THE MATHEMATICAL ASSOCIATION OF VICTORIA

Algorithmic thinking resources

Year 6: Tracking triangular numbers

In this lesson students investigate the properties of triangular numbers. This lesson follows from the lesson Scrutinsing square numbers. The students investigate the question 'How many triangular numbers might there be in the first 100 numbers?'

Level 6 - Number and Algebra | Patterns and Algebra | Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (VCMNA219)
Design algorithms involving branching and iteration to solve specific classes of mathematical problems (VCMNA221)

MATHEMATICAL LANGUAGE

Triangular number, difference, sequence, sum, product.

MATERIALS

- A large number chart on display.
- A large collection of counters.

Warm up

- Catch 22 National Literacy and Numeracy Week Resource.

LAUNCH

- Ask students to 'fish bowl' around a table.
- Using counters start to create triangular numbers. Ask students if they can think of a way to record or explain what you are doing.
- Caution: Be very careful not to be too prescriptive at the start of the lesson. If you say we are hoping to find some patterns, the students won't have a chance to explore triangular numbers and 'discover' the seeds of the pattern themselves. Instead try and stimulate curiosity. You might begin by saying nothing to the class but begin making the first couple of triangles as students sit in a circle.
- Can you make the next triangle in the sequence?

EXPLORE

- After a short time, invite students to explore this pattern.
- As the pattern emerges, ask the students to describe how they see the maths. For example: *I can see one added to the row and column each time (just like with the square numbers). I notice that the shape stays the same but gets larger each time. I notice that as the shape enlarges, you have to add more and more objects. I think there is a pattern...*
- Invite the class to consider: **How many triangular numbers might there be in the first 100 numbers?** Given what we are beginning to see with the pattern, what is an estimate that is too low? Too high?
- Ask the students to return to their work areas to explore the problem themselves.

SUMMARISE

- In the final discussion, invite students to share their solutions and strategies. Stress the importance of students being able to communicate and defend their solutions by having written and/or pictorial evidence to go with their explanations.

ENABLING PROMPTS

- Provide individual number charts to help students highlight square and/or triangular numbers (or have a few students highlight the large one).
- Work with a partner. Can you find the 10th triangular number?

EXTENDING PROMPTS

- Write a sequence of steps to help a fellow mathematician find the 20th triangular number.
- If each counter in the triangle was a 5c piece, which triangular number would give you enough money to buy a movie ticket? Find two methods to show your thinking.

SOLUTIONS

- There are 13 triangular numbers in the first 100 numbers. These are 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91.
- In continuing the sequence students may make a table to help them find the triangular numbers up to 100. A table such as this will also help them recognise and describe the pattern whereby the difference increases by 1.

	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12	+13
1	3	6	10	15	21	28	46	45	55	66	78	91

- Another way to demonstrate the sequence visually is to physically model the triangular numbers.
- Ask the students to find two ways to show their understanding and invite them to explain how the pattern works. Though mathematical 'proof' is extremely difficult to obtain, the students can reasonably conclude that they have found a pattern that works at least for the first 13 triangular numbers.

1	
3	1 + 2
6	1 + 2 + 3
10	1 + 2 + 3 + 4
15	1 + 2 + 3 + 4 + 5
21	1 + 2 + 3 + 4 + 5 + 6
28	1 + 2 + 3 + 4 + 5 + 6 + 7
36	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8

QUESTIONS TO ENCOURAGE VISUALISING AND/OR DEEPER THINKING

- What do you see? What do you notice? Can you prove it?
- What is the relationship between consecutive terms in the sequence?
- What is the rule that describes the sequence?
- Can you provide me with a convincing explanation?

SUPPORTING RESOURCES

- Fuse Resource - Triangular Numbers: Beyond Handshakes (https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=df087826-ab29-4c9e-aa74-2d7268a602f7&SearchScope=Teacher)
- BBC Bitesize - Triangle Numbers (http://www.bbc.co.uk/bitesize/ks3/maths/algebra/number_patterns/revision/5/)

EXTENDED VICTORIAN CURRICULUM LINKS MATHEMATICS

Level 5 - Number and Algebra
Number and place value

- Identify and describe properties of prime, composite, square and triangular numbers (VCMNA208)

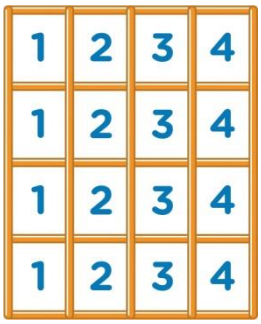
Pattern and algebra

- Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (VCMNA219)

NATIONAL Literacy and Numeracy WEEK

Catch 22

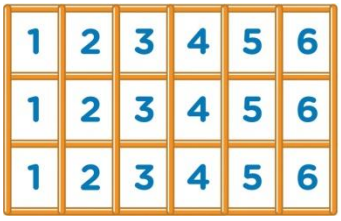
In this two-player game, students compete to reach 22 or force their opponent to go over 22. The first player crosses out a number and says the total crossed out. The second player crosses out another number, adds it to the first number and says the new total. The winner is the first student to reach 22 or the student who forces their opponent to go over a total of 22.

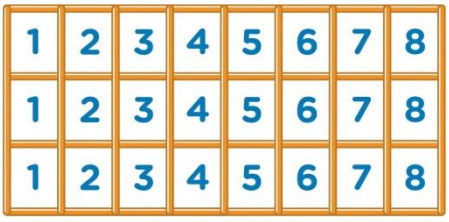


Use the numbers 1, 2, 3 and 4 as illustrated above. Playing cards and counters could also be used.

Options

- Play Catch 31 using four sets of the numbers 1 to 6.
- Play Catch 40 using four sets of numbers 1 to 8.
- Record the cumulative totals.





Considerations

- Explore the difference between being able to explain how the game is played and how to describe the winning strategy.
- Look for misconceptions that may be present in the children's descriptions of the strategy.

Key Questions

- How can you win if you make a total of 12?
- How can you win if you make a total of 17?
- Is it a good idea to start with 2? Explain your reasons.
- Is there a pattern that you can use to help you win?
- Can you explain the strategy to someone else?
- Why is the game called Catch 22?

LANGUAGE

- strategy, verify, deduce, explain, reason, game
- random, pattern

CONCEPTS

- winning strategy
- visible thinking
- problem solving, working backwards
- reasoning
- logical thinking
- chance
- number patterns
- deduction

CURRICULUM LINKS

Key Ideas - The proficiency strands are:

- Understanding
- Fluency
- Problem-solving
- Reasoning.

The proficiency strands describe how content is explored or developed; that is, the thinking and doing of mathematics.

THE AUSTRALIAN ASSOCIATION OF MATHEMATICS TEACHERS INC.