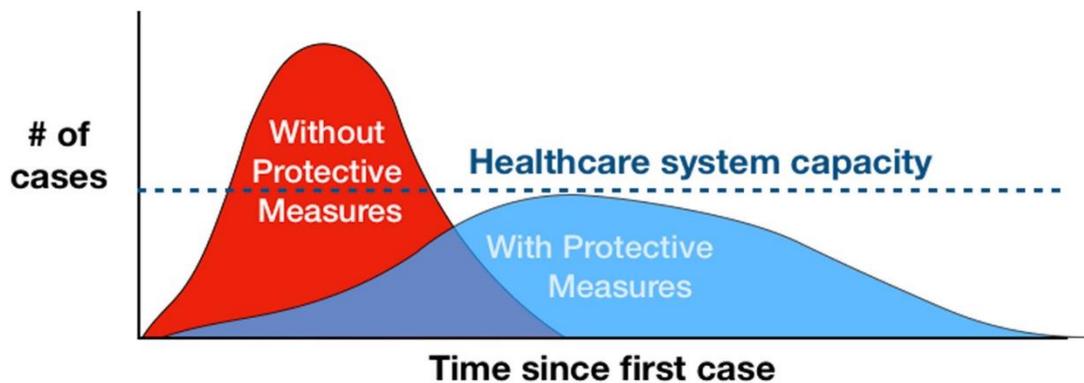




What does flattening the curve actually mean?



Adapted from CDC / The Economist



2020

AIM

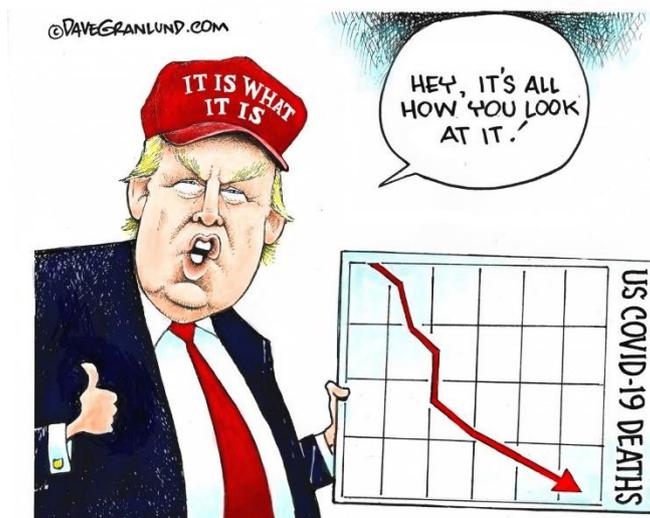
With the recent coronavirus outbreak health officials have stated the importance of flattening the curve, but what does flattening the curve mean? I am hoping to learn how the curve is calculated and predicted. How those curves represent what has happened in the COVID-19 pandemic so far. I also want to see how the safety precautions like social distancing and lockdown have affected the curve so far.

Prediction

I decided to choose a COVID-19 topic because I kept hearing how we need to flatten the curve and how the news cases are growing exponentially. I wanted to learn the maths behind how the curve was calculated and how realistic the curve actually is. I predicted at the start of looking into the curve that there wouldn't be any structure to the curve and that it is just how well we do to combat COVID-19 and as we get better at preventing the virus from spreading the curve will decrease.

Maths used

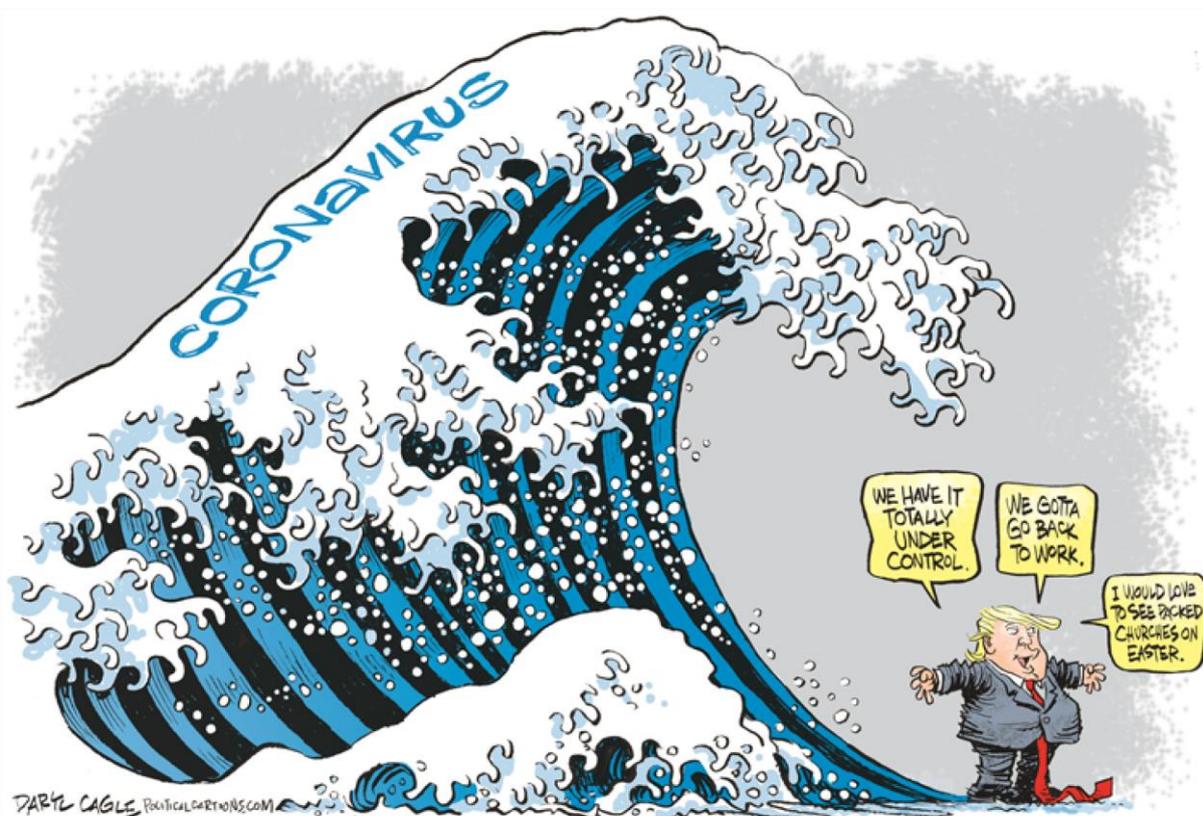
- Functions and graphs
- Algebra
- Exponent laws
- Substituting and solving
- Modelling
- Manipulating expressions and seeing what effect this has on the model
- Applications and applying real world scenarios to real world
- Interpreting results
- Recognising characteristics of data which suggest a certain exponential function is an appropriate model
- Identify relationships between functions



Approaching the investigation

To truly understand what it means by flattening the curve I had to first figure out how the curve is calculated. I approached this task by planning out the following steps:

1. Find what “the curve” is
2. Find articles that reference the curve and the maths behind the curve
3. Collect and highlight explanations that I understand and reference the maths used.
4. Search for YouTube videos that teach the concepts
5. Clarified my confusions at lunch time training
6. Gather relevant information that supports the math.
7. Broke the question down into smaller questions and focus on each question.
8. Make graphs by applying the explanations and formulas I found.
9. Write my report for my investigation and summarise my findings.
10. Peer review and check for any typos and confusion
11. Editing and finalising my investigation



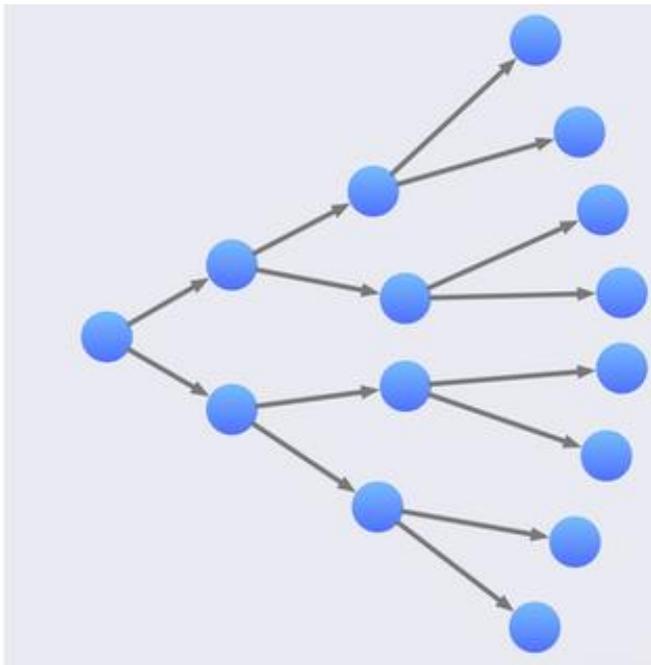
Exponential growth

In the early days of coronavirus health officials were worried about the exponential growth of the new COVID-19 cases each day. But what does growing exponentially actually mean?

A number that is growing exponential is experiencing exponential growth.

What is Exponential Growth?

Exponential growth is a way a number can increase over time. Exponential growth is when a number multiplies everyday by a constant. If the constant is 2 with a starting number of 1 then on the second day the number would be 2, then 4, then 8 and so one doubling every day.



Another way of describing exponential growth is by saying that if you give a good deed to 2 people, then they both go and give 2 different people a good deed. Then they go and give another 2 people good deeds. Before long in just 8 days 256 people will have gotten a good deed in one day. This is a lot faster than if you try to give everyone a good deed.

Formula

A very simple equation to work out how big that number will be after an amount of time is:

$$y = a^x$$

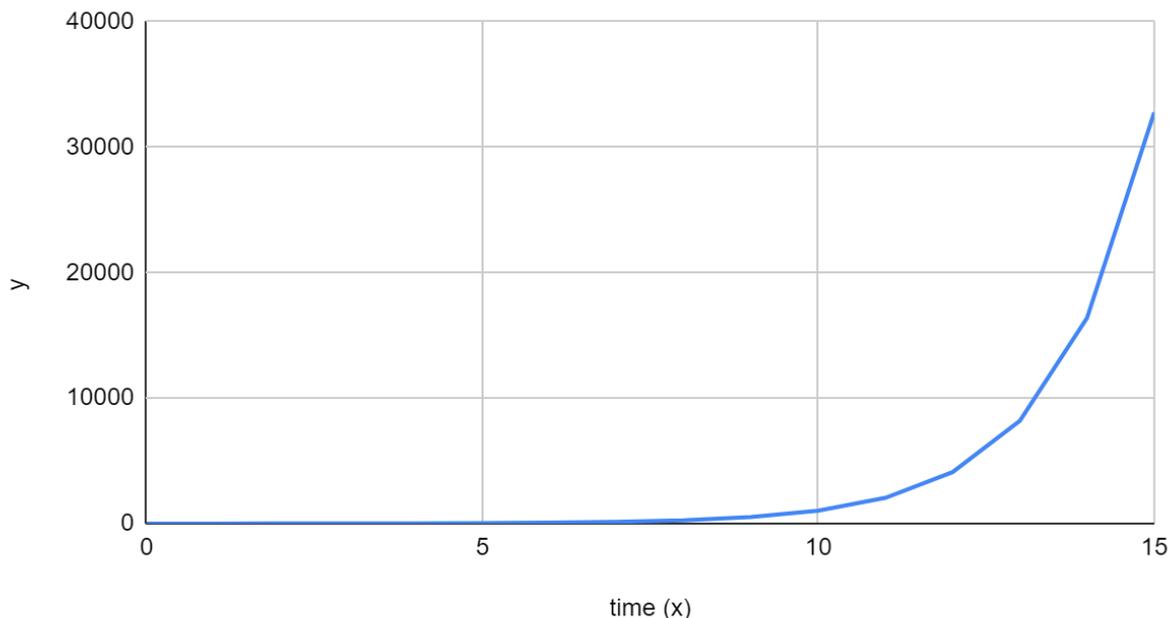
Where y is the amount of cases

a is the constant that multiplies y each day

If $a = 2$ and $x = 15$, then $y = 32768$

If we use the same formula as above and plot each of points onto a graph we should get what's called an exponential curve. Using the same formula as above and having $a = 2$. We would get a graph like below.

Exponential curve



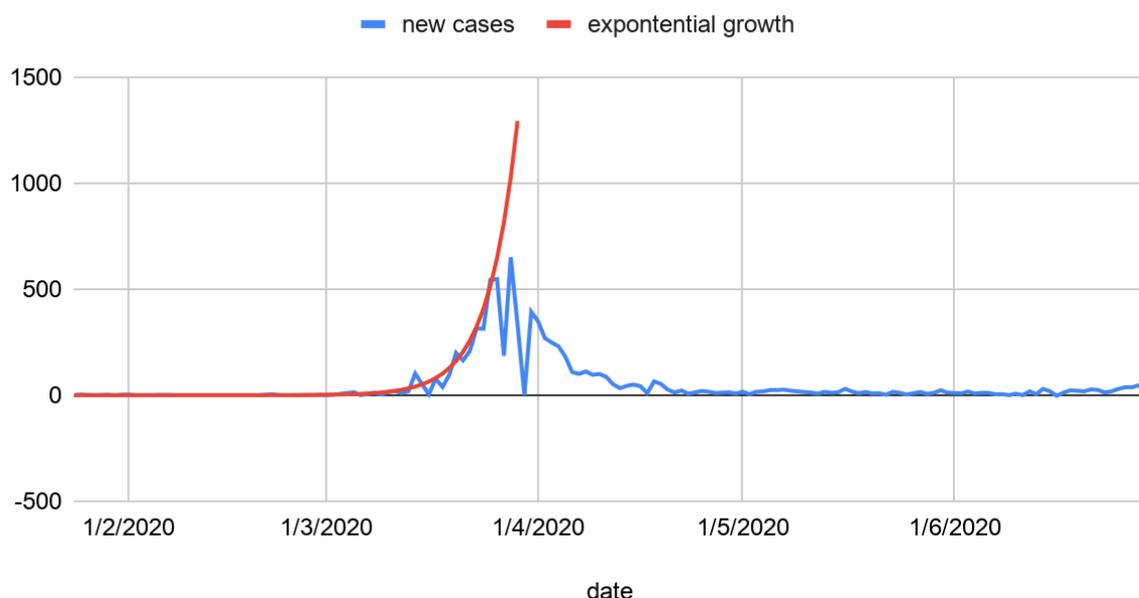
In the graph above you can see that y starts off increasing at a slow rate and the curve gradually gets steeper and steeper as time goes on. The higher the constant a the steeper the curve and the lower the constant a the slower y increases each day.

Where does an exponential curve show up in the real world COVID-19 pandemic?

There have been numerous safety precautions put in place to reduce the spread and infection rate of COVID-19 to slow the exponential growth of coronavirus like social distancing and quarantine. But have they slowed the spread of COVID-19.

If we compare an exponential curve using the same formula as shown above where $a = 1.26$ with day 0 being the 22nd of February we can see that the exponential curve closely resembles the new COVID-19 cases each day graph as shown below .

Australian daily new cases vs exponential growth



The exponential curve above resembles the initial growth of the Australian daily new cases until the 27th of March when the daily amount of new cases declines and the exponential curve doesn't fit the daily corona cases anymore.

Logistic curve

A logistic curve is much like an exponential curve but instead of continuing on increasing forever like an exponential curve a logistic curve will find a balance and level off at a carrying capacity. At the start of a logistic curve the number of cases grows exponentially while in the second half of the logistic curve the curve starts to level off and the rate of increase starts to drop off.

What is a logistic curve?

A logistic curve is a way of predicting a population growth over time..Logistic growth is very similar to exponential growth in the first half, but unlike an exponential curve a logistic curve's rate of change will slow down and will level out at a carry capacity.

Formula

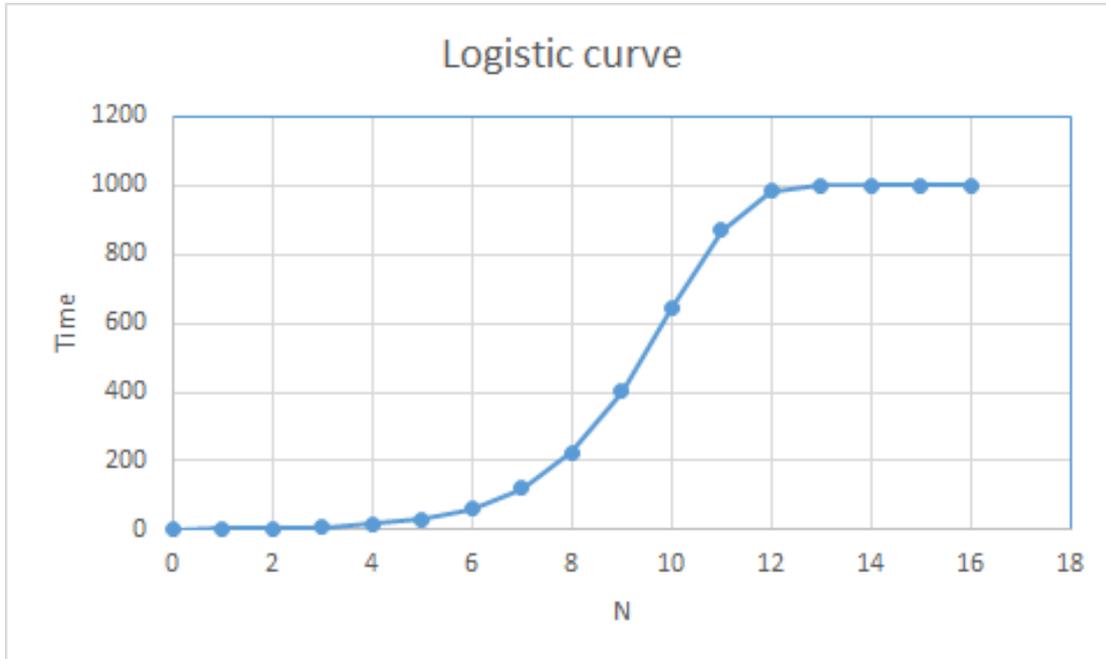
$$\frac{dN}{dt} = rN\left(\frac{K-N}{K}\right)$$

This formula calculates the rate of change of N over time

Where N is the total number or population
r is the rate of transmission. If $r > 0$ than the total number will increase in a slanted S shape. If $r = 1$ than N will double exponentially if there wasn't a carry capacity.
K is the carry capacity or the maximum amount that is supported.

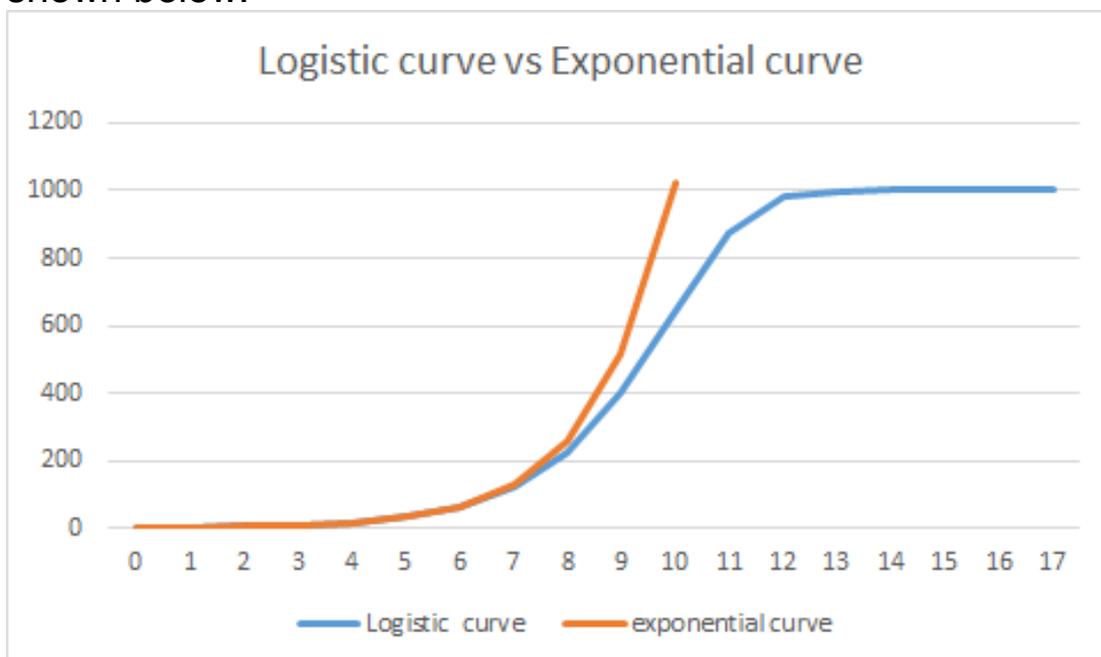
Modelling a Logistic curve

Using the formula above we can put a logistic curve onto a graph when $r = 1$, $K = 1000$ and the initial $N = 1$



As you can see N starts off increasing at a slow rate and gradually grows faster and faster like an exponential curve and then slowing down as it gets closer to the carrying capacity and then levels off when it reaches the carrying capacity.

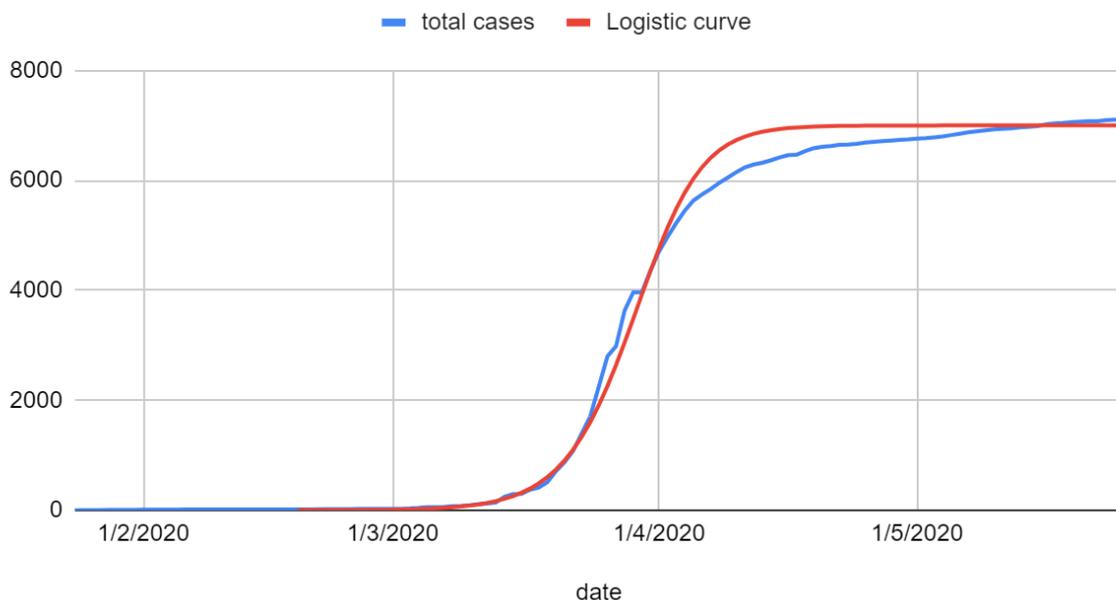
If we compare a logistic curve to an exponential curve you should see they both grow at around the same pace during the first half but a logistic curve slows down and stops and levels out. As shown below.



Where does a logistic curve come up in the coronavirus pandemic?

Unlike the exponential curve a logistic curve doesn't show up in the Australian daily new COVID-19 cases but a logistic curve does show up in the total COVID-19 cases in Australia after the first wave.

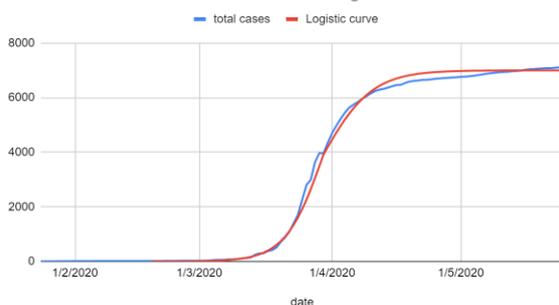
Total Australian COVID-19 cases vs Logistic curve



The graph above shows a logistic curve when $r = 0.25$, $K = 7000$ and the initial $N = 1$ compared to the total amount of cases. The logistic curve fits the total number of cases up to around April 1st.

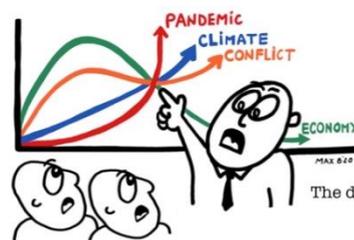
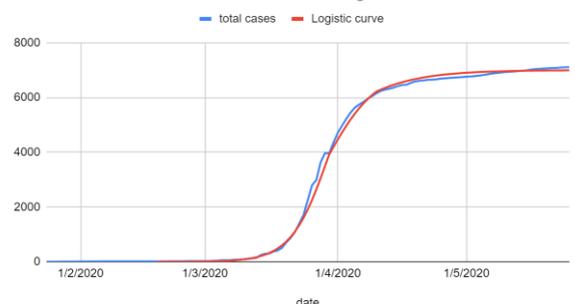
On March 30th Australia entered stage three lockdown. This reduced the transmission rate (r). If we reduce r by $.09$ on March 30th we can see that the logistic curve fits the total cases a lot better as shown below.

Total Australian COVID-19 cases vs Logistic curve



We can go even further and reduce the transmission rate when social distancing was being first being enforced by an additional 0.06 and have $r = 0.1$

Total Australian COVID-19 cases vs Logistic curve



The disasters are collaborating better than we are!

SIR model

A SIR model is used in Epidemiology to try and predict the spread of infections in a small group or population. A SIR model keeps track of the amount of people who are susceptible, who haven't already had the infection and can get the infection. A SIR model keeps track of how many people have the infection and the amount of people who have recovered from the disease and have developed immunity from the infection. Like an exponential curve and a logistic curve a SIR model also has an infection rate that is used to control the speed of the infection.

Formulas

$$\frac{dS}{dt} = -aSI$$

$$\frac{dI}{dt} = aSI - bI$$

$$\frac{dR}{dt} = bI$$

The formulas above calculates the rate of change of S, I and R over time

Where S is the amount of susceptible.

I is the amount of infectious people at one time.

R is the amount of recovered people.

a is the infection rate.

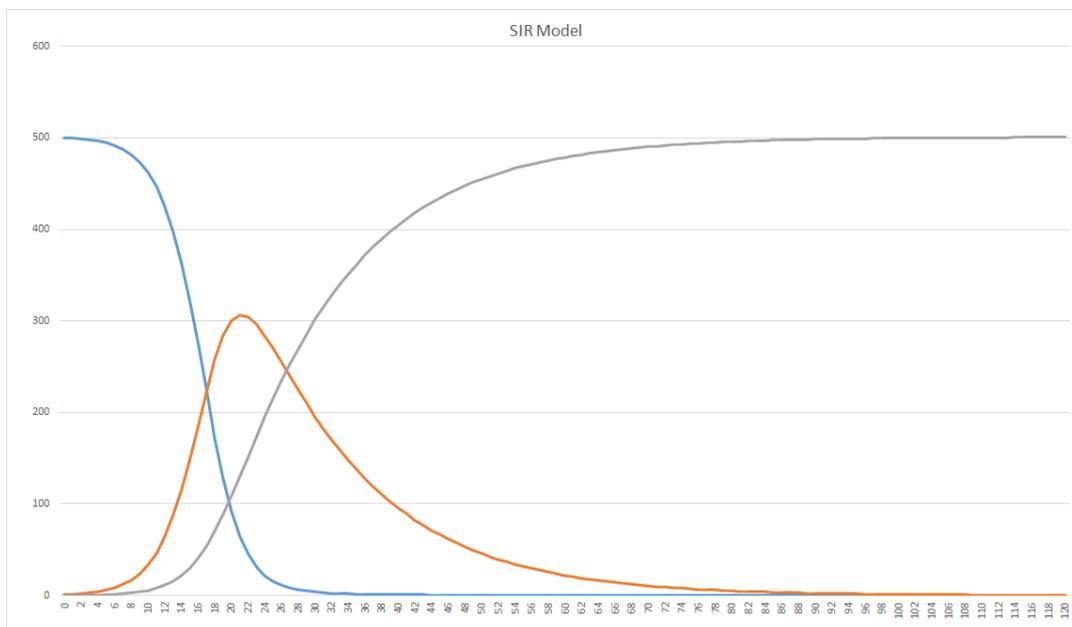
b is the recovery rate.

How a SIR model works?

A SIR model expects that everyone in the population will get the infection as long as there are some infectious people. In the formulas above there are two recurring expressions that are aSI and bI . In the formulas above aSI calculates the new infections people each day by multiplying the amount of susceptible and infected people by the infection rate (infection rate is a percentage). This works because the SIR model thinks that everyone comes in contact with everyone and every susceptible person that comes in contact with an infected person has a change to become infected (a). This number is added to the infection people and taken away from the susceptible people. The other expression is bI , this expression is the amount of infected people that recover each day. This works by having the amount of infected people divided by the recovery rate (recovery rate is a fraction e.g. $1/14$).

To show how the formulas work:
If $a = 0.001$
Initial $S = 500$,
Initial $I = 1$,
Initial $R = 0$,
And $b = 1/14$,
If we use the formula and the values above and put them onto a graph

In the chart below you can see how everyone in this population has been infected and has recovered. The amount of infected people at one point peaked at having $3/5$ of the population sick and then after the peak there was barely any susceptible people left and the new infected people was less than the amount of people recovering so the amount of infected people declines



What is a SIR model used for?

A SIR model is used to try and predict how many people are infected at one time. If we compare a SIR model using real life data we can start to see the benefits of using a SIR model. If we have the starting population of people the total number of cases after the first wave which is around 6000 people and have 1 of them infected and no one recovered and the rest of them susceptible.

$$S = 5999$$

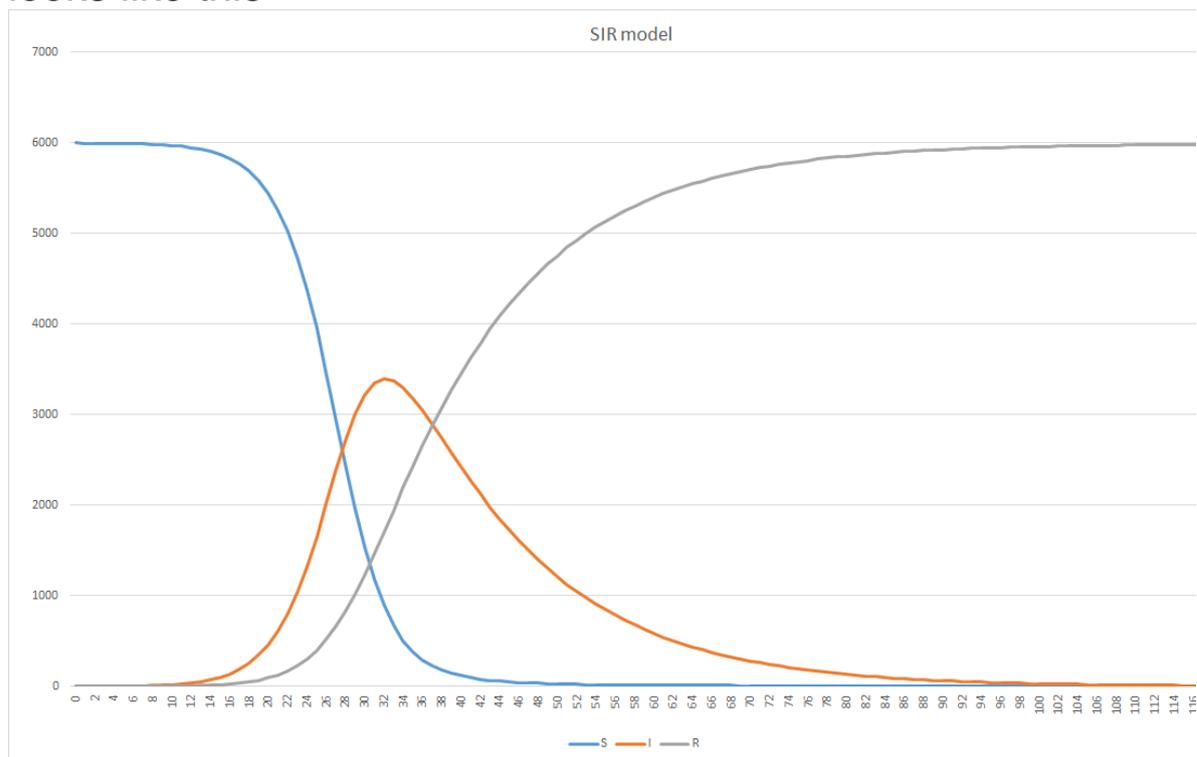
$$I = 1;$$

$$R = 0$$

$$a = 0.0000072$$

$$b = 1/14$$

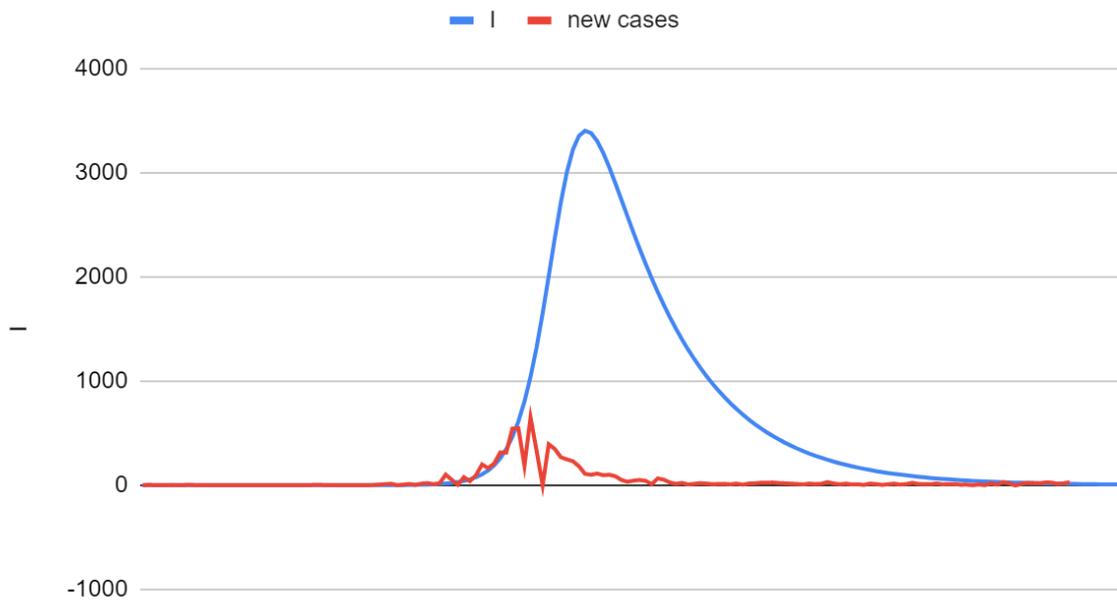
Using these numbers in a SIR model we get a graph that looks like this



SIR model compared to real world data

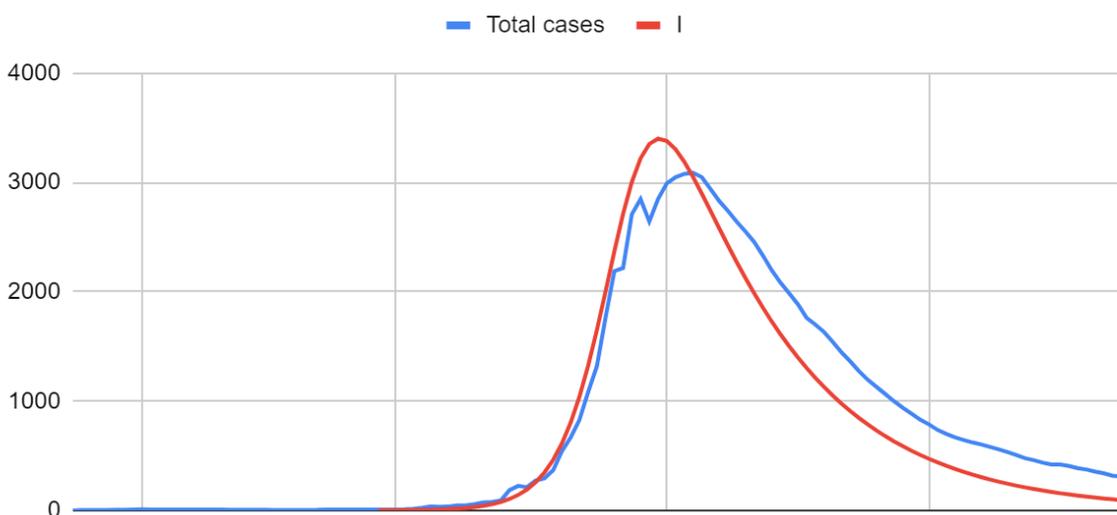
If we compare the SIR model from before and compare them to the daily new cases in Australia.

New daily cases vs SIR model



We can see the SIR model doesn't remotely resemble the new cases. This can be explained because the new cases are the new cases each day while the SIR model is the amount of infected people at one time. This can be fixed by adding the new cases and then subtracting that number by the bl like the SIR model. Than if we compare this to the SIR model.

Total cases vs SIR model

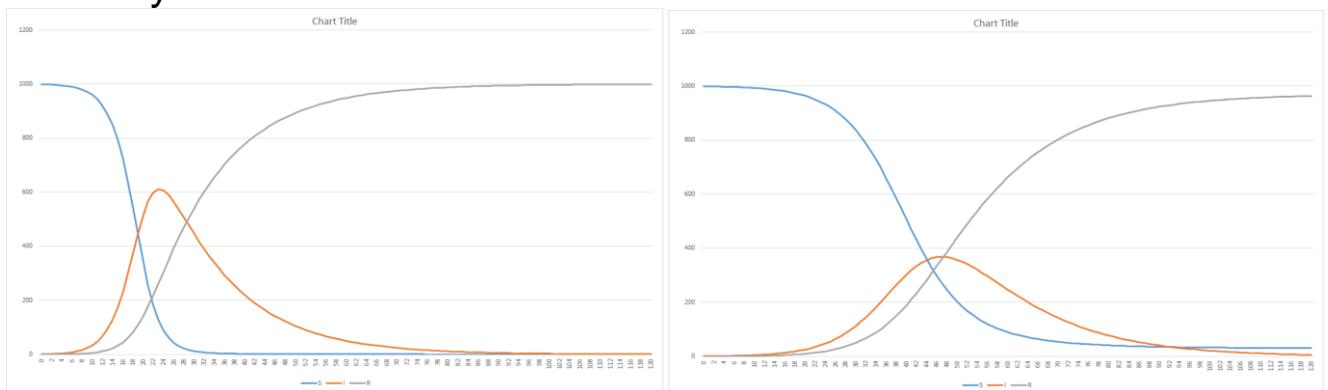


SIR model compared to real world data

As you can see in the graph above the SIR model fits the real world data a lot better, but it isn't perfect. In the graph above the SIR models started to drop off on the 27th of March which was the day that only 187 people were infected which was a noticeable dip in new cases. After 27th of March the graph follows the SIR model closely until the 30th of March which was when quarantine started. During the decline of cases the SIR model has a lot steeper decline, this could be explained by the infection rate after quarantine declining and the peak of the COVID-19 cases was lower so that more people got infected later.

What does flattening the curve actually mean?

So what does flattening the curve mean, and why is it so important? Flattening the curve basically means that you reduce the infection rate. Reducing the infection rate means that less people will be infected and sick at the same time, but the same amount of people will get sick but over a longer time. To demonstrate this below is 2 identical SIR models but the one on the left has $a = 0.5/1000$ and the one on the right has $a = 0.25/1000$ and the same infected, susceptible, recovery rate and recovered.



What does flattening the curve mean?

Above in the left graph the peak amount of infected people at one time was 610 people and in the graph on the right the peak amount of infected people was 468 people. Another noticeable difference in the two graphs is the time it takes for the infection to infect everyone, after 90 days the graph on the left has infected everyone but in the graph on the right side after 120 days only 970 had been infected. Another way to show this is how long it takes for half of the population to become infected. On the left graph it only takes 20 days and on the right side it takes 40 days to infect half the population.

But why is flattening the curve important? Flattening the curve is important because it means that less people are sick at one time and less people need help from our health care.

Hospitals can only support so many people or a carrying capacity, so if a new sudden surge from a pandemic comes in and thousands of people need to be in a hospital to have the treatment they need to survive at once it doesn't give our hospitals and health care enough time to prepare for the peak of the curve. COVID-19 has an added challenge because the patients with COVID-19 have to be separate in an isolated intensive care unit (ICU) to stop the spread in the hospitals with patients how are suffering from other diseases and problems like a heart attack or stroke who are likely to die is exposed to coronavirus. This means that there isn't infrastructure in place before the pandemic starts and if there is a massive peak of the curve there won't be enough ICU for everyone who needs it unless the curve is flatter and it gives the hospitals time to prepare for the peak. This is why it is so important to stay at home and follow restrictions so less people die unnecessarily.

Conclusion

In this investigation I have found what exponential growth is and how an exponential curve can be found in world data from Australia's coronavirus pandemic. I have found that the daily new cases in Australia closely fits with an exponential curve with $a = 1.26$. I have also explained that an exponential curve only fits during the first part when the new cases are growing, after the new cases each day starts to drop the exponential curve doesn't fit anymore. To try and fix this I then look into a logistic curve because a logistic curve does not keep increasing like an exponential curve but finds a balance at a carrying capacity. I went through the equation and how the equation works to give a logistic curve. I also went into detail on how a logistic curve shows up in real world data and how a logistic curve when $a = 0.25$ and $k = 7000$ gives a result that resembles the total cases. I went into detail on how the inconsistencies on the logistic curve and the total cases was because of precautions like social distancing and quarantine that reduced the infection rate of COVID-19.

I explained what a SIR model is and how a SIR model is calculated and how it works. I showed how accurate the SIR model was when comparing a SIR model to real world data. I then explain what flattening the curve means and why it is important.

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