THE MATHEMATICAL ASSOCIATION OF VICTORIA

Algorithmic thinking resources

Year 3: Number paths

In this lesson students use their understanding of directional language to investigate number paths through a number grid. They use basic addition facts to complete the task.

Level Three | Number and Algebra | Patterns and Algebra | Use a function machine and the inverse as a model to apply mathematical rules to numbers or shapes (VCMNA138)

Level Three | Number and Algebra | Number and Place Value | Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation (VCMNA133)

MATHEMATICAL LANGUAGE

Total, row, column, rule, vertical, horizontal, diagonal.

MATERIALS

- Students need access to materials where they can share their solutions and strategies. Individual whiteboards are great for this purpose.
- Have the grid (pictured) displayed prominently in the room.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

WARM UP - CELEBRITY HEAD

- Allow two students to volunteer to sit on the chairs in front of the class with their backs to the whiteboard. It's a good idea to begin this game with your more confident mathematicians. These students need to be the first player to work out what their number is.
- Write two different numbers above each of the students. The idea being that the audience can see the numbers but our "celebrities" cannot.
- Players take it in turns to ask the audience a question with a yes or no answer about the number above their head.
- You might agree on a range of numbers (e.g., between 20 and 70) especially with younger students or when you are first starting out.
- Play the game with three students.
- Give each student some options that they can use at any time to help them:
- 'Phone a Friend' might allow one student to give them a clue.
- 'Number Flip' reveals one of the digits in the number.
- As an alternative, play the game as a 20 questions/ hangman style where the teacher has a secret number and the students ask yes/no questions to work out what it is. You might wish to use a number line to support student thinking. For example, if a student asks 'Is the number greater than 50?' and you answer 'Yes', you might cross out the section of the number line from 0 to 50.

LAUNCH

Introduce the grid, the key language and the rules for solving the problems that will be focused on.

Rule 1: when you travel through or land on an even number you must add all the numbers in the sequence
Rule 2: a move can be vertical, horizontal or diagonal.
Rule 3: moves must remain inside the grid unless otherwise stated.

Share an example problem with the students:
Example problem

- I started on a multiple of three.
- I moved to the east two squares and then moved one more square to end on a prime number.
- What might my total be?

Ask students to share some solutions. Demonstrate the correct path 9 → 10 → 11 ↑ 7 and the equation.
9 + 10 + 11 + 7 = 37
The total is 37.

EXPLORE

Provide the students with the Student Resource Sheet Number Paths. Read through the sheet with the students highlighting challenging language. Ask students to work in pairs to solve each problem. Encourage students to record their working after each problem. State the focus is on the process not just the correct answer.

SUMMARISE

- In the summary, invite pairs of students to share the strategies they used.
- Have the students evaluate the different strategies and explanations based on how convinced they are that the pair of students have found all of the possible solutions.
- Spend some time collaboratively trying to solve any of the problems that the students have written for each other.

ENABLING PROMPTS

- Provide individual mini grids for students to use; ask only for the path taken and not the sum.
- Provide counters to cover the path each time.

EXTENDING PROMPTS

- Use the grid to write two problems for another student to solve.
- Expand the size of the grid and/or use larger numbers (Task 1 only).

SOLUTIONS

Refer to the Student Resource Sheet Number Paths.
Problem 1. There are two possible starting numbers creating two sequences to make 20 and 36.
The two solutions can be found using a process of elimination. The move south-east immediately eliminates the fourth column of even numbers as well as the number 14 (see rule 3). This leaves 2,6 and 10. It cannot be 6 because the move south-east lands on 11, therefore the second move will move away from 11. The possible starting number could either be 2 or 10.
A sequence starting from 2 would be: 2, 7, 11 making a total of 20.

A sequence starting from 10 would be: 10, 15, 11 making a total of 36.

Problem 2. The possible finishing numbers are 1 and 3. Students may decide to make a table to locate these numbers.

Start	2	2	2	2
Step 1	1	1	1	3
Step 2	2	2	5	2
Step 3	1	3	1	1
Total	6	8	9	8

The 'working backwards' strategy can be a superior strategy for solving a problem with this structure, as it concentrates on whether a finishing number is possible as opposed to finding all of the possible ways. A student who adopts this strategy will quickly rule out numbers close to, and higher than 10. They will probably begin by looking at the smallest numbers on the grid which will help them locate 1 and 3.

Problem 3. There are four possible totals that can be made following this sequence. If the distance travelled North-East was one step then the possible totals are 21 (1,5,9,6) and 25 (2,6,10,7) and 29 (3,7,11,8). If the distance travelled North-East was two steps then the possible totals are 24 (1,5,9,6,3) and 29 (2,6,10,7,4). Though there are 5 possible sequences, and two of these five sequences make the total of 29. Hence there are four possible totals.

Problem 4. There are seven possible totals that can be made from nine different sequences. These are: 24, 26, 27, 28, 30, 32, 34. A simple table might help students organise the information (see table in right column)

Starting number	Step 1	Step 2	Finishing number	Total
3	2	7	12	24
3	4	7	12	26
3	4	8	12	27
3	6	7	12	28
3	6	11	12	32
3	7	8	12	30
3	7	11	12	34
3	8	7	12	30
3	8	11	12	34

QUESTIONS TO ENCOURAGE DEEPER THINKING

- Can you convince 1. yourself, 2. a friend, 3. a skeptic that you have found all the possible solutions?

EXTENDED VICTORIAN CURRICULUM LINKS MATHEMATICS

Level 3 - Number and Algebra
Number and Place Value

- Investigate the conditions required for a number to be odd or even and identify odd and even numbers (VCMNA129)
- Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation (VCMNA133)

Pattern and algebra

- Use a function machine and the inverse machine as a model to apply mathematical rules to numbers or shapes (VCMNA139)

Location and Transformation

- Create and interpret simple grid maps to show position and pathways (VCMMG143)

Level 3-4 - Critical and Creative Thinking
Meta-cognitions

- Examine an increased range of learning strategies, including visualisation, note-taking, peer instruction and incubation, and reflect on how these can be applied to different tasks to reach a goal (VCCCTM019)
- Investigate a range of problem-solving strategies, including brainstorming, identifying, comparing and selecting options, and developing and testing hypotheses (VCCCTM020)

Design and Technologies – Levels 3 & 4
Creating Designed Solutions

- Plan a sequence of production steps when making designed solutions (VCDSCD032)

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PROBLEM 1. START UNKNOWN

I started on an even number.
I travelled south-east. Then I travelled one more space to land on the number 11.
What might my total be?

PROBLEM 2. FINISH UNKNOWN

I started on the number 2.
I travelled across three squares to make a total less than 10.
What square might I have finished on?

PROBLEM 3. ADDING AND SUBTRACTING ODD AND EVEN NUMBERS WITH THE START AND FINISH UNKNOWN.

I started outside the grid.
I entered the grid, going South 3 spaces.
I went North-East and finished on a number within the grid.
What might my total be?

PROBLEM 4. START AND FINISH KNOWN, JOURNEY UNKNOWN

I took 3 steps to get from 3 to 12.
How many different totals could I make?