Mathematics with a spirit of inquiry

Kaye Stacey
University of Melbourne

MAV 2019 Primary and Early Childhood Mathematics Education Conference

Leading whole school improvement in mathematics education

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k.stacey@unimelb.edu.au

reSolve: Mathematics by Inquiry

- Free resources from www.resolve.edu.au
- Provide lesson plans, teaching advice, student materials, slideshows, handout masters, professional development
- Aim to assist teachers in moving towards teaching mathematics with a spirit of inquiry.
- Funded by the Australian Government Department of Education and Training; based at the Australian Academy of Science.
- Champions program run in collaboration with Australian Association of Mathematics Teachers

re(Solve) MATHS BY INQUIRY

www.resolve.edu.au

TEACHING RESOURCES | PROFESSIONAL LEARNING | OUR CHAMPIONS | OUR VISION | OUR WORK

Promoting a Spirit of Inquiry

reSolve: Maths by Inquiry is an innovative national program that promotes relevant and engaging mathematics teaching and learning from Foundation to Year 10. It is a collaboration of the Australian Academy of Science and the Australian Association of Mathematics Teachers.

The reSolve Protocol underpins all aspects of the project and sets out a vision for teaching and learning mathematics.

The reSolve: Maths by Inquiry Champions Showcase was held at the Shine Dome, Australian Academy of Science on 15 August 2018. On the right you can watch a highlights video showcasing the ten thoughtful and inspiring presentations given by Champions from across Australia and sharing the Champions' insights.

Read more about the reSolve Champions Showcase here.

To keep updated on stories about the program, join the reSolve community.

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Teaching Resources – Primary (at June 2019)

Year	Number & Algebra	Measurement & Geometry	Statistics & Probability
	Lessons	*Lessons*	*Lessons*
F	11	2	1
1	13	1	4
2	17	3	3
3	11	4	2
4	9	13	1
5	13	15	3
6	13	8	3

re(Solve) MATHS BY INQUIRY

* Includes classroom resources and special topics

Professional Learning Modules

The Professional Learning Modules provide the link between the Protocol and the Teaching Resources.

They are designed to inform and stimulate individual teacher and whole school change.

reSolve Professional Learning Modules promote a shared understanding of the reSolve Protocol and the reSolve approach to promoting a spirit of inquiry.

reSolve: Mathematics by Inquiry

This module introduces the reSolve: Maths by Inquiry project, describing its underlying principles, its key elements and looking in detail at some sample resources.

Purpose and potential

The key parts of the module include participants considering the nature of the mathematics to be learned and reviewing the way the choice of task connects to the goals of the learning.

Including all students

Participating teachers will consider the nature of difference and the challenge of including all students in the learning, and ways that such strategies can be part of planning, interacting with students, and reviewing their work.

Challenging mathematical experiences

This module presents both a rationale for and examples of challenging tasks from reSolve classroom resources that are accessible to all students and which create opportunities for them to learn mathematics.

Using student strategies to promote learning

This module examines the purposes and processes for effective use of students' strategies and solutions of mathematics inquiries to enhance their own learning and the learning of others.

Creating mathematical inquiries for your students

This module suggests four strategies that teachers can use to generalise the reSolve approaches to other mathematical foci, and hence to create their own inquiry learning sequences.

Consolidating learning

Examples of task sequences will be reviewed to consider ways that the subsequent experiences consolidate learning.

Leading a culture of excellence

This module is designed to support leaders and potential leaders to enable others to incorporate inquiry approaches into mathematics teaching repertoires.

Promoting a spirit of inquiry in school mathematics

reSolve Protocol

reSolve mathematics is purposeful
reSolve tasks are inclusive and challenging
reSolve classrooms have a knowledge-building culture

Promoting a spirit of inquiry in school mathematics

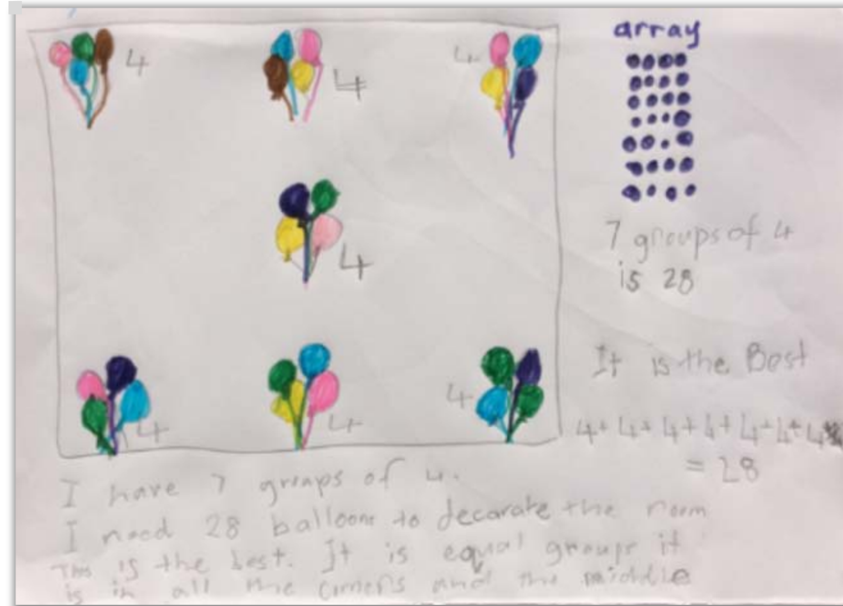
- A spirit of inquiry through all lessons – from one minute to a whole unit.
- Characteristics:
  - Asking questions, trying to find out, explaining what you find
  - Natural emphasis on problem solving and reasoning proficiencies
  - Fits ‘classrooms with knowledge-building culture’
- Varied styles of teaching within the teaching resources
  - Something for everyone to use

Example: Part-lesson inquiry
Reasoning with 2D Shapes (Yr 2)

<table>
<tr><td>There is only one way that two equilateral triangles can be joined together. As the sides are equal in length the shape will not change if different sides of the triangle are joined together.</td><td colspan="3"></td></tr>
<tr><td>There is only one way that three equilateral triangles can be joined together.</td><td colspan="3"></td></tr>
<tr><td>There are three ways that four triangles can be joined together.</td><td>
A triangle can be added to the top to create a larger triangle</td><td>
A triangle can be added to one of the sides to create a rhombus</td><td>
A triangle can be added to the base of the trapezium</td></tr>
</table>

Today's focus

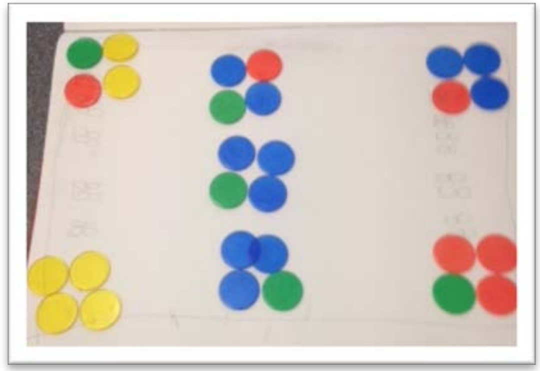
- Sustained units of work
- Mostly multi-year themes – moving maths teaching forward
- Developed by reSolve in conjunction with Australian and overseas experts
- Five sets of related teaching units
  - Bringing the Real World into Algebra (Chicken Box Patterns)
  - Modelling Motion
  - The Bar Model Method
  - Assessing Reasoning
  - Mathematical Inquiry into Authentic Problems



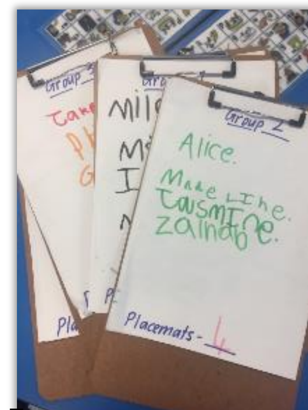
MATHEMATICAL INQUIRY INTO AUTHENTIC PROBLEMS

Created with Associate Professor Katie Makar, Dr Jill Fielding-Wells, Sue Allmond (University of Queensland)

Overview



- Aim is
  - to build students' skills in collaboratively addressing complex, non-routine problems
  - supplement the teaching of designated maths topics
  - build inquiry skills, with a focus on mathematical evidence
- 10 units, spread across F – 6

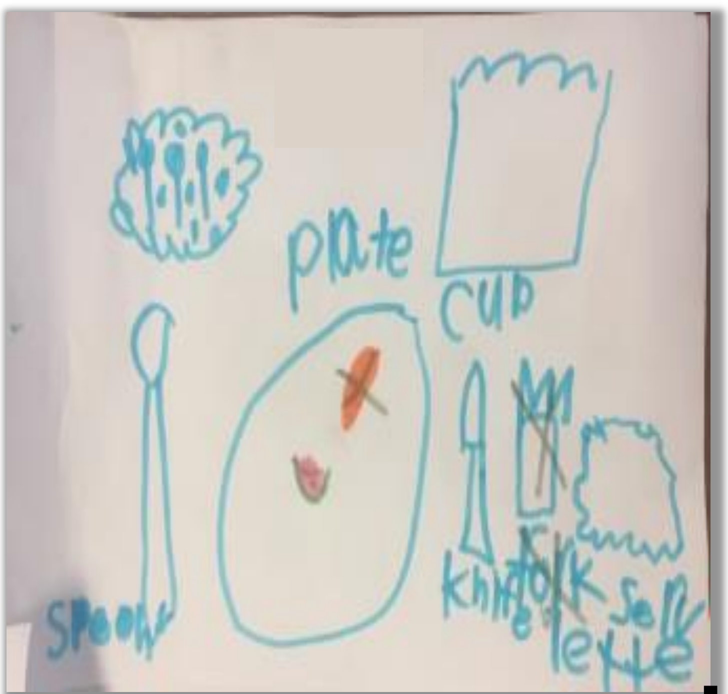


- Engaging contexts
  - assist students to link everyday knowledge to maths
  - improve access for students at different performance levels

- Provides a model for "inquiry learning" that is appropriate for teaching mathematics

Foundation – Tea Party

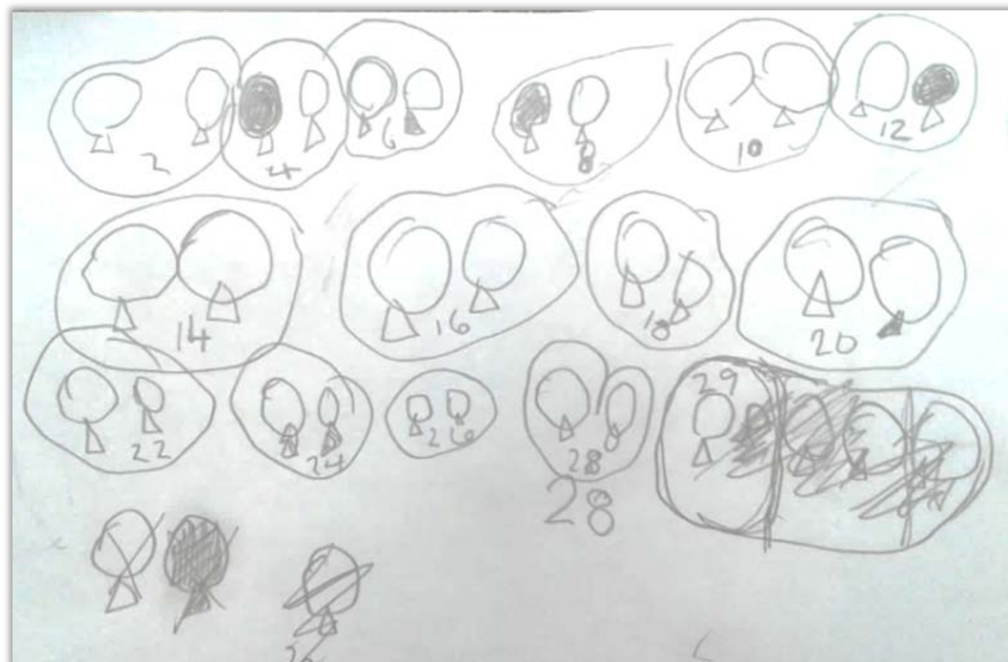
- **Foundation**: How can we plan a tea party for our friends?
- **Focus**: counting, one-to-one correspondence



Year 2 – Bunches of Balloons

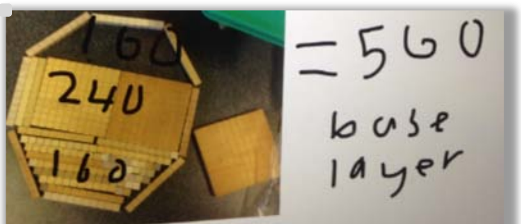
- **Year 2**: What is the best way to decorate our room with bunches of balloons? (Packet of 29)

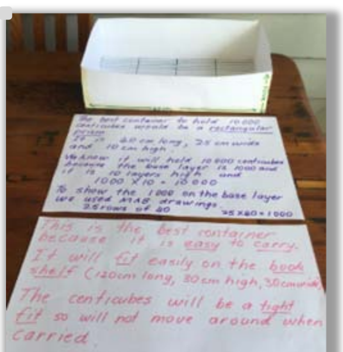
- **Focus**: arrays, division into equal groups

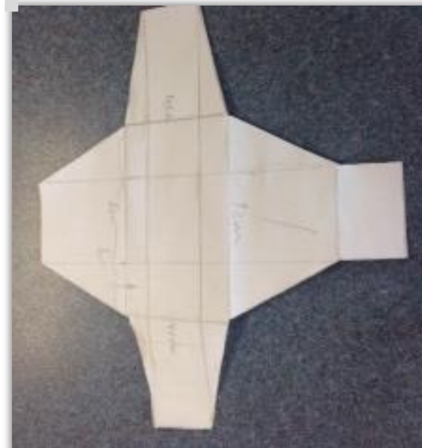


Examples: Upper Primary

- **Year 4**: What is the best container to hold 10 000 centicubes?
- **Focus**: partitioning large numbers, volume





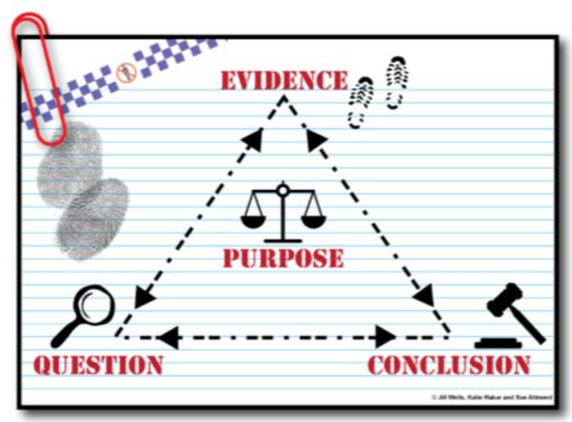


- **Year 6**: What is the best box to hold 2 different sized items that are packaged as pyramids?

- **Focus**: prisms, pyramids, nets

Features of Guided Inquiry

- Explained in the lesson plans and in the Teacher's Guide
- Many examples shown for creating a classroom with a knowledge-building culture
  - Teaching students to listen attentively and offer constructive feedback
- Focus on gathering and presenting mathematical evidence, and justifying conclusions
- Lesson plans feature regular checkpoints
  - Accountability for students
  - Chance to get feedback and help if stuck
  - Ensures a direction in students' inquiry
  - Examines quality of mathematical evidence



Different places teachers found to use for the 1 second drop

MODELLING MOTION

Created with Associate Professor Susie Groves, Dr Brian Doig, Dr John Cripps-Clark (Deakin University) and Professor Julian Williams (University of Manchester, UK)

Overview - this is STEM !

- 7 well-documented lessons for Years 5 or 6
- Very strong links to Science
- Contexts include rolling balls, falling balls pulleys
- Careful development of mathematical aspects of speed and acceleration/deceleration
- Touches on friction, gravity, pulleys and Newton's First Law of Motion

A typical collection of Purple graphs

Instead of rolling to the end, the ball will stop just before the end because of friction.

Ribbon	Distance rolled
Carpet	58 cm
Streamer	53 cm
White Ribbon	70 cm
Yellow Sports Band	85 cm

A sample completed table

The ribbon slows the ball down by gripping the ball down, creating friction.

Modelling Motion Lessons

- Lesson 1: Constant Speed
  - Students challenged to walk at a constant speed.
  - Use ‘streamer graphs’ to represent distances walked per second.
  - Discover that faster speed corresponds to travelling a greater distance in unit time.
- Lesson 2: Rolling Downhill
  - Use streamer graphs to find what happens to the speed of a ball rolling downhill.

Measuring time with stopwatches and tablets

- Lesson 3: Falling Balls
  - Students find the height from which a ball needs to be released for it to take 1 second to hit the ground. (Predict from lower heights)
- Lesson 4: Rolling Uphill
  - Students discover what happens to the speed of a ball when it rolls uphill and when it rolls along the track covered in ribbon.

Modelling Motion Lessons (continued)

- **Lesson 5: Measuring Forces**
  - Students make and calibrate their own forcemeter and use it to measure pushes and pulls.
  - Students discover what happens when two forces pull in opposite directions and draw force diagrams to represent the results.
- **Lesson 6: Modelling Force and Motion**
  - Students investigate the motion of a car pulled in opposite directions.
- **Lesson 7: Complex Motion**
  - Students investigate the motion of a ball in two dimensions, as it travels across a sloping table. They trace its path on butcher’s paper. They change the angle of the launcher to make the ball hit a specified target on the table.

re(Solve) MODELLING MOTION

Modelling Motion – Speed
Lesson 1: Walking at Constant Speed

Name: ____________________

Can you walk at constant speed?

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Resources supplied

- 4-page student workbook for each lesson
- Detailed lesson notes, including equipment lists with advice on purchase, practical advice for conducting lessons, sample student work etc
- Explicit links to Australian Curriculum Mathematics and Science

Walking at a slow constant speed

1. Lay out the Red streamer in a long straight line for the walk.
2. Your teacher will choose a person to be the Walker, some people to be the Markers, and a person to be the Timer.
3. The Walker should practise walking at a slow constant speed.
4. Each Marker needs a block. The Markers line up beside the streamer. Everyone needs to know their place in the line.
5. The Timer starts the metronome and says 'Ready! Set! Go!' in time to the ticks.
6. The Markers put down the blocks when it is their turn – they must put them down exactly where the walker is when the metronome ticks.
7. Repeat until everyone is happy with the way the blocks are being placed.
8. Use the Red paper streamer to record the distances between the blocks.
9. Number the strips in order before cutting the streamer.
10. Make a streamer graph using butcher's paper and glue-sticks to glue the streamers onto the paper.
11. Sketch the Red streamer graph in the space on the next page. (Only use the bottom half of the space.)
12. Add a title and label the axes on your Red streamer graph.
13. Use this space to sketch what you think the graph for constant speed should look like.

Walking at a faster constant speed

14. You are going to make a Blue streamer graph for someone walking at a faster constant speed.
    Do you expect the length of the Blue streamers to be:
    shorter ☐ longer ☐ or about the same ☐ as the Red ones?
15. Sketch your predictions for the Blue streamer graph next to your sketch of the Red Graph.

Red Graph	Prediction for Blue Graph	Actual Blue Graph

16. Using different people from last time, make a Blue streamer graph for a faster walker.
17. Sketch the actual Blue Graph in the space above.
18. Use the space below to explain how you could use graphs like these to find out who is walking faster.

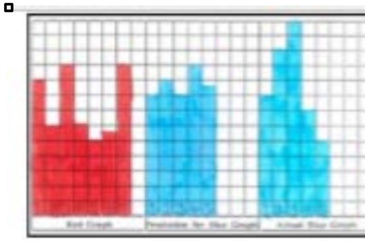
19. Another class doing this activity produced the graph shown.

    In which second was the student walking fastest?

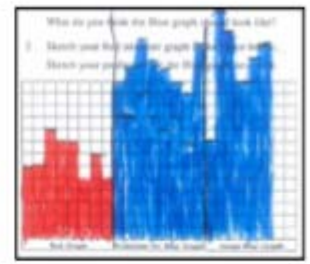
20. Use the space below to explain how you could use graphs like these to find the speed at which someone was walking?

21. Write down what you have learned about graphs by doing this activity.

Sample student work in lesson plans



A prediction showing similar heights for Red and Blue strips



A prediction showing longer Blue strips

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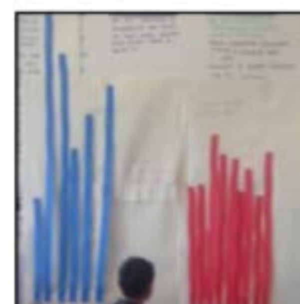

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Comparing the Red and Blue Graphs

BRINGING THE REAL WORLD INTO ALGEBRA: CHICKEN BOX PATTERNS (YEARS 5 &6)

Created with Associate Professor Robyn Pierce and Dr Duncan Symons (University of Melbourne)

Lesson 1: A single row of bird boxes (sample slides)

A Row of Chicken Boxes

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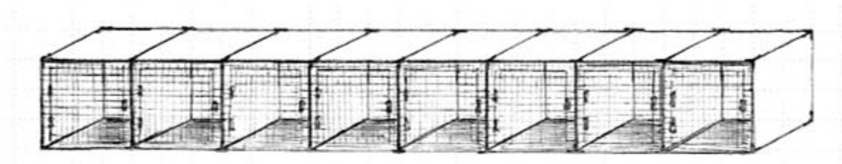

- How many wire screens do we need for the row?
- How is this number related to the number of boxes?
- Think about how many solid panels we need for the row (top, base, back, left and right sides). Why is this harder to work out?
- First, how many back panels do we need?

Two Different Types of Rules

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- One type of rule tells us how to get from one step to the next. If we know how many toothpicks for 3 squares we can find out how many for 4 squares.
- Another type of rule takes us straight from the number of squares to the number of toothpicks. If we know we want 200 squares we can find the number of toothpicks quickly.

What Have We Learned?

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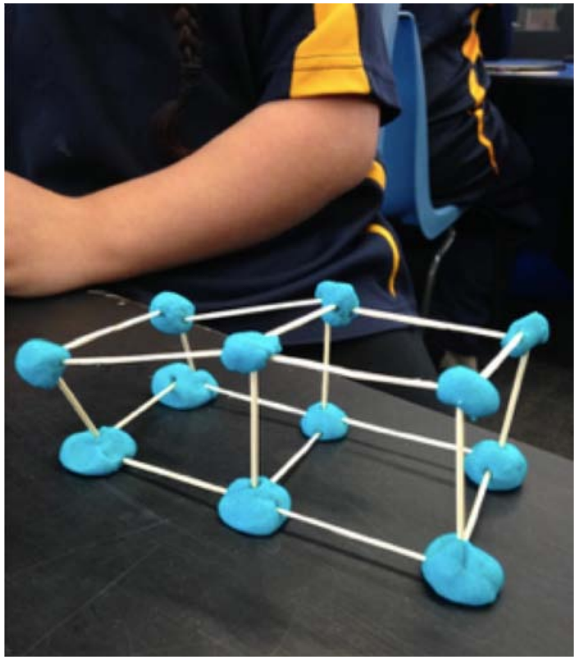
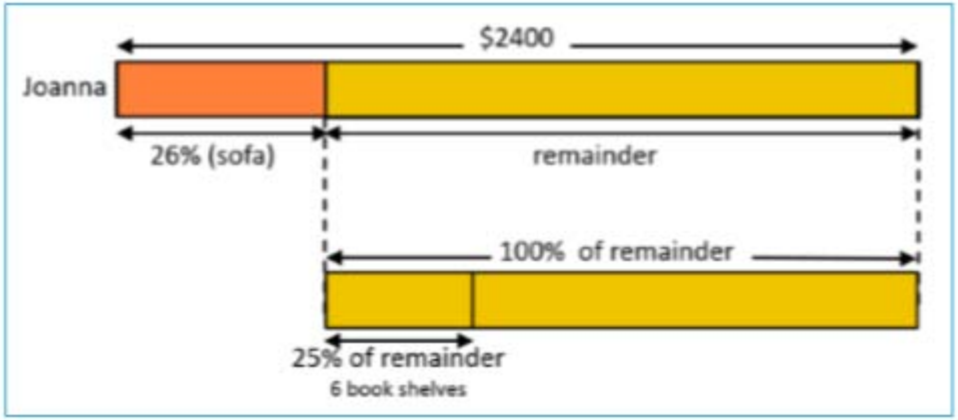
- I can find patterns in sequences of numbers.
- I can describe a rule that continues a pattern.
- I know that there can be different correct rules for describing one pattern.
- Some rules work step-by-step and other rules go straight to the answer.

Year 5/6: Chicken Boxes

- Lesson Sequence
  - Lesson 1: A single row of bird boxes
  - Lesson 2: Box designs with other shapes
  - Lesson 3: An array of chicken boxes
  - Lesson 4: 3D chicken boxes
- Main ideas - pre-algebra
  - Seeing patterns in numbers and in structures
  - Describing the pattern in a general way
  - Explaining the link between the pattern and the structure
  - Writing a general rule





Bar model for finding 25% of 74% of $2400.

THE BAR MODEL METHOD

Created with Associate Professor Lee Ngan Hoe, Dr Ng Kit Ee Dawn, Dr Cheng Lu Pien, Dr Yeo Kai Kow Joseph and Dr Paul Maurice Edmund Shutler

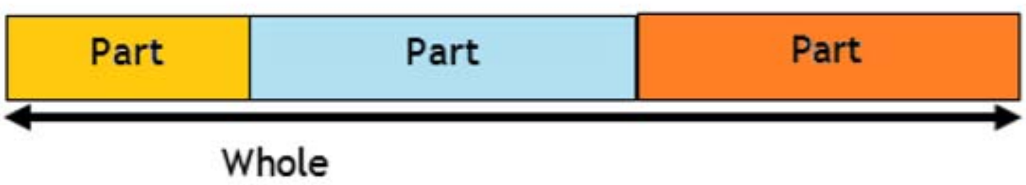
(National Institute of Education, Singapore)

The Bar Model Method

- A pedagogical strategy that is widely used in Singapore to help students solve word problems.
  - Highly visual.
  - The Concrete-Pictorial-Abstract (C-P-A) pedagogical approach – introduced because of the multiple languages in Singapore schools
  - Useful across many topics in Number and for introductory algebra
- Problem solving method that
  - Displays and organizes the information in a problem
  - Displays the hidden relationships in the problem
  - Reduces cognitive load
- Three units, for Years 5, 6 and 7/8
- Each unit takes about 8 hours – can be spread across year
- Very detailed resources, including problem sets and slideshows with problem solutions for each lesson
- Units 2 and 3 aim for creativity and use Polya's four steps for problem solving

Unit 1:
Introduction to the Bar Model Method

- Whole Number Problems
  - addition, subtraction, equal groups multiplication, partition and quotition division
- Fraction problems
  - Addition and Subtraction



Part-whole Model

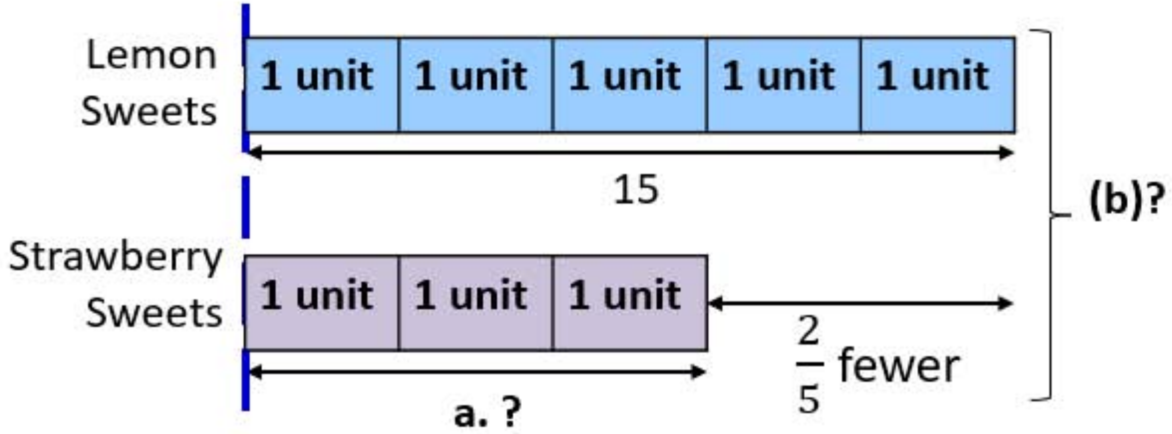
Comparison Model

Example 4 (Solution)

Jennifer has 15 lemon sweets. She has $\frac{2}{5}$ fewer strawberry sweets than lemon sweets.

a. How many strawberry sweets does Jennifer have?

b. How many lemon sweets and strawberry sweets does Jennifer have in total?



5 units = 15

1 unit = 15 ÷ 5 = 3

3 units = 3 x 3 = 9

a. Jennifer has <u>9</u> strawberry sweets.

15 + 9 = 24

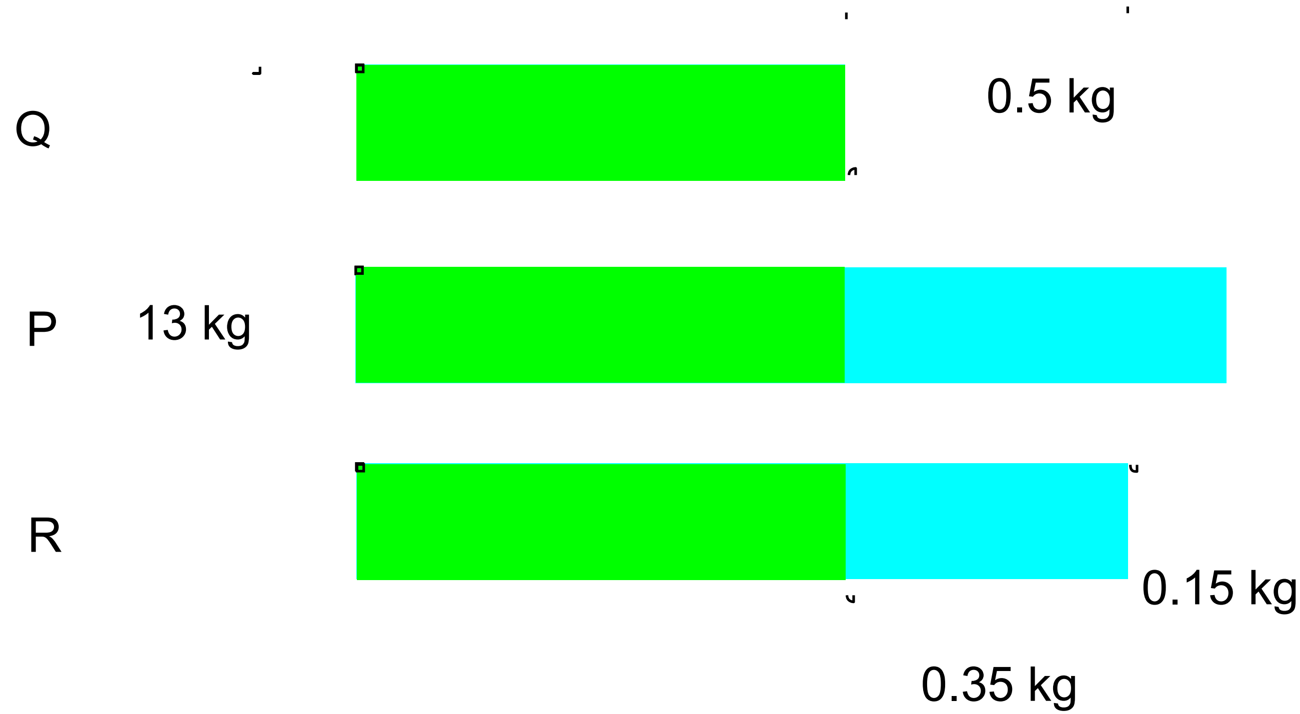
b. Jennifer has <u>24</u> lemon sweets and strawberry sweets in total.

Example:

The total mass of three parcels P, Q and R is 13 kg. P is 0.5 kg heavier than Q and 0.15 kg heavier than R.

a. How much heavier is R than Q?

b. What is the mass of Q?



a. R is heavier than Q by (0.5 – 0.15) kg, i.e. 0.35 kg.

b. 3 units = (13 – 0.5 – 0.35) kg
3 units = 12.15 kg
1 unit = 4.05 kg
So, Q weighs 4.05 kg

Connecting the Key Reasoning Actions

ASSESSING REASONING

Created with the Mathematical Reasoning Research Group at Deakin University (Professor Colleen Vale, Dr Wanty Widjaja, Dr Esther Loong, Dr Leicha Bragg, Dr Sandra Herbert and Ms Aylie Davidson)

Shapeshifter: The Lesson

Introducing "The Greedy Triangle"

- Read or listen to "The Greedy Triangle" by Marilyn Burns, Syd Hoff and Gordon Silveria (1995 Scholastic Press). There are several readings of the book on the internet e.g. https://www.youtube.com/watch?v=kPuI4XyyZUE
- This lesson is best conducted by students working in pairs but it can also be done in small groups.
- Record the names of the two-dimensional shapes and discuss the properties of these two-dimensional shapes.

Reasoning Task

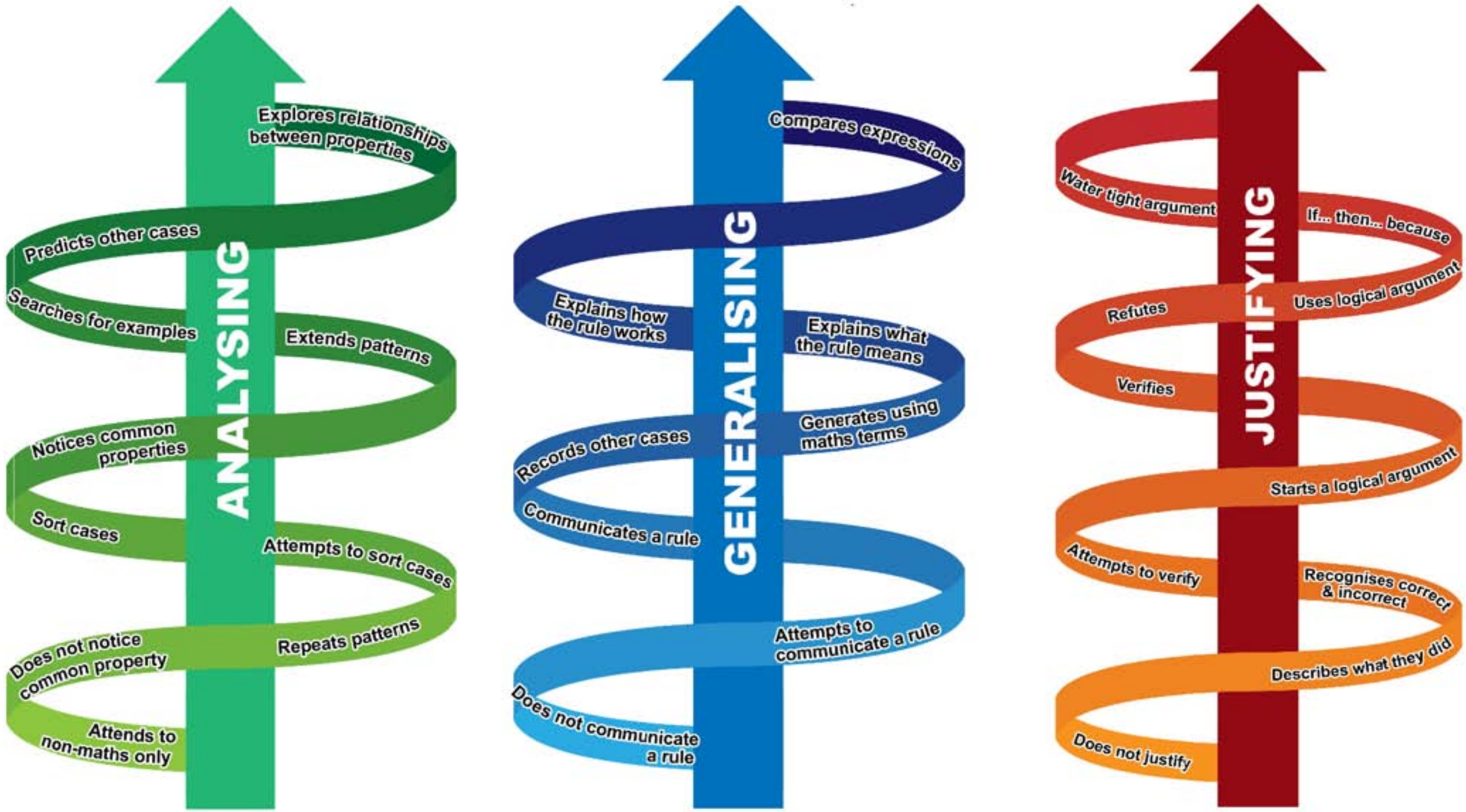
Imagine you are the Shapeshifter in "The Greedy Triangle". You have only one tool that you can use. It cuts one straight line only. Work out **a rule** for cutting shapes that the Shapeshifter could use to change the triangle to a quadrilateral and then to a pentagon and so on. Create a poster to show that your rule works and explain why your rule works.

- Hand out Student Sheet 1 - Shapeshifter and multiple triangles for students to use for their trials. Triangles can be cut from coloured paper (use a variety of shapes), or printed in greyscale or colour from Triangles for Printing.
- Encourage students to look at all their shapes and compare and contrast the shapes they have created by cutting in different orientations.
- Students can cut triangles, or fold, or just draw lines with a ruler. They need to keep the pieces of any split triangles together in order to compare and record the outcomes. Sometimes students end up with a pile of shapes and do not know which two started off as joined together. One way to get around this problem is to encourage students to explore by folding or drawing and then when they have found the rule to use the scissors to cut and then paste onto their poster.
- When roaming the classroom prompt students to tell you what they think the rule is and explain it. This will help students to verbalise the rule to make a quadrilateral (e.g. "To make a quadrilateral, cut a corner

Professional development resource with classroom exemplars

- Aims
  - to improve teachers’ capacity to notice and assess students’ reasoning
  - develop greater insights into the relationship between students' reasoning and mathematical learning
  - identify areas of reasoning for classroom or individual attention
- Resources for use by teachers as part of their everyday planning, teaching and assessment practices
  - Reasoning assessment rubric
  - Reasoning prompts
  - (8 exemplar tasks with lesson notes, Years 3 – 6)
- Professional learning resources
  - Slideshow for presenter, with audio
  - Quick guide (handout) to reasoning learning trajectories for teachers, detailed guide for presenter

Reasoning Trajectories



Rubric – full and abbreviated

Student Name: Reasoning Task: Date:

Observation of student's reasoning:

	ANALYSING	GENERALISING	JUSTIFYING
NOT EVIDENT	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify.**
BEGINNING	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
DEVELOPING	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
CONSOLIDATING	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
EXTENDING	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument.** • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

	Analysing	Forming Conjectures and Generalising	Justifying and Logical argument
Not evident	• **Does not notice** numerical or spatial structure of examples or cases. • Attends to **non-mathematical** aspects of the examples or cases.	• **Does not communicate** a common property or rule for pattern. • **Non-systematic** recording of cases or pattern. • **Random** facts about cases, relationships or patterns.	• **Does not justify**. • **Appeals** to teacher or others.

Beginning	• **Notices** similarities across examples
	• **Recalls** random known facts related to the examples.
	• Recalls and **repeats patterns** displayed visually or through use of materials.
	• Attempts to **sort** cases based on a common property.
		• Uses body language, drawing, counting and oral language to **draw attention** to and **communicate**:
		  ○ a single common property
		  ○ repeated components in patterns.
		• **Adds** to patterns displayed verbally and/or visually using diagrams or through use of materials.
			• **Describes** what they did and why it may or may not be correct.
			• **Recognises** what is correct or incorrect using materials, objects, or words.
			• Makes **judgements** based on simple criteria such as known facts.
			• The **argument may not be coherent** or include all steps in the reasoning process.

Developing	• **Notices** a common numerical or spatial property.
	• **Recalls**, **repeats and extends** patterns using numerical structure or spatial structure.
	• **Sorts and classifies** cases according to a common property.
	• **Orders** cases to show what is the same or stays the same and what is different or changes.
	• **Describes** the case or pattern by labelling the category or sequence.
		• **Communicates** a rule about a:
		  ○ *property* using words, diagrams or number sentences.
		  ○ *pattern* using words, diagrams to show recursion or number sentences to communicate the pattern as repeated addition.
		• **Explains** the meaning of the rule using one example.
			• **Verifies** truth of statements by using a common property, rule or known facts that confirms each case. May also use materials and informal methods.
			• **Refutes** a claim by using a counter example.
			• Starting statements in a **logical argument** are correct and accepted by the classroom.
			• **Detecting and correcting errors** and inconsistencies using materials, diagrams and informal written methods.

Sample assessment of 'Magic V'

- several samples of student work
- each assessed with explanation given

Sample assessment

A Magic V is a number puzzle. The aim is to arrange five consecutive numbers so that the sum of each 'arm' of the V is the same. Below are two Vs. The left V is a Magic V (4+2+3=9 and 5+1+3=9); the V on the right is not. Each number can be used only once.

Sam said, "It is impossible to make a Magic V with an even number at the bottom with the set of numbers 1 to 5."

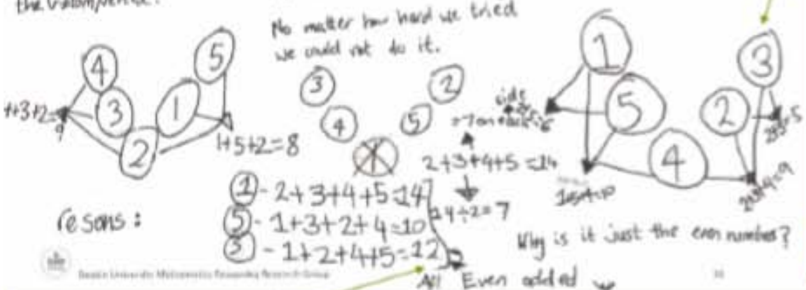
Is Sam right? Explain why or why not.

JUSTIFYING: Starting statements in a **logical argument** are correct and accepted by the class.

ANALYSING: Sorts and classifies according to a common property.

The student is analysing the total of the arms and sorting according to even and odd numbers at the vertex.

Sam said "It is impossible to make a Magic V with an even number at the bottom with the set of numbers 1 to 5".
Is Sam right? Explain why or why not? [You can use sentences, number sentences and drawings in your explanation.] We both think Sam is right/correct.
When we tried to make the V with the 2 or the 4 at the vertex We couldn't find a way to make a magic V with the 2 or the 4 at the bottom/vertex.
No matter how hard we tried we could not do it.



ANALYSING: Developing
GENERALISING: Beginning
JUSTIFYING: Developing
Teacher Prompt:

You have made a very interesting remark that if we have an even number at the bottom then the total of the two arms are not equal.

Is it true for all different Magic V with 2 and 4 at the bottom? Why?

What do we need to have at the bottom to make the two arms equal?

JUSTIFYING: Attempts to verify by testing cases or explaining the meaning of a conjecture using one example.

GENERALISING: Detecting and correcting errors and inconsistencies using materials, diagrams and informal written methods.

The next step is for the student to think about the properties of odd and even numbers.

Student Name: Work Sample 2 **Reasoning Task:** MAGIC V **Date:**

Observation of student's reasoning:

* Adding total of arms & comparing 'Vs' with an odd/even number at vertex.
* developing a logical argument but no 'why'

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and ✓ **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct. ✓
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases. * 'what will...'	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. * • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument.** • **Verifies** that the generalisation holds for all cases using logical argument.

Comments (feedback, reasoning prompts for further development):

* Look closer at properties of odd/even numbers
* Develop justification → look at reasoning prompts

Draft Levels of Reasoning & Rubric, Mathematical Reasoning Research Group, Deakin University & ReSolve STS, AAMT & Academy of Science, December, 2018

Reasoning Prompts – poster or cards

re(Solve) MATHEMATICAL REASONING PROMPTS

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

ANALYSING

- What is the same and different about ...?
- What stays the same and what changes?
- Sort or organise the following according to ...
- Alter an aspect of something to see an effect. If we change this what will happen?
- What follows from this? What do you think will happen next if we do this?
- What do you notice...?
- When is it true?
- Is it just sometimes true, or is it always true?

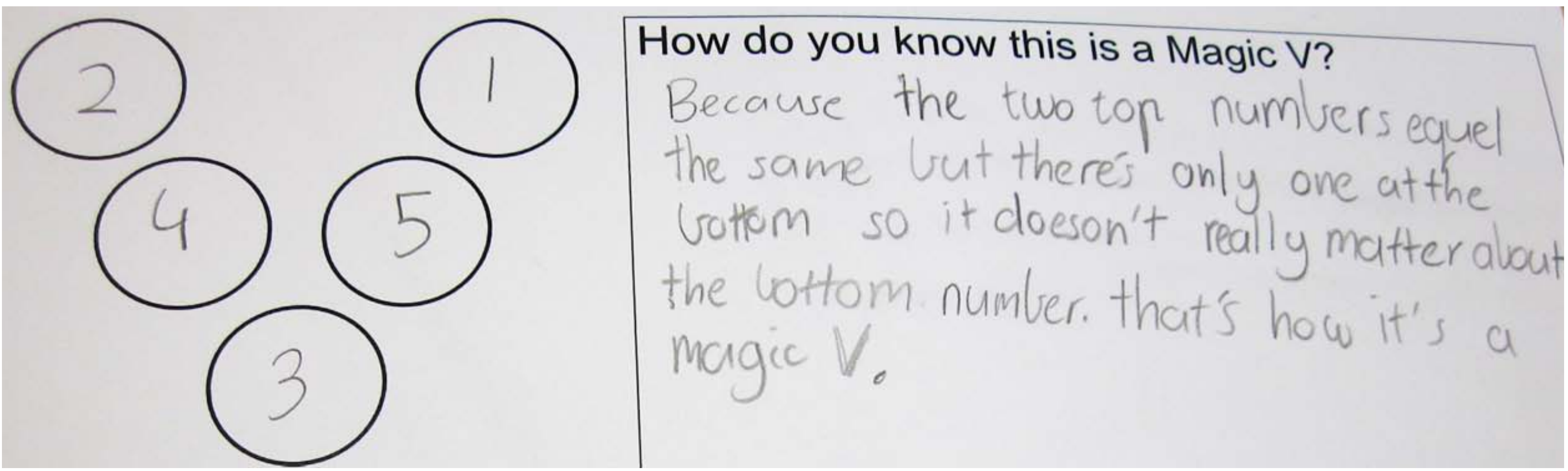
GENERALISING

- How can you describe what is the same?
- What is the rule?
- What is the pattern here?
- How can you describe the pattern?
- What happens in general?
- Is that ... (pattern) always going to work?
- Are there other examples that fit the rule?
- How could you explain the rule to someone else?

JUSTIFYING

- Is this conjecture just sometimes true, or always true?
- How do you know?
- How could we show or prove that it is true?
- True or false? Why? Let's justify.
- Convince me...
- How can we be sure...?
- Tell me what is wrong with....
- Explain - why does this (process/rule/result) work?
- Can you go through that step by step?
- Can you explain that step by step?
- Why?
- If...then...

Exemplar Tasks



	Analysing	Forming Conjectures and Generalising	Justifying and Logical argument
Not evident	• **Does not notice** numerical or spatial structure of examples or cases. • Attends to **non-mathematical** aspects of the examples or cases.	• **Does not communicate** a common property or rule for pattern. • **Non-systematic** recording of cases or pattern. • **Random** facts about cases, relationships or patterns.	• **Does not justify**. • **Appeals** to teacher or others.

Beginning	• **Notices** similarities across examples
	• **Recalls** random known facts related to the examples.
	• Recalls and **repeats patterns** displayed visually or through use of materials.
	• Attempts to **sort** cases based on a common property.
		• Uses body language, drawing, counting and oral language to **draw attention** to and **communicate**:
		  ○ a single common property
		  ○ repeated components in patterns.
		• **Adds** to patterns displayed verbally and/or visually using diagrams or through use of materials.
			• **Describes** what they did and why it may or may not be correct.
			• **Recognises** what is correct or incorrect using materials, objects, or words.
			• Makes **judgements** based on simple criteria such as known facts.
			• The **argument may not be coherent** or include all steps in the reasoning process.

Developing	• **Notices** a common numerical or spatial property.
	• **Recalls, repeats and extends** patterns using numerical structure or spatial structure.
	• **Sorts and classifies** cases according to a common property.
	• **Orders** cases to show what is the same or stays the same and what is different or changes.
	• **Describes** the case or pattern by labelling the category or sequence.
		• **Communicates** a rule about a:
		  ○ *property* using words, diagrams or number sentences.
		  ○ *pattern* using words, diagrams to show recursion or number sentences to communicate the pattern as repeated addition.
		• **Explains** the meaning of the rule using one example.
			• **Verifies** truth of statements by using a common property, rule or known facts that confirms each case. May also use materials and informal methods.
			• **Refutes** a claim by using a counter example.
			• Starting statements in a **logical argument** are correct and accepted by the classroom.
			• **Detecting and correcting errors** and inconsistencies using materials, diagrams and informal written methods.

Thank you

k.stacey@unimelb.edu.au

www.resolve.edu.au

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