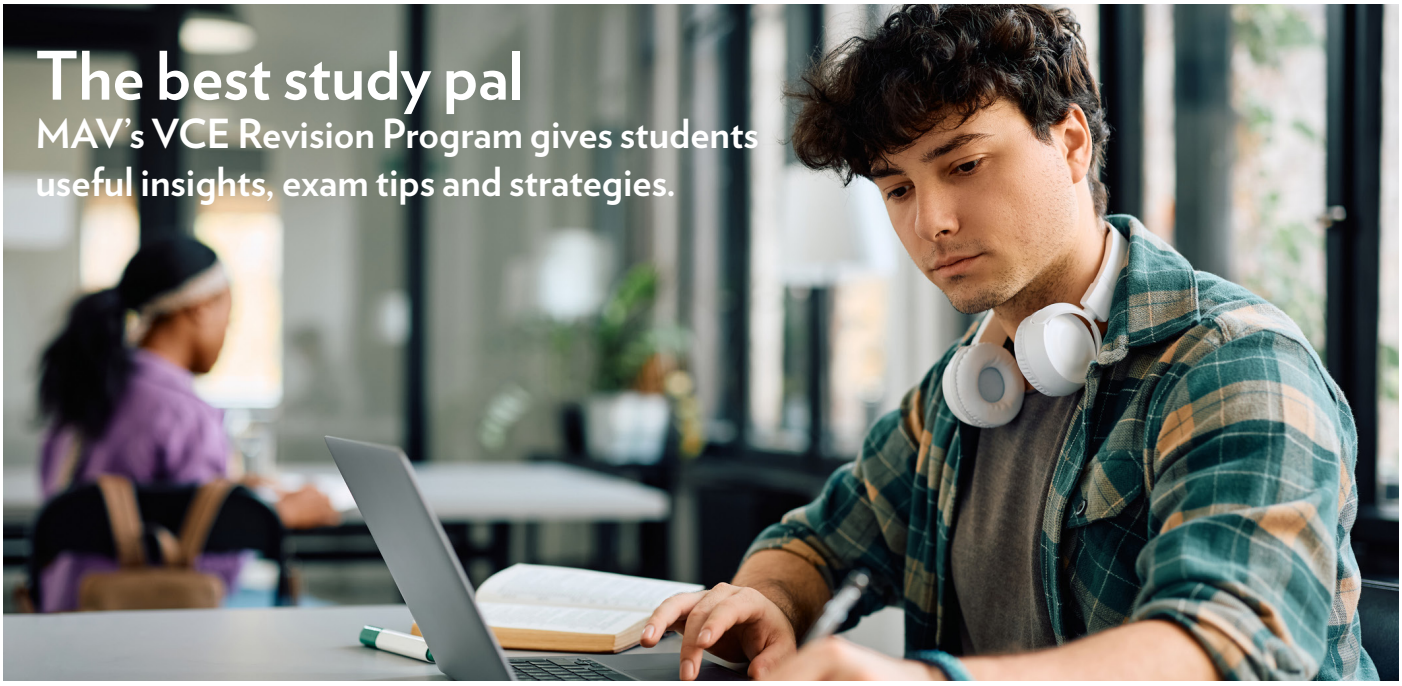
When implementing this task or any mathematical modelling task, the focus should be on the modelling process rather than particular mathematical content. Communication of the process will be oral and verbal, however the intention is to clearly communicate the task and solution 'in writing'. This of course includes words, diagrams, numbers, tables, models, etc. The mathematical content and the mathematical processes can and should inform future teaching and learning.

Part 2 of this article will include some sample approaches to the task and further insight for teachers. If you want to contribute and be a co-author, send your solution and/or that of your students and any insights you are willing to share from your experience in implementing this task. Email Jill, jill.brown@deakin.edu.au or via LinkedIn www.linkedin.com/in/jill-brown4a1946248.

Many additional facts can be sourced from www.tennis.com.au/wpcontent/uploads/2025/02/2025-02-03-Australian-Open-2025-by-the-numbers-1.pdf that could prompt additional problems worth exploring.

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CULTURALLY RESPONSIVE CLASSROOMS

Hajin Park

ENHANCING CULTURAL DIVERSITY FOR ENHANCED UNDERSTANDING

The increasing diversity in Australian society, shaped by migration and immigration, is reflected in our classrooms (Tartwijk et al., 2012). According to the Australian Bureau of Statistics (2022), 29.1% of Australia's population in 2021 was born overseas, with Australia ranking ninth internationally for the proportion of migrants in its population as of 2020. This diversity presents both opportunities and challenges for mathematics education.

While some studies suggest a potential decrease in the concentration of ethnically diverse students within individual classrooms (Abacioglu et al., 2023), culturally responsive mathematics teaching remains crucial. Birnbaum et al. (2020) argue that multicultural classrooms can contribute to reducing racial achievement disparities in mathematics. Conversely, 'colour blind' approaches and standardised curricula may inadvertently widen these gaps, highlighting the need for practical, culturally responsive pedagogies in mathematics classrooms (Birnbaum et al., 2020).

In this context, the mathematics classroom becomes not just a space for knowledge transmission but for social interaction, development of interpersonal skills, and sharing of individual cultural perspectives on mathematical concepts. Understanding and implementing culturally responsive teaching and learning in mathematics is vital for fostering inclusive learning environments. This is particularly significant as mathematics, often perceived as a universal language, is deeply influenced by cultural contexts and perspectives.

The ever-changing value of culture in a diverse society (Tielman et al., 2012) offers rich opportunities for mathematics education. We can create a more inclusive and effective learning environment by acknowledging and incorporating various cultural approaches to mathematical thinking. This paper will explore the concept of cultural hybridity in mathematics education, examining how the interplay of diverse cultural perspectives can enhance mathematical understanding and reduce achievement gaps in Australian classrooms.

CULTURAL HYBRIDITY THEORY IN MATHEMATICS CLASSROOM

The cultural hybridity theory believes that classrooms appear as a unique blend of identities, as students and educators combine their distinctive cultural backgrounds and interconnect to construct a diverse and unique classroom culture (Casinader & Manathunga, 2020). Learning in this environment is collaborative, and diverse cultural elements are welcomed and celebrated for their richness as they create a unique learning experience (Valdiviezo & Nieto, 2015).According to cultural hybridity theory, the classroom is a space where different cultural identities come into contact, overlap, and blend. Cultural métissage, the process of mixing and connecting cultural forms and identities, is a critical concept in hybrid cultural theory, particularly when applied to classroom dynamics (Banciu et al., 2013). When students and educators bring their diverse cultural perspectives on mathematics to the learning environment, they give voice to these backgrounds and create a new classroom culture that blends individual and collective mathematical traditions (Worley, 2006).

In mathematics education, cultural messaging supports students in building or solidifying their mathematical identity by contributing to and blending into a hybrid mathematical culture. It provides opportunities for students to confront their preconceptions about mathematics, reflect on different cultural approaches to mathematical problems, and ultimately develop a broader understanding of mathematical concepts (Vieira, 2014). For example, students might explore different counting systems, problem-solving strategies, or geometric patterns from various cultures, integrating these diverse approaches into their mathematical knowledge. This could involve comparing mental math techniques from East Asian educational systems with Western algorithmic approaches or examining the mathematical principles behind traditional games and practices from students' varied cultural backgrounds.

CULTURAL MÉTISSAGE IN THE CLASSROOM

Cultural métissage is a concept rooted in cultural anthropology and sociology that describes the intricate process of cultural blending and hybridisation when different cultures come into contact (Vieira, 2014). This concept extends beyond simple cultural exchange by encompassing the dynamic of reshaping and synthesising cultural elements to create entirely new and unique cultural expressions (Scott, 2021). In this context, the educator actively integrates aspects from these distinct cultures into the curriculum. In today's multicultural mathematics classrooms, educators actively integrate aspects of their students' varied cultural backgrounds into the curriculum. This integration manifests in various forms, such as:

1. Digital technology integration: Incorporating smartphone apps or social media platforms that students from various cultures use daily to teach data analysis or statistics. For example, analysing TikTok view counts or Instagram engagement rates to understand large numbers and percentages.

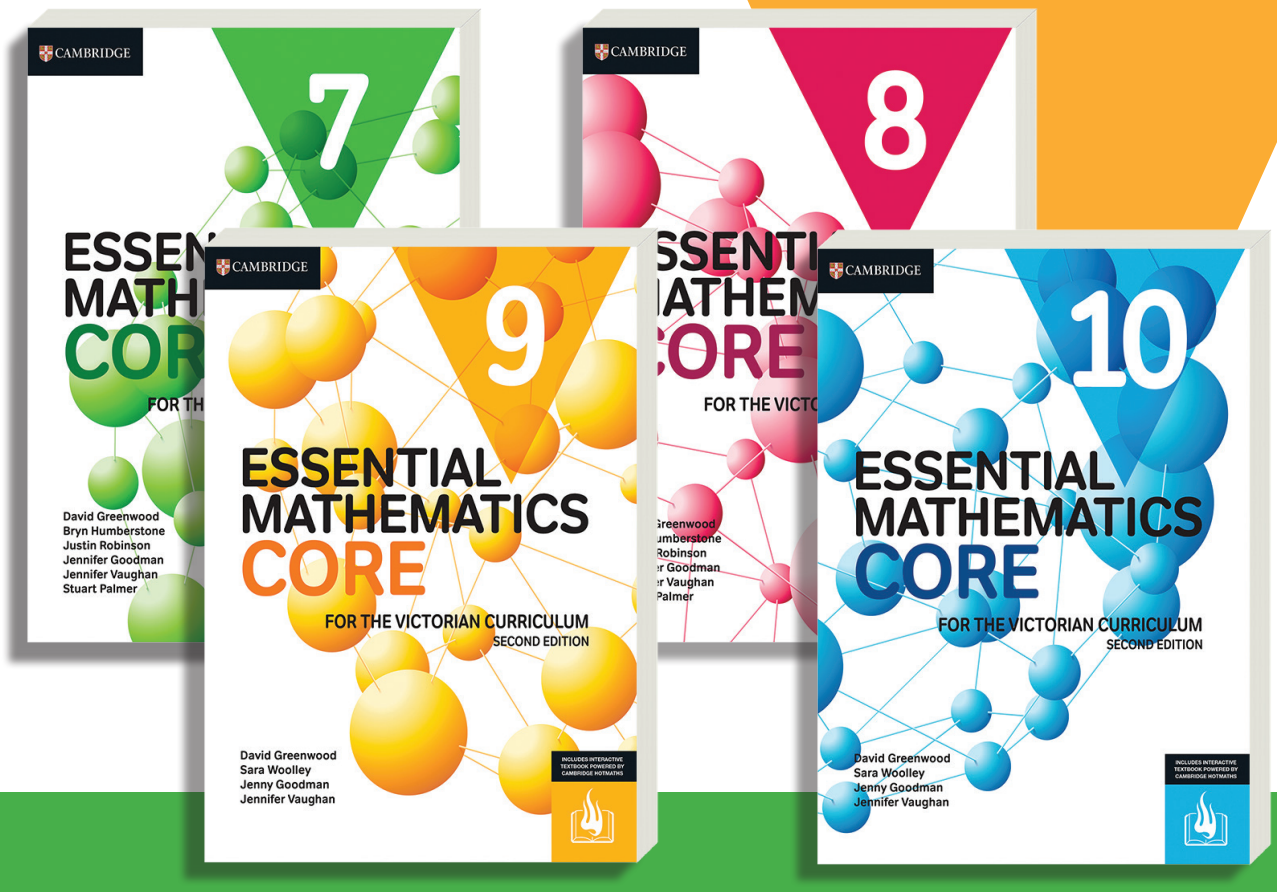
2. Sports and games: Using popular sports from different cultures to teach probability and statistics. This could involve calculating batting averages in cricket for students from South Asian backgrounds or using soccer (football) statistics for students with European or South American heritage.

3. Music and rhythm: Exploring mathematical patterns in diverse musical styles, such as hip-hop beats, K-pop choreography or Bollywood dance sequences, to teach concepts like fractions, ratios, and sequences.

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CULTURALLY RESPONSIVE CLASSROOMS (CONT.)

Hajin Park

4. Food and cooking: Utilising recipes from students' home cultures to teach measurement, proportion, and scaling. This could involve converting between metric and imperial units or scaling recipes for different serving sizes.

5. Cultural festivals and celebrations: Incorporating mathematics into discussions of cultural celebrations. For instance, exploring geometry through the designs of Diwali rangoli patterns or using the lunar calendar calculations for Chinese New Year to teach cyclical patterns.

6. Environmental concerns: Addressing global issues like climate change through a mathematical lens, considering how it affects different regions represented in the classroom. This could involve analysing sea level rise data for island nations or temperature changes in various countries.

By integrating cultural métissage, mathematics educators can create a learning environment that respects their students' diverse backgrounds and uses these varied perspectives to deepen mathematical understanding. This approach helps students see the relevance of mathematics in their daily lives and cultural contexts, fostering greater engagement and comprehension of mathematical concepts.

CONCLUSION

The integration of cultural hybridity and cultural métissage in mathematics education mirrors the dynamic nature of culture itself, which is constantly evolving and developing. This challenges the traditional views of culture as a fixed, culture-free discipline and instead presents it as a rich, culturally influenced field (Scott, 2021). By embracing cultural hybridity in mathematics classrooms, educators play a crucial role in creating more inclusive learning environments that value diverse mathematical traditions, engage students from all backgrounds, and foster a deeper, more nuanced understanding of mathematical concepts. This approach has several benefits:

- **Enhanced engagement:** By connecting mathematical concepts to students' diverse cultural experiences, educators can increase their interest and participation in mathematics.
- **Broader perspective:** Exposing students to various cultural approaches to mathematics fosters a more comprehensive understanding of mathematical concepts and their real-world applications.
- **Reduced achievement gaps:** Cultural hybridity in mathematics education helps to bridge achievement gaps by ensuring that all students see their cultural identities reflected in the curriculum.
- **Preparation for a globalised world:** Students exposed to diverse mathematical traditions are better equipped to navigate and contribute to an increasingly interconnected global society.

By embracing cultural hybridity in mathematics education, we can create a more equitable, engaging, and effective learning environment that prepares all students for success in our multicultural society. As educators, it is our responsibility to recognise the power of diversity in our classrooms, turning it into a strength that enhances mathematical understanding for all students.

REFERENCES

Abacioglu, C. S., Epskamp, S., Fischer, A. H., & Volman, M. (2023). Effects of Multicultural Education on Student Engagement in Low- and High-Concentration Classrooms: The Mediating Role of Student Relationships. *Learning Environments Research*, 26(3), 951–975.

Australian Bureau of Statistics. (2022). Australia's population by country of birth. www.abs.gov.au/statistics/people/population/australias-population-country-birth/latest- release

Banciu, V., Leah, C., & Jireghie, A. (2013). Métissage and Multiculturalism, the New Dimensions of Globalization. *Journal of Advanced Management Science*, *1*(2), 184–189. https://doi.org/10.12720/joams.1.2.184-189

Birnbaum, H. J., Stephens, N. M., Townsend, S. S. M., & Hamedani, M. Y. G. (2021). A Diversity Ideology Intervention: Multiculturalism Reduces the Racial Achievement Gap. *Social Psychological and Personality Science*, *12*(5), 751–759. https://doi.org/10.1177/1948550620938227

Casinader, N., & Manathunga, C. (2020). Cultural hybridity and Australian children: speaking back to educational discourses about global citizenship. *Discourse*, *41*(6), 940–952. https://doi.org/10.1080/01596306.2019.1593108

Scott, B. (2021). *Reconciliation through métissage in higher education.*

Tielman, K., Bolhuis, S., den Brok, P., & Vallejo, B. (2012). Collaborative learning in multicultural classrooms: A case study of Dutch senior secondary vocational education. Journal of Vocational Education and Training, *64*(1), 103–118. https://doi.org/10.1080/13636820.2011.622448

Valdiviezo, L. A., & Nieto, S. (2015). Culture in Bilingual and Multilingual Education. *The Handbook of Bilingual and Multilingual Education*, 92–108. Wiley. https://doi.org/10.1002/9781118533406.ch6

Vieira, R. (2014). Life stories, Cultural métissage, And personal identities. *SAGE Open*, *4*(1). https://doi.org/10.1177/2158244013517241

Worley, V. (2006). Revolution is in the everyday: Métissage as place of education. *Discourse*, *27*(4), 515–531. https://doi.org/10.1080/01596300600988853

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OUTDOOR ADVENTURES WITH PYTHAGORAS

Corey Campbell, Parkdale Secondary College

Measurement units allow teachers to get to know students in a topic where students feel more confident whilst completing hands-on activities. This provides teachers opportunities to help students develop their skills and their confidence whilst exploring mathematical modelling (Van de Walle et al., 2019; Goos et al., 2019; Brown, 2024). This investigation is centred around the powerful tool of peer-collaboration and mathematical modelling; both with evidence based research showing these methods can increase student learning outcomes in high school settings (Cornell University, 2024; Laal & Ghodsi, 2011).

Mathematical modelling allows students to evaluate real-life problems and explore different processes to find solutions; gaining important skills such as estimating, evaluating, and communicating (Brown, 2024). Connecting our classrooms with these challenges also increases student engagement and motivation, student outcomes, and supports students with skills for the workforce, such as team work, and creative thinking (PISA, 2023).

Vygotsky's 'Sociocultural Theory' (1987) and Bandura's 'Social Learning Theory' support student learning, retention and outcomes when learning is undertaken in social settings with persons with knowledge, especially in trusted settings. Bandura's (1987) findings includes students observing and imitating others as a form of learning, thus enabling students to work and learn together whilst building their sense of belonging, self-efficacy and their perceived sense of possible achievement and motivation.

TAKING PYTHAGORAS OUTSIDE

This activity has students working outside and within the quadrangle space in Figure 1. The wooden decking areas are used in during break times and assembly points throughout the day, and will be referred to as 'the decking' here-on-in. These scaffolded activities build students to accurately measure the decking (prompting a trade-based context), and supporting workable and practical proofs with Pythagoras Theorem.

TASK 1: MEASURING TO TEST CALCULATIONS

Student groups are taken outside (see Figure 1) and create and measure three different right-angled triangles with the decking, measuring all three sides to each right-angled triangle. This allows students to practice accurate measuring – with teachers



Figure 1. Parkdale SC student quadrangle decking (supplied).

considering using consistent decks for accuracy discussions; and supply different measuring tools for experience and discussion. Students enter measurements in Table 1 and calculate; discussion later in class.

TASK 2: BUILDERS 3, 4, 5 RULE

Teacher to demonstrate a key construction technique in the building trade – the '3, 4, 5' rule where builders construct triangles in 3m, 4m and 5m (hypotenuse) ratios to create accurate right-angle triangles. Students use their decks to apply the rule and measure if right-angle triangles present, recording results in Table 2.

BUILDERS 3, 4, 5 RULE CHECK	90° PASS?
Deck 1	
Deck 2	
Deck 3	
Whiteboard wall (classroom)	
Door (classroom)	

Table 2. Task 2 results.

DECK TRIANGLE	MEASUREMENT 1	MEASUREMENT 2	PYTHAGORAS THEOREM CALCULATION	HYPOTENUSE MEASURED	ARE THEY SIMILAR? YES/NO
Deck 1					
Deck 2					
Deck 3					

Table 1. Task 1 results.

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TASK 3: TAKING ON THE TREES

Students measure the small trees found planted in the middle of the decking to examine Pythagoras' Theorem proofs. This activity still allows the students to measure the three sides of the triangle (measuring the height of the tree with their calculation) and allowing for a rich discussion as to why their measurements vs calculations could be slightly different (such as rounding calculations too early or inaccurately at the end, or measuring techniques or inaccurate measuring tools). Students will take the measurements of the opposite and adjacent side of the tree and perform the calculations outside, before measuring the height of the tree and comparing the results.

TASK 4: UP AND BEYOND

The final task that allows students to calculate the height of something with a true unknown factor. Students will measure the height of the shadow coming from a building, a water fountain or water hydrant (if no shadows – always have your backup such as the flag pole or building height). Students will decide what to do, how to do it, and present their findings in a small write-up on a worksheet that had the first three tasks heavily scaffolded. They will submit their findings in a prac-report style, showing their estimation, method of collecting results and results, calculations and discussion.

BENEFITS FOR STUDENT LEARNING

Following research-based studies for 21st century workers, measurement is a unit that provides many opportunities for learning beyond the mathematical concepts (PISA, 2022). Many jobs that our students face have not been created yet, or are evolving, thus requiring teachers to pivot from focusing on content driven lessons to engaging, hands-on, and interdisciplinary lessons with mathematical modelling (Brown, 2024; Foster, 2023).

Teachers producing authentic learning opportunities that challenge students to think critically and creatively with mathematical modelling and problem-solving creates situations where students can develop their communication skills, building toolkits with analogue and digital skills, supporting persistence and adaptability in learners, and expanding their metacognition with the context and content (Goos & Geiger, 2012; AAMT & AiGroup 2014; Van de Walle et al., 2019; Cowan, 2006; Foster, 2023; Oblinger, 2007). These Pythagoras tasks support students real-life experiences with measurement and using measuring tools; furthering discussions in real-life contexts such as navigating the seas, surveying, the construction industry, sports and measurements, and sizes of television and device screens. Goos' *Model for Numeracy in the 21st Century* (2007) highlights the importance of students working with these contexts in classrooms to improve student connection to their learning and outcomes, challenging students to conceptualise and apply their mathematics in real settings, and further developing strategies to check the 'reasonableness' of their results whilst problem-solving (Goos et al., 2019).

Further to this, students benefit from working collaboratively. This allows them to learn from each other in an environment that replicates the 21st century workforce that demands positive work dispositions; for some students this building of trusting relationships between teacher and student due to previous negative experiences can take considerable time (Goos & Geiger, 2012; Goos et al., 2019; Australian Association of Mathematics Teachers (AAMT), 2015). Teachers can support student dispositions by providing collaborative settings with real-life contexts and demonstrating their enthusiasm for mathematics, showing self-belief in their students and promoting the idea that each student can improve their growth mindset and progress (Goos & Geiger, 2012; AAMT, 2015). This also allows teachers to secretly provide opportunities to students to make connections with mathematics outside of their classroom and textbook experiences (much like parents hiding veggies in the pasta sauce).

PRIOR CLASS-BASED WORK

Prior to working with students outside, the teacher can develop and support students for this investigation with the following ideas and practice:

Engaging with students to ensure they understand how to measure accurately/remove misconceptions

I have known senior students who rush this process or confuse a standard school ruler with a trade-based rule/measuring tape that starts measuring right at the tip. So to begin this measurement unit, we brainstorm all the different measuring tools and list examples of where they are used, what they measure, and what units they use.

TREE	MEASUREMENT 1	MEASUREMENT 2	PYTHAGORAS THEOREM CALCULATION	HYPOTENUSE MEASURED	ARE THEY SIMILAR? YES/NO
Tree 1					
Tree 2					
Tree 3					

Table 3. Results table for Tasks 3 and 4.

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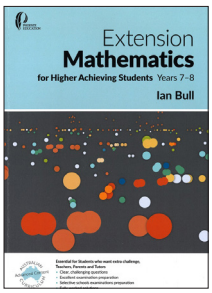
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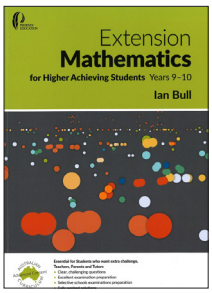
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EXTENSION MATHEMATICS FOR HIGHER ACHIEVING STUDENTS

7–10

These books are for students who need and enjoy the challenge of working with material which encourages the application of higher order thinking. The problems and exercises in these books encourage students to:

- think creatively
- be flexible in the use of new ideas and approaches
- be fluent in the creation of an array of ideas
- elaborate on similar ideas
- be original in thinking of new approaches or new ways of seeing problems.

The problems and tasks are based on the topic and concept areas studied in the upper middle curriculum with extra task to allow challenge, application and extension of ideas.

YEARS 7-8:	$39.20 (MEMBER) $49 (NON MEMBER)
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MAV SACS SUGGESTED STARTING POINTS

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MAV SACS SUGGESTED STARTING POINTS 2025

VCE

The MAV 2025 VCE Mathematics SACs materials are an online resource that is accessed through our learning management system Thinkific.

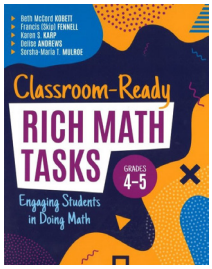
The materials are designed to provide suggested starting points for VCE Mathematics teachers for their School Assessed Coursework (SACs).

Resources include SAC starting points, alternative tasks, solution guides, marking criteria and data sets. A recording of the SAC session from the VCE MAV conference is also included to assist teachers with creating appropriate VCE SACs.

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3–6

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OUTDOOR ADVENTURES WITH PYTHAGORAS (CONT.)

Corey Campbell, Parkdale Secondary College

Many of the tools students list are already in the school (such as rulers (15cm, 30cm, 100cm), 30m measuring tapes, trundle wheels (maths/PE cupboards), callipers, and digital distance readers (thanks woodwork / Dads), and apps on mobile phones with school permission. Bring them into the classroom and have students 'play and explore' with them in a simple and effective task - estimate then measure the dimensions of a nominated window, table, whiteboard to ensure consistency, and some glue sticks, whiteboard markers and student diary to also be measured (same product for consistency, but spread around the room to stop crowds near the bigger items being measured).

This creates a friendly competition in the classroom with students trying to determine which tool gives the most accurate reading, and which pair of students can provide the most accurate reading. Students working in pairs allow for collaboration and discourse where peer-learning, role-modelling and coaching informally. We know that this peer-learning improves student outcomes (Cornell University, 2024; Laal & Ghodsi, 2011) and furthers student understanding with not only the 'how' but the 'why' within the concept that they are working on (Sullivan, 2011).

Explicit teaching of Pythagoras Theorem

Students will require explicit teaching of Pythagoras Theorem to ensure they experience success with these tasks,

The explicit teaching prior to the investigation allows multiple exposures, which supports and improves student learning outcomes (DET, 2017). Creating different situations and tasks where students are engaging in new knowledge and skills over several tasks consolidates mastery of skills, provides opportunities to apply skills in different contexts when teachers clearly making those links and allows deeper learning for retention (DET, 2017).

CONCLUSION

This set of investigation tasks scaffolds Pythagoras' Theorem application and proofs after some classroom practice, supporting students to gain practical skills and relatable learning experiences to make deeper connections with their learning. Using the decking allows plenty of space for the students, easy supervision for the teacher, practical experience with measuring tools, peer coaching, and practical work used often in construction industries. Working in collaborative settings, students can mimic others or work together, developing interpersonal and communication skills, with other 21st century skills learner skills also being developed (Goos & Geiger, 2012; Goos et al., 2019; PISA, 2023).

REFERENCES

Australian Association of Mathematics Teachers (AAMT) & Australian Industry Group (AiGroup). (2014). *Tackling the School–Industry Mathematics Divide.* Commonwealth of Australia. www.chiefscientist.gov.au/sites/default/files/Quantitative-Skills-of-21st-Century-Workers-Summary.pdf

Australian Association of Mathematics Teachers Inc. (2015). AAMT Position Statement: Promoting Positive Attitudes towards Mathematics. https://aamt.edu.au/wp-content/uploads/2020/07/positive-attitudes.pdf

Bandura, A. (1971). *Social learning theory.* General Learning Press.

Brown, J. (2024). Mathematical Modelling: If the Gloves Fit. *Vinculum*, *61*(2),16-17.

Cowan, P. (2006). *Teaching Mathematics A Handbook for Primary and Secondary School Teachers.* Taylor and Francis.

Department of Education and Training (DET) Victoria. (2017). *High Impact Teaching Strategies.* www.education.vic.gov.au/Documents/school/teachers/support/Expired/0000highimpactteachstrat-expired.pdf

Foster, N. (2023) *21st century competencies: Challenges in education and assessment.* www.oecd-ilibrary.org/education/innovating-assessments-to-measure-and-support-complex-skills_3637901c-en

Goos, M. (2007). *Developing numeracy in the learning areas (middle years)*. Keynote address delivered at the South Australian Literacy and Numeracy Expo, Adelaide.

Goos, M., & Geiger, V. (2012). Connecting social perspectives on mathematics teacher education in online environments. *ZDM*, *44*(6), 705–715. https://doi.org/10.1007/s11858-012-0441-y

Goos, M., Geiger, V., Dole, S., Forgasz, H., & Bennison, A. (2019). *Numeracy across the curriculum : research-based strategies for enhancing teaching and learning*. Allen & Unwin.

OECD (2023) *PISA 2022 mathematical framework*. PISA 2022 assessment and analytical framework. OECD Publishing. https://doi/10.1787/7ea9ee19-en.

Van de Walle, J., Karp, K., Bay-Williams, J. M., Brass, A., Bentley, B., & Ferguson, S. (2019). *Primary and Middle Years Mathematics*. Pearson Education Australia.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge University Press.

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GRAPHS IN OUR GEOMETRY CURRICULUM

Robert Money

The geometry curriculum in Victoria has changed. Proof in Euclidean geometry (congruent triangles etc.) is not included in VCE courses, and 'graphs and networks' has been included in Level 10 VCAA content and in VCE General Mathematics and Specialist Mathematics Areas of Study. School mathematics programs from Year 7 upwards need to respond to these changes, perhaps spending less time on 'Euclidean' geometry and more on graphs and networks. In particular schools should consider including 'graphs and networks' in their Level 7 to 9 programs. Suggestions are made here how this might be done.

GRAPHS AND NETWORKS IN LEVELS 7 TO 10

In summary, suggested topics are:

Level 7: Simple graphs, directed graphs and bipartite graphs

Level 8: Planar graphs, map colouring and Euler's Rule

Level 9: Trees, Euler trails and Hamilton paths

Level 10: Weighted graphs: introductory applications

YEAR 7

1. Let the vertices represent each of the students in a group. The teacher says 'Tell me something interesting about yourself' and a student might respond 'I play football.' Students then draw a simple graph in which the edges represent links between pairs of students *who play the same sport*. The teacher introduces key terms - vertex, edge, degree, connected, simple and complete. Students use these terms to describe and interpret the graph they have built.
2. The teacher asks 'Which other sports do you all play as well?' and leads the class through construction of a bi-partite graph where source vertices represent the students, sink vertices represent the various sports, and edges are drawn between a student and each of the sports they play. Students interpret the graph and discuss how many extra edges would be required for a complete bi-partite graph.
3. The teacher asks 'Do you contact other students after school time?' and students draw directed edges for *a contacts b and b contacts a*. The teacher introduces the terms out degree and in degree and students interpret the directed simple graph that has been created.
4. Further activities can grow out of the interests of the students and the teacher. The use of real data increases interest but the teacher must be aware of potential embarrassment to individual students, for example students isolated on a 'birthday invitations' graph.

YEAR 8

1. Students explore a range of graphs to decide which can be redrawn as planar graphs, for which faces can be identified as well as the vertices and edges. Students are led to the conclusion that any graph that contains the K_5 simple graph or the $K_{3,3}$ bi-graph is not a planar graph.
2. Students examine the five regular polyhedra and their Schlegel representations as regular planar graphs in which all vertices are of the same degree. Students count vertices, edges and faces on all their planar graphs to discover Euler's rule $V + F = E + 2$.

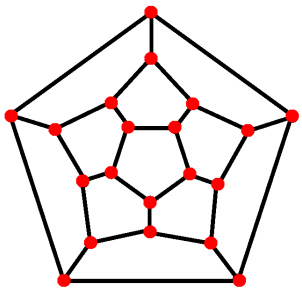


Figure 1. In this Schlegel diagram the 12th face of the dodecagon is represented by the outside of the figure.

3. Students discuss the key elements of a proof of Euler's rule, that the rule works for the simplest possible graph and that adding an extra edge to a planar graph requires an addition of 1 to either the number of edges or the number of faces.
4. Students add maps to their collection of planar graphs and explore the question of how many colours are required so that no adjoining faces/regions are of the same colour. The teacher can use the opportunity to explain that the former four colour conjecture is now a theorem.

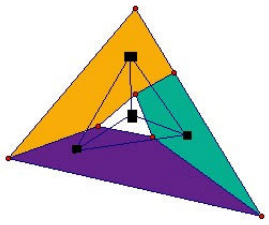


Figure 2. A map requiring four colours.

YEAR 9

1. Students collect examples of tree diagrams and discuss their usefulness. These should include family and kinship trees, the Dewey Decimal tree, a home or school local area network tree, an internet searching tree and tree structures they have met in other school studies.

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2. Students develop their own definition of a tree and note that the number of vertices in a tree is one more than the number of edges. They count the number of different trees with up to 5 edges, as exemplified in the different carbon alkanes.

3. Students learn the different methods for reducing a weighted graph to its minimum spanning tree. They discuss how this might be useful, such as in a distribution network.

4. Students investigate Euler trails that cycle through all edges of a graph without repetition. They recognise that for this to happen the vertices of the graph must all be of even degree.

5. Students tackle problems in which extra repeat edges are added to Eulerize a graph. In the graph shown in Figure 3:
   a. How many of the edges of this route could be covered without covering any edge more than once?
   b. How many extra edges are required for a bus to cover this route?
   c. How many edges would a garbage truck need to cover both sides of the road in this route?

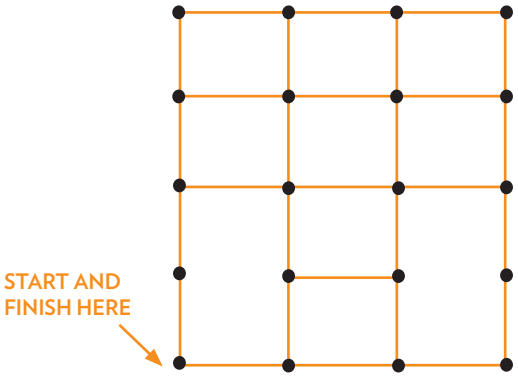


Figure 3.

6. Students investigate Hamilton paths that cycle through all vertices of a graph without repetition. They use simple graphs to find Hamilton paths and recognise that there is no simple rule to decide whether a Hamiltonian path exists.

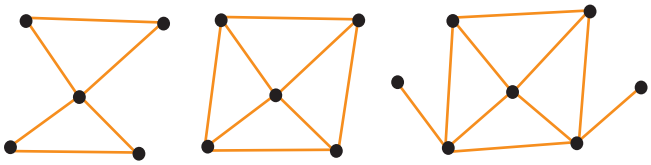


Figure 4. Non-Hamiltonian, Hamiltonian, semi-Hamiltonian

YEAR 10

If as suggested, the above content is covered prior to Year 10 then a Year 10 course can focus on some of the more sophisticated applications of weighted graphs. These could include:

- Travel applications: Postman and travelling sales people problems.
- Shortest path problems
- Maximum flow problems
- Scheduling problems and critical path analysis

CONCLUSION

The activities outlined here for Year 7 to 9 are generally accessible at those year levels and their completion would be an excellent preparation for the more demanding and time-consuming' graphs and networks' content of Years 10 and beyond. Less time covering Euclidean geometry should allow for this.

VCAA is to be congratulated for introducing graphs and networks into the senior mathematics curriculum but now is the time to extend coverage of this content into the lower year levels. An early introduction to graphs and networks can widen students' understanding of how such material might be of value to them in areas both within and beyond mathematics – such as in biological sciences, business and communications. Hopefully Australian developers of curriculum advice will look again at whether they should suggest a 'graphs and networks' element in future Year 7 to 9 courses – providing encouragement and an element of 'official legitimacy' for teachers to include coverage of the 'graphs and networks' topic.

REFERENCES

COMAP. 2015. *For All Practical Purposes: Mathematical Literacy in Today's World*. Macmillan Education

Victorian Curriculum Assessment Authority 2023 *Mathematics Victorian Curriculum F – 10*. Accessed from https://victoriancurriculum.vcaa.vic.edu.au

Victorian Certificate of Education Study Designs, Accessed from www.vcaa.vic.edu.au/curriculum/vce/vce-study-designs

Wilson R.J., *Introduction to Graph Theory.* Accessed Nov 4 2022 from www.maths.ed.ac.uk/papers/wilsongraph

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INVESTIGATIONS

MELBOURNE'S ICONIC FERRIS WHEEL by Lindy Sharkey

The Melbourne Star is the 120m tall Ferris wheel located in Docklands. It has been plagued with controversy since opening in 2008 before eventually closing permanently in 2021. A ride on the Southern Star boasted 30 minutes of 360 degree panoramic views of the city, Port Phillip Bay and the Dandenong Ranges. The Skyline Melbourne is a newly constructed Ferris wheel located at the Melbourne Convention and Exhibition Centre. It is 35m tall and the ride is 8-10 minutes with views of the city.



ROTATION RATES

Draw a diagram of each Ferris wheel. Determine the relationship between the height of the passenger cabins, the radius of the Ferris wheel and the distance travelled by the passengers.

Compare the time and speed of travel in each Ferris wheel.

As an extension, determine the theoretical travel speeds of each Ferris wheel if a passenger were to be travelling at the same maximum height of the other Ferris wheel.

MAPPING MOTION

Graph and compare the functions modelling the height of a passenger (or passenger cabin) of both the South Wharf Skyline Ferris Wheel and the Melbourne Star, extend by comparing the frequency and amplitude of the functions.

As an extension, analyse and compare ticket prices, waiting times, number of passenger carts and operating hours to determine which is the most profitable opportunity and strategies to maximise capacity.

CURRICULUM CONNECTIONS

Level 7: VC2M7M03 describe the relationship between π and the circumference, radius and diameter of a circle.

Level 8: VC2M8M05 recognise and use rates to solve problems involving the comparison of 2 related quantities of different units of measure.

Level 10: VC2M10ASP03 use the unit circle to define the simple trigonometric functions of $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$ as functions of a real variable, and graph them with and without the use of digital tools.

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