

# Level 10: What is Mathematical Modelling?

### Lesson overview

In this lesson, students review their understanding of the process of mathematical modelling. A messy real-world context is presented, and the lesson lays the foundation for students to be able to describe key aspects of mathematical modelling.

This lesson's focus is for students to articulate some of this detail *with understanding*. Hence, the focus is on **considering and discussing a messy real-world situation** rather than solving a specific task.

## Victorian Curriculum 2.0 Content Description

Use mathematical modelling to solve applied problems involving inverse proportion, growth and decay, including in financial contexts to establish the compound interest formula as repeated applications of simple interest; formulate problems, choosing to apply linear, quadratic or exponential models; interpret solutions in terms of the situation; evaluate and modify models as necessary and report assumptions, methods and findings (VC2M10A15)

### Important background to consider prior to the lesson

This lesson uses small group and whole class discussions is mainly focused on the process of mathematical modelling. It is expected students have previously solved several modelling tasks. In this lesson, the focus is on articulating thinking about the process. The expectation is conveyed to students that they do need to read information in mathematics and make decisions about what is most relevant.

The beginning of modelling is can be most difficult part, this lesson provides opportunities for student to articulate and develop their understanding of the process and some of the key language involve (which will vary depending on prior knowledge and experiences).

As mathematical modelling involves the messy real-world, in genuine modelling problems, it is important to activate knowledge of the context, preferably before considering the specific problem if one is already posed for student modellers. As the lesson continues, relevant contextual knowledge can be furthered via discussion or access to relevant resources. In the beginning and other stages of the lesson, it is important to appreciate that learner's knowledge of the context will develop.

It is important to encourage students to have the expectation that they read text as part of their mathematical lessons.

Some function or statistical related tasks, encourage students to simply test function types one by one. When the real-world is involved, as in modelling, student attention should be directed to consider what a specific function type might be used (i.e., constant rate – try a linear model. Constant rate of rate of change – try a quadratic model.)

Further information about the RTBC can be sourced via <u>https://www.swifft.net.au/cb\_pages/sp\_red-tailed\_black-cockatoo.php</u> or <u>https://www.redtail.com.au/</u> however it is recommended to use the information in the Newspaper article and other details provided in this lesson. The Age article contains a great deal of information. This lesson is a great opportunity for student to *sift* through this.

#### Materials required for the lesson

#### Student

- Looking Up in Wonder (The Sunday Age,1 Sept, 2024, pp. 25-27)
- Alternatively <a href="https://www.swifft.net.au/cb">https://www.swifft.net.au/cb</a> pages/sp</a> red-tailed <a href="https://www.swifft.net.au/cb">black-cockatoo.php</a>

#### Teacher

• Teacher resource (Supporting Resource 1) – use for teacher reference and display



Activate knowledge of the key player in the context in focus	Display teacher resource (Supporting Resource 1 – slide 1)
	Ask students what they notice, (Expect students to recognise the key player is a bird. Some students will recognise the Red-tailed black Cockatoo (RTBC))
	Play the audio ( <u>https://www.redtail.com.au/audio.html</u> that helps distinguish the calls of the RTBC and the more common yellow tailed black cockatoo.
	Facilitate by prompting: <i>Can you distinguish the calls of the RTBC and the more common yellow tailed black cockatoo?</i>
	Ask students to consider why we might be interested in this bird? (Australian animals, habitat, climate changes)
Establish the real-world is a messy place	Expanding knowledge of the real-world context.
	Provide the article Looking Up in Wonder (The Sunday Age,1 Sept, 2024, pp. 25-27) or access the website provided. Slides 3-4 includes images from article and task, to prompt your discussion as required.
	Ask students to read and record their important information. (Allow time for students to read independently and write their own important information
	Facilitate a class discussion recording key ideas on the board.
	If needed, pose these questions:
	<ul> <li>What is the problem? (The RTBC is endangered)</li> <li>What is the significance of the RTBT to Travis Lovett?</li> <li>Do all the birds look the same? (No, the male has the bright red tail, the females and juvenile birds have an orangey banded or barred tail.)</li> <li>In the photograph on page 27, how many birds can you count? Of these how many are male  barred  unable to determine?</li> <li>What are the current numbers? (1000-1500, or 1200-1300)</li> <li>What do we know about their eating habits? [Fussy, only eat seed from two types of stringybark and buloke seeds. Stringybarks have reduced by about 50% and buloke 97% in the locations where the RTBC is found.]</li> </ul>
	What <b>factors</b> are contributing to the endangering of the RTBC?
	<ul> <li>Loss of habitat</li> <li>Reduction in number of tree hollows for nesting]</li> <li>Reduction in food supply</li> <li>Bushfires</li> <li>Predators</li> <li>Illegal birding</li> </ul>
	How many birds were counted in 2024? [1303], 2023 [99 fewer, so 1204], and 2015? [1545]
	Why does one part of the article suggest 1000-1500 RTBC and another part exact values? [it is hard to count the birds, the yearly data is only an <i>estimate</i> . Some birds may be missed, others may be counted twice although attempts are made to avoid this.]



	Conclude this stage of the lesson by asking students, 'Do you agree this is a messy real-world situation?' (Expect student to agree the situation is complex and/or a <b>messy real-world situation</b> .)
Focus on the real-world situation	Facilitate a discussion, alternating whole class discussion and small groups/pair discussion, using the following as a guide.
	Explain that as part of mathematical modelling, once we <i>understand</i> the messy real-world and have <i>noticed</i> some of the complexity, we need to decide what to focus on.
	Highlight that many factors are contributing to the situation.
	To <i>simplify</i> the situation, we can focus on one aspect of the <b>real-world</b> <b>situation</b> , namely the population of RTBC over recent years. This <i>simplifying</i> of the situation helps us proceed with the mathematical modelling. We could have decided to focus on some of the factors contributing to the endangering of the RTBC, but first we will consider the number of birds.
	To reach this phase of the modelling process, we have transitioned from a messy real-world situation to a solvable real-world mathematical problem situation through processes such as <i>structuring</i> and <i>simplifying</i> . Mathematical modellers (i.e., the students) have developed an <i>understanding</i> of the situation, and have been <i>noticing</i> aspects that are relevant to a sub-problem.
	In our simplified real-world situation, we are setting aside many of the aspects involved and focusing only on some. It is important to record this thinking, that is, specify which aspects are being considered and which are not. In many modelling tasks, modellers will revisit the process after finding their initial solution. In addition, the solution is only to the simplified real-world situation.
Posing Questions	Next mathematical modellers, need to pose questions that can be answered mathematically and hence determine what sort of mathematical model will be useful in solving the problem. This involves <i>identifying variables, making assumptions, mathematising</i> and <i>justifying choices</i> in such a way that it is possible to work mathematically to an appropriate mathematical solution. The mathematical outputs should be verified as mathematically correct at this point during the modelling process.
	In the RTBC situation, we might ask, what is the population of the RTBC? This relates to concerns as to if the population is stable, declining, or recovering. Such questions are relevant when interventions are implemented to see if the interventions are making a difference.
Supporting metacognitive activity required for successful modelling	At this stage of modelling, it is important to <i>think forward</i> as to how we might use data to help answer the question. Thinking forward (and back) during modelling is important for successful modelling.
	Ask students what data might be helpful and what they might do with such data. (Expect students to suggest bird counts over time (possibly yearly).
	Expect students to suggest plotting the data (i.e., a scatterplot) which can be used to visually show how the data is changing over time and if any trends are apparent. Thinking forward here is considering 'if I have data, how can I make use of the data. We might ask questions including, if I plot the data, how might that help in solving the problem, if I plot the data what might that then allow me to do? [In this mathematical modelling task, a plot may allow student modellers to notice trends and possibly consider representing the



data via a function model. Both a plot and a function may allow us to make predictions about the future population of the RTBC and or consider how successful current interventions are.]

- When students suggest finding a function that best represents the data, probe to see if they are expecting a line/curve of good fit or an exact mathematical model (highly unlikely).
  when students suggest the plot and or approximate function may be used to make future predictions, encourage discussion about how
  - used to make future predictions, encourage discussion about how confident they would be about such predictions. [Response should relate to how closely the data follows an identifiable pattern.

In the discussion, the reasonableness of values representing the number of RTBCs should be discussed.

- Clearly the numbers used must be positive integers.
- As you can imagine, counting birds is a challenge. Hence any bird count data, no matter how carefully collected, are estimates.
- Students should consider to what level of accuracy any solution to this task should be communicated (e.g., to the nearest multiple of 10? 100? Etc)

The data provided in the article is minimal.

Looking at some data Present the slide with the data tabulated and plotted (SR1 slide 5). Two plots are presented to help student consider the importance of the domain and range (or viewing domain and viewing range). Encourage students to consider the benefits of considering different views of the same data.

- How useful is this data?
- The two plots have differing viewing ranges, what are these and how do the two plots differ in what we might notice?

In looking at data, remember the focus in this lesson is the process of mathematical modelling and hence thinking about data should focus on collection issues. How thew data might be represented and what we might reasonably infer about the real-world problem.

Focus on Interpreting Finally, the mathematical output is interpreted to establish its real-world meaning which is then used to critique the modelling so far for adequacy in solving the problem posed in the real-world situation. If the modelling is deemed satisfactory, the solution can be accepted, communicated and justified in a report. If the modelling is considered unsatisfactory, the model can be revised and the modelling process revisited.

In this real-world problem, modellers might revisit the simplified real-world situation and consider additional factors involved.

Ask student to suggest what factor(s) might be investigated next.

(Student may suggest looking at the number of nesting sites to see if there is a relationship between these and or their locations and population.

**Reflect Reflecting on** mathematical modelling by having students write, draw diagrams to 'Explain what mathematical modelling is.'