THE MATHEMATICAL
ASSOCIATION OF VICTORIA

Algorithmic thinking resources

Year 6: Scrutinising square numbers

In this lesson students investigate the properties of square numbers and seek to investigate the question 'How many even square numbers are in the first 1000 numbers?'

Level 6 - Number and Algebra | Patterns and Algebra | Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (VCMNA219)
Design algorithms involving branching and iteration to solve specific classes of mathematical problems (VCMNA221)

MATHEMATICAL LANGUAGE

Square, sum, array, product, difference, even, odd

MATERIALS

- A large number chart on display.
- A large collection of square or circular counters.

Warm up

- 2 Max National Literacy & Numeracy Week Resource.

LAUNCH

- Have the students observe you make the first two square numbers $1(1^2)$ and $4(2^2)$ using counters or cubes. Invite the students to consider, what might the next number in the sequence look like? Students may make a collection of 7, anticipating that the rule could be +3. Another student may make 16, suggesting that the rule will be x4.
- Make the third square number 9 (3^2).

EXPLORE

- Invite students to continue this pattern working individually or in small groups.
- Once the patterns have emerged, ask the students to describe how they 'see the maths'.

Invite different responses. For example: I can see one added to the row and column each time. I notice that the shape stays the same but gets larger each time. I notice that as the shape enlarges, you have to add more and more objects. I think there is a pattern...

- Once the students have had time to share and listen to different perspectives on how they see the maths, pose the following question: How many even square numbers are in the first 1000 numbers? Find two ways to demonstrate your solution.
- Students are to work in small groups to investigate the problem posed.
- Given what we know about how the pattern grows, what is an estimate that is too high? Too low?

SUMMARISE

- In the final discussion, invite students to share their solutions and strategies. Stress the importance of students being able to communicate and explain their solutions by having written and/or pictorial evidence to go with their explanations.

ENABLING PROMPTS

- How many square numbers are in the first 100 numbers?
- Provide individual number charts to help students highlight square numbers (or have a few students highlight a large number chart).

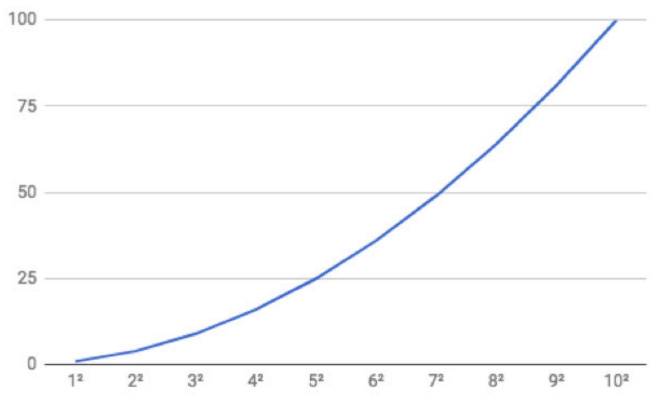
EXTENDING PROMPTS

- Can you describe the rule to create the sequence?
- Are some digits more likely to appear in square numbers than others?
- Find two or more ways to show how cubic numbers change?

SOLUTIONS

1. Continuing the sequence

- The students may identify that the difference between the numbers increases by 2 each time (e.g., the difference between 1 and 4 is 3; 4 and 9 is 5; 9 and 16 is 7). A simple table, supported by a visual representation like the one pictured above, can demonstrate understanding.
- Students can create graphs as another means to demonstrate how the sequence grows. The growing steepness of the curve of this graph demonstrates how quickly multiplicative patterns can increase in size.



2. How many even square numbers in the first 1000 numbers?

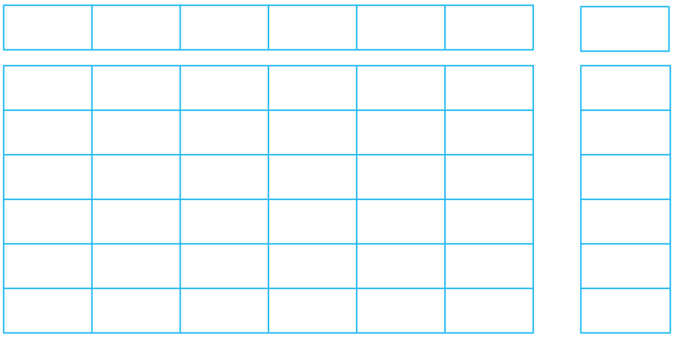
- The answer to this problem is 15.
- A 'make a table' strategy is one way to demonstrate this (pictured.
- A student that presents their solution in this way will undoubtedly have recognised that only even numbers will result in an even number when squared.
- In recognition of the odd, even sequence in square numbers, another students may recognise that if there are 31 square numbers in the first 1000 (31^2 (961) being the highest number) they may identify that this is made up of 16 odd square numbers and 15 even.

2^2	4^2	6^2	8^2	10^2
4	16	36	64	100
12^2	14^2	16^2	18^2	20^2
144	196	256	324	400
22^2	24^2	26^2	28^2	30^2
484	576	676	784	900

3. Describe the rule to create the sequence

- While the sequence is easy enough to identify and describe, the rule is somewhat more sophisticated. Your students may use the following algebraic expression to explain the rule to describe the next number in the sequence, where n is the previous number in the sequence: $(n+1)^2 = n^2+2n+1$.
  - This can be rewritten as: $(n+1)^2 = n^2+n+n+1$.
  - To show how the rule continues the sequence from 6^2 to 7^2 would be: $(6+1)^2 = 6^2+6+6+1$

A visual representation that demonstrates the rule could look like this:



The visual representation of the rule can increase conceptual understanding. Students can see the rule applied with 6^2 to 7^2 and test whether it works with all square numbers in a sequence.

4. Extending Prompt Solutions

Students will discover that square numbers can only end in 0, 1, 4, 6, 9 or 25. Conjecture and explanations about why this occurs can be found at The Math Forum (http://mathforum.org/library/drmath/view/63510.html)

QUESTIONS TO ENCOURAGE VISUALISING AND/OR DEEPER THINKING

- What do you see? What do you notice? Can you prove it?
- Can you provide me with a convincing explanation of what is happening?
- What is an estimate that is too high? Too low?
- Did you find/think of something that hasn't come up yet?

SUPPORTING RESOURCES

- Fuse Resource - Two Primes Make One Square (https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=d4d1cb38-a17d-4580-b58d-bf23798f3e1c&SearchScope=Teacher)
- Wild Maths - Odd Square (https://wild.maths.org/odd-square)
- Think Maths - Square Numbers (http://thinkmath.edc.org/resource/square-number)

EXTENDED VICTORIAN CURRICULUM LINKS MATHEMATICS

Level 5 - Number and Algebra

Number and place value

- Identify and describe properties of prime, composite, square and triangular numbers (VCMNA208)

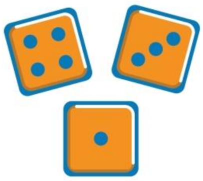
Pattern and algebra

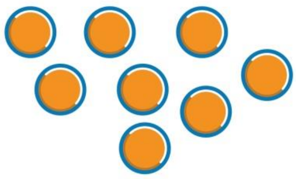
- Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (VCMNA219)

NATIONAL Literacy and Numeracy WEEK

1 2-Max

In this two-player game, students take turns to remove either 1 or 2 counters until there are none left. Whoever takes the last 1 (or 2) counters wins the game.





To decide how many counters to start with, roll a die three times adding the numbers rolled together to get the total number of counters.

Options

- Demonstrate the game by playing against a student.
- Set the number of counters for successive games.
- Ask the children to record the game play.
- Explore variations of the game:
  - Limit 3 in which players can take 1, 2 or 3 counters per turn.
  - Limit 4 in which players can take 1, 2, 3 or 4 counters per turn.
- Contrast the concept of a winning strategy to the concept of chance.
- Conduct a tournament in which the 'Limit' is randomly selected each round.
- Have a child commentate as the game is played, describing the move made and the strategy.

Considerations

- Explore the difference between being able to explain how the game is played and how to describe the winning strategy.
- Look for misconceptions that may be present in the children's descriptions of the strategy.

Key Questions

- When you get to your second-last turn, is there a way you can be sure you will win on your last turn?
  - Does it always work?
  - What is your explanation of this?
- How can you be sure to make the right second-last turn so that you will win?
  - Does it always work?
- What difference does the number of starting counters have on who can win the game?
- Can you explain the winning strategy to someone else?
- How can the second player win?

LANGUAGE

- strategy, verify, deduce, explain, reason, logic
- random, multiples, game of chance

CONCEPTS

- winning strategy
- visible thinking
- problem solving, working backwards
- reasoning
- logical thinking
- chance
- multiples
- probability
- fairness

CURRICULUM LINKS

Key Ideas - The proficiency strands are:

- Understanding
- Fluency
- Problem-solving
- Reasoning

The proficiency strands describe how content is explored or developed; that is, the thinking and doing of mathematics.