Mathematical Inquiry into Authentic Problems Years F - 2

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MAV 2019 Primary and Early Childhood Mathematics Education Conference
Leading whole school improvement in mathematics education
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reSolve: Mathematics by Inquiry

- Free resources from www.resolve.edu.au
- Provide lesson plans, teaching advice, student materials
- Aim to assist teachers in moving towards teaching mathematics with a spirit of inquiry.
- Funded by the Australian Government Department of Education and Training; based at the Australian Academy of Science.
- Champions program run in collaboration with Australian Association of Mathematics Teachers
- MAV organised contracts for some contributed materials

www.resolve.edu.au

TEACHING RESOURCES | PROFESSIONAL LEARNING | OUR CHAMPIONS | OUR VISION | OUR WORK

Promoting a Spirit of Inquiry

reSolve: Maths by Inquiry is an innovative national program that promotes relevant and engaging mathematics teaching and learning from Foundation to Year 10. It is a collaboration of the Australian Academy of Science and the Australian Association of Mathematics Teachers.

The reSolve Protocol underpins all aspects of the project and sets out a vision for teaching and learning mathematics.

The reSolve: Maths by Inquiry Champions Showcase was held at the Shine Dome, Australian Academy of Science on 15 August 2018. On the right you can watch a highlights video showcasing the ten thoughtful and inspiring presentations given by Champions from across Australia and sharing the Champions' insights.

Read more about the reSolve Champions Showcase here.

To keep updated on stories about the program, join the reSolve community.

Teaching Resources – Primary (at June 2019)

Year	Number & Algebra	Measurement & Geometry	Statistics & Probability
	Lessons	*Lessons*	*Lessons*
F	11	2	1
1	13	1	4
2	17	3	3
3	11	4	2
4	9	13	1
5	13	15	3
6	13	8	3

re(Solve) MATHS BY INQUIRY

* Includes classroom resources and special topics

reSolve Protocol

reSolve mathematics is purposeful

reSolve tasks are inclusive and challenging

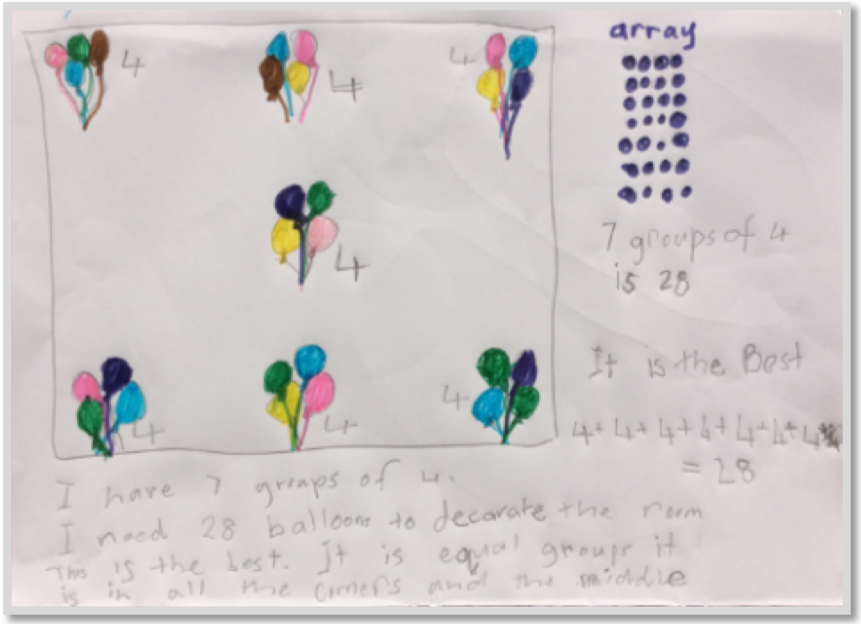
reSolve classrooms have a knowledge-building culture

Promoting a spirit of inquiry

- A spirit of inquiry through all lessons – from one minute to a whole unit.
- Characteristics:
  - Asking questions, trying to find out, explaining what you find
  - Natural emphasis on problem solving and reasoning proficiencies
  - Fits ‘classrooms with knowledge-building culture’
- Varied styles of teaching within the teaching resources
  - Something for everyone.
- Today – sustained units of work within the teaching resources; guided inquiry adapted for mathematics

Example: Part-lesson inquiry
Reasoning with 2D Shapes (Yr 2)

There is only one way that two equilateral triangles can be joined together. As the sides are equal in length the shape will not change if different sides of the triangle are joined together.			
There is only one way that three equilateral triangles can be joined together.			
There are three ways that four triangles can be joined together.	 A triangle can be added to the top to create a larger triangle	 A triangle can be added to one of the sides to create a rhombus	 A triangle can be added to the base of the trapezium

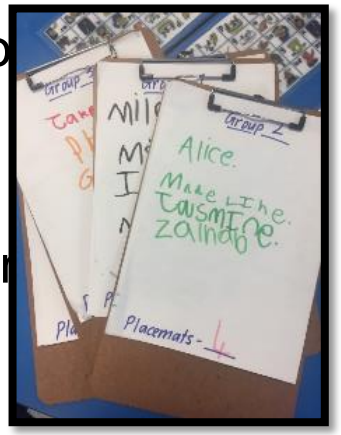


MATHEMATICAL INQUIRY INTO AUTHENTIC PROBLEMS

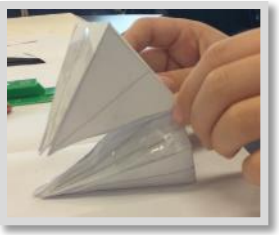
Created with Associate Professor Katie Makar, Dr Jill Fielding-Wells, Sue Allmond (University of Queensland)

Overview

- Aim is
  - to build students' skills in collaboratively addressing co
    routine problems
  - supplement the teaching of designated maths topics
  - build inquiry skills, with a focus on mathematical evide



- 10 units, spread across F – 6
- Each unit follows the same 4D 'lesson' plan
  - Discover - Devise – Develop – Defend
  - might take longer than 4 class periods



- Engaging contexts
  - assist students to link everyday knowledge to maths
  - improve access for students at different performance levels

Ten Units F - 6

Unit Name	Year Level	Australian Curriculum Links
Tea Party	Early Foundation	ACMNA001, ACMNA002, ACMNA003, ACMNA289
Grandma's Soup	Year 1	ACMNA012, ACMNA013, ACMNA014, ACMMG019
Target Ball	Year 1	ACMNA013, ACMNA019, ACMSP262, ACMSP263
Bunches of Balloons	Year 2	ACMNA031, ACMNA032
What's for Lunch?	Year 2	ACMNA027, ACMNA028, ACMNA030, ACMNA031, ACMSP049
Bottle Flipping	Year 3	ACMNA058, ACMMG061, ACMSP068, ACMSP069, ACMSP070
10 000 centicubes	Year 4	ACMNA073, ACMNA076, ACMMG079, ACMMG084, ACMMG290
Expanded Squares	Year 4	ACMNA077, ACMMG087, ACMMG088, ACMMG091
Reaction time	Year 5	ACMNA104, ACMNA105, ACMSP118, ACMSP119, ACMSP120
Pyramids in a Box	Year 6	ACMMG140

Five Units F - 2

Unit Name	Year Level	Brief Description of Activities
Tea Party	Early Foundation	Children plan a tea party relying on representation, counting to 10 and beyond, and one-to-one correspondence.
Grandma's Soup	Year 1	To improve a failed recipe for soup, children estimate how many macaroni are in a handful using various methods of skip counting and grouping strategies to count 100 macaroni.
Target Ball	Year 1	Informal measurement strategies are developed to select and test the best ball to use for a game of Target Ball and to decide where to place the target. Measurements are represented on a number line as mathematical evidence to justify findings.
Bunches of Balloons	Year 2	Partitioning a packet of balloons into groups is investigated to decorate the room. Arrays are introduced as a method to find numbers that can be put into groups with no left overs.
What's for Lunch?	Year 2	Prior to using standard multiplication, students develop and represent strategies with larger numbers to estimate how many sandwiches (or other lunch items) they eat in a year.

Target Ball

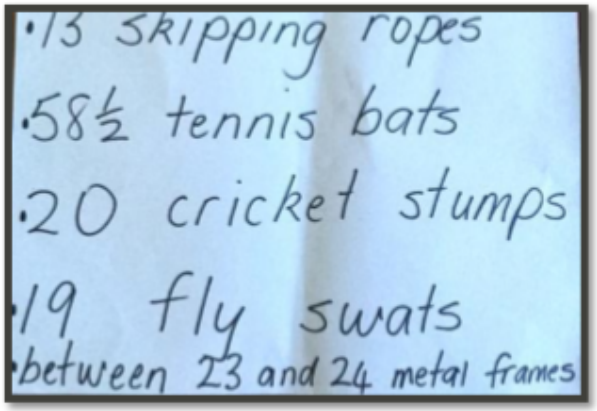
- Challenge: Always ‘authentic’ and engaging
  - Advise the sports teacher on how to organise a game of ‘target ball’ for children about our age.
- Rules for Target Ball:
  - A target is placed at a nominated distance from the starting line. Whoever rolls the ball that stops closest to the target wins.

- Main mathematical content
  - introducing uniform informal units of measurement for distances
  - a lot of practice of measuring
  - measuring requires selecting an appropriate unit and counting repeats
  - accurate measurement requires no gaps, no overlaps, straight path etc
  - dealing with measures between two whole numbers of units
- Inquiry skills
  - gather, organise and interpret evidence
  - draw justified inferences

Target Ball - Lesson Summary

- Lesson 1: Discover Phase
  - Students use direct comparison of distances rolled to explore what type of ball rolls best.
  - Students decide on the best ball and appreciate the need to measure.
- Lesson 2: Devise Phase
  - Students gather first measurement evidence on how far the ball rolls using an informal unit of their own choice.
  - Checkpoint to reduce error when measuring, improve their recording methods and to address the need to record lengths involving part of the informal unit.

- *Two of us were counting how many fly swats so we could check we had the right number.*
- *We measured from the start at the edge of the cricket pitch to where the identification marker was placed.*
- *Thomas made sure we measured in a straight line to the marker.*
- *We made sure there were no gaps and overlaps.*



Target Ball - Lesson Summary (cont.)

- Lesson 3: Develop Phase
  - Groups repeat measuring of Lesson 2, gathering better data – e.g. no gaps.
  - All groups use same informal unit (e.g. adult foot).
  - Groups display distance measures on number line and write mathematical statements to say how far the ball rolls.

Students measuring with foot cut-out.

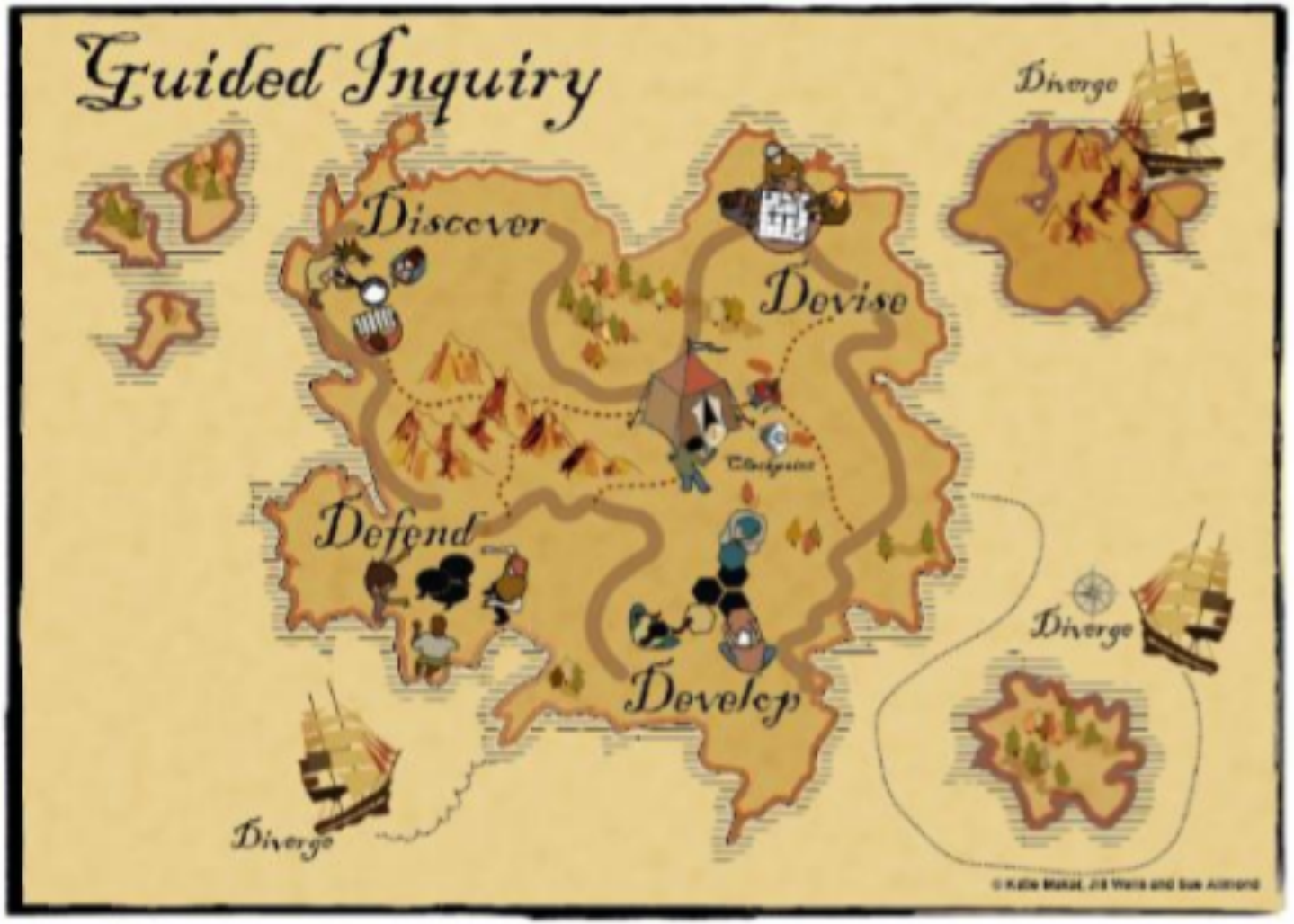
- Lessons 4: Defend Phase
  - Class discusses how to position the target fairly (e.g. use the lowest measurement?)
  - All group measurements categorised into intervals and displayed in table.
  - Students decide on a recommended position of the target and explain their reasoning.
  - Play the game and use the results as supporting evidence for their justification. They reflect on the whole inquiry.

Making an inference from data

- Teacher: *Only seven students rolled more than 50 footsteps. Is this the best place to put the target?*
- Kai: *Three rolls in our group were 15 footsteps, so that would be a good place to put it.*
- Tim: *No. More students rolled in the 20s, 30s and 40s. I think 40 footsteps would be a good place for the target.*
- Leticia: *30 is in the middle and lots of students rolled 30-39 footsteps.*
- Teacher: *Leticia and Tim are using our class data to decide where to place the target and Kai is using data that his group collected. These statements are definitely more mathematical now! I would like each of you to come up with your own mathematical statements for where to position the target using the class data. The class data will make sure our inferences apply to the whole class, not just one group.*

Organised with
4D Guided Inquiry Model

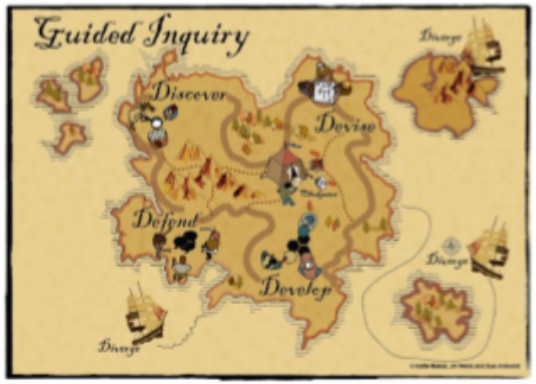
Discover - Devise – Develop - Defend



re(Solve) MATHS BY INQUIRY

MATHEMATICAL INQUIRY INTO AUTHENTIC PROBLEMS TEACHERS' GUIDE

Contents



This guide introduces you to Special Topic 8: *Mathematical Inquiry into Authentic Problems* and provides you with general guidance on teaching the units. Inquiry covers a range of practices that incorporate open-ended problems. This Special Topic focuses on *extended* mathematical inquiry problems that: (1) are driven by an inquiry question; (2) contain ambiguities that require negotiation; and (3) require mathematical evidence to address the problem and justify a proposed solution.

The units have been developed around a 4D Guided Inquiry model with four phases—*Discover*, *Devise*, *Develop and Defend*. Each unit aims to develop content knowledge as well as assisting students to understand the process of inquiry. Mathematical inquiry is not intended to replace other teaching approaches, but rather complement them; the aim is to rebalance students' experiences so that they build procedural skills and conceptual understanding, but also learn to apply mathematics to solve and provide insight into messy, everyday problems. Teaching mathematics through inquiry can be quite different than conventional approaches to teaching mathematics. This Guide briefly provides advice for getting started and a list of resources for further development.

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

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Guided inquiry features of these units explained in Teacher's Guide

(detailed models with the lesson plans)

Guided Inquiry

- Problem characteristics
  - Problems are driven by an inquiry question to be addressed (as opposed to a task or an activity) with some authenticity
  - There are ambiguities in the inquiry question and/or method of solution that require negotiation;
  - Mathematical evidence is needed to address the inquiry and persuade an audience of a proposed solution.

- Value of these questions
  - Showing use of mathematics, in conjunction with other thinking
  - Motivation from attractive context
  - Ambiguities are common in real world uses of mathematics
  - Negotiation allows for some ownership

Teaching for Mathematical Inquiry

- Develop classroom talk: Encourage active listening and explaining thinking
  - these skills need to be explicitly taught and practised (many examples given in the lesson plans)
- Build in frequent whole-class checkpoints

Checkpoints allow students to:	Checkpoints allow teachers to:
• Account for their progress by explaining what they have done and answering any requests for clarification. • See how other groups have worked on the task • Present any challenges they are facing and seek ideas to help them move forward. • Revisit the inquiry question to consider what they will need to do next. • Ensure sufficient appropriate mathematical evidence is being gathered. • Analyse other groups' ideas and provide constructive feedback, which has the potential to improve thinking and ideas.	• Highlight ideas which have the potential to improve the quality of the mathematical ideas. • Model clarifying questions and feedback that focuses on the mathematics. • Encourage students to refine plans or build further on their ideas. • Validate challenges as a normal part of problem solving. • Prompt students to consider an alternative pathway if the current one is unproductive. • Refocus the inquiry to maintain momentum.

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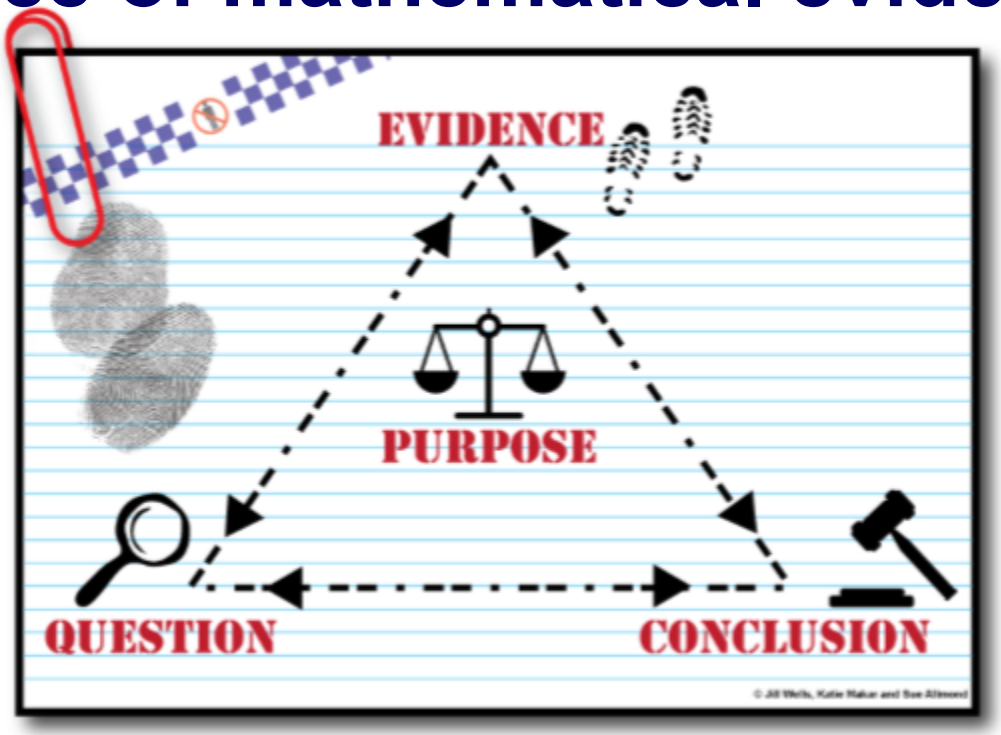
Importance of mathematical evidence



- The need to gather and present mathematical evidence encourages
  - thinking about the problem mathematically
  - using maths words and explaining thinking
  - recording carefully and analysing data
  - justifying conclusions logically

A Maths Investigator

- FINDS THE MATHS
- DOES THE MATHS
- SHARES THE MATHS

Poster for explaining to students what their role is in these inquiries.

Five Units F - 2

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What's for Lunch?	Year 2	Prior to using standard multiplication, students develop and represent strategies with larger numbers to estimate how many sandwiches (or other lunch items) they eat in a year.

Grandma's Soup (Year 1)

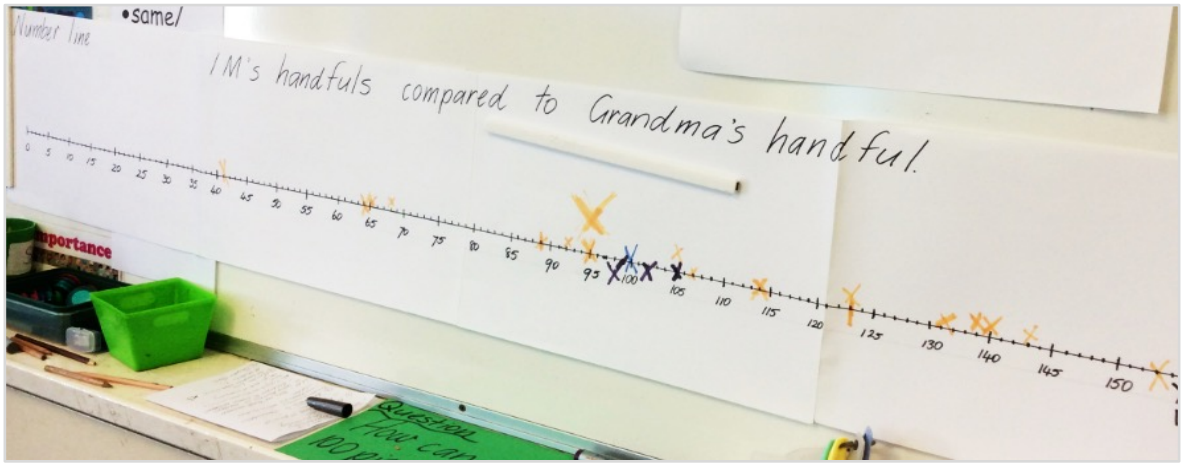
- Lesson 1: Discover
  - Students hear the story and discover that the problem with the recipe is the ambiguity of the term 'handful' as a unit of measurement. They count the pieces of macaroni in their own handfuls. They trace their hands and write the handful numbers on them. They order them from smallest to largest handfuls.

- Lesson 2: Devise
  - Grandma's handful was 100 pieces of macaroni. Grandma's cut-out hand is added to the display of hands. Students write their handful numbers on a class number line, and compare the order to the hands display.
  - Now students make several attempts to grab 100 pieces of macaroni. They record attempts on a number line. They attempt to find the most efficient way to count.

Grandma's Soup (Year 1)

- Lesson 3: Develop
  - Students use the evidence they have gathered to have a last 'grab' of macaroni. Groups select the one closest to 100 and re-count using several strategies. They decide on their most efficient strategy.
- Lesson 4: Defend
  - Groups present their best handful and the counting methods to the class with prompting questions from the teacher where required. They demonstrate counting a collection using skip counting and justify their most efficient method.
  - Students actively listen to others, ask clarifying questions and provide feedback.

Using number lines to compare handfuls

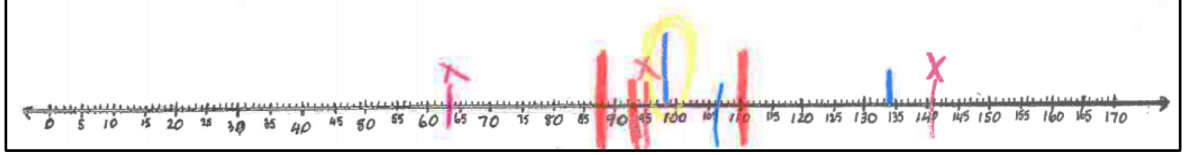


Recording sheet: Devise stage

How many pieces of macaroni in my handful?

Name	Grab 1	Grab 2	Grab 3	Grab 4	Grab 5	Grab 6	Grab 7
Sam	63	141	95				
Jess	107	134	98				
mario	9E	1X10	88				

Number Line:

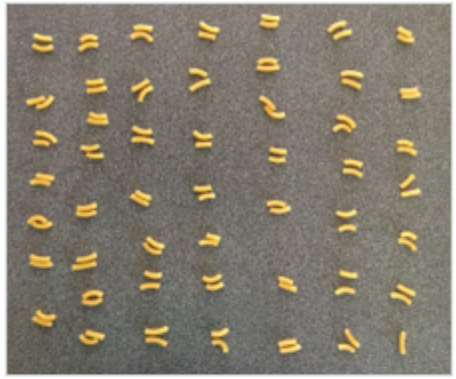


Grandma's Soup (Year 1)

- Mathematics goals
  - ACMNA012: Develop confidence with number sequences to and from 100 by ones from any starting point. Skip count by 2s, 5s and 10s starting from zero.
    - Developing fluency with forwards counting in a meaningful context.
  - ACMNA013: Recognise, model, read, write and order numbers to at least 100. Order these numbers on a number line.
    - Modelling numbers with a range of materials.
    - Identifying numbers that are represented on a number line and placing numbers on a number line.
  - ACMNA014: Count collections to 100 by partitioning numbers using place value.
    - Understanding partitioning of numbers and the importance of grouping in tens.

Different ways of counting 99 macaroni

99 pieces of macaroni



Counted in 2s to 98 and counting on 1 to 9

Counted in 1s to make bundles of 20. Made bundles of 5 with the left-overs. Counted in 20s to 80, then 5s, then ones.

20, 40, 60, 80, 85, 90, 95...96,97,98,99

Counted in 1s to make lines of 10. Counted in 10s to 90 and ones from 90-99.

Counted in 5s to 95 and counted on to 99

Counting in 1s to make a bundle of 50 and a bundle of 49. Counted from 50 in 1s but there weren't 2 bundles of 50.

Making groups of 5 to count left overs.

Year 2 – What's for Lunch?

- **How many [sandwiches] does our class eat in a year?**
  - **ACMNA027:** Recognise, model, represent and order numbers to at least 1000
  - **ACMNA028:** Group, partition and rearrange collections up to 1000 in hundreds, tens and ones to facilitate more efficient counting
  - **ACMNA030:** Solve simple addition and subtraction problems using a range of efficient mental and written strategies
  - **ACMNA031** Recognise and represent multiplication as repeated addition, groups and arrays
- Sandwiches represented for counting
  - Counters
  - Hundreds board
  - MAB blocks, unifix cubes (stick represented week)
  - Grid paper

Students use grid paper to help calculate the large numbers

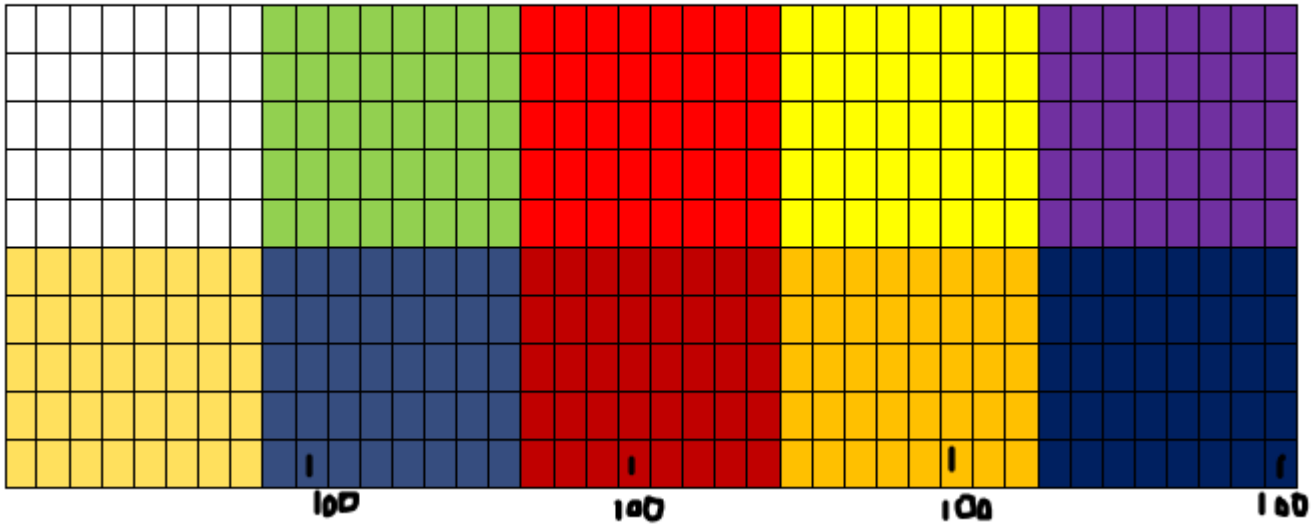
Student representation (Elizabeth)

Discussion

Elizabeth:	*In a week, we had 40 pieces of fruit.*
Teacher:	*How did you calculate the weekly total? Show me where you worked that out.*
Elizabeth:	*There are 5 days in a school week and we ate 8 pieces of fruit each day. This shows every day in different colours. We did 8+8+88+8+8=40 8+8=16 16+8=24 24+8=32 32+8=40*

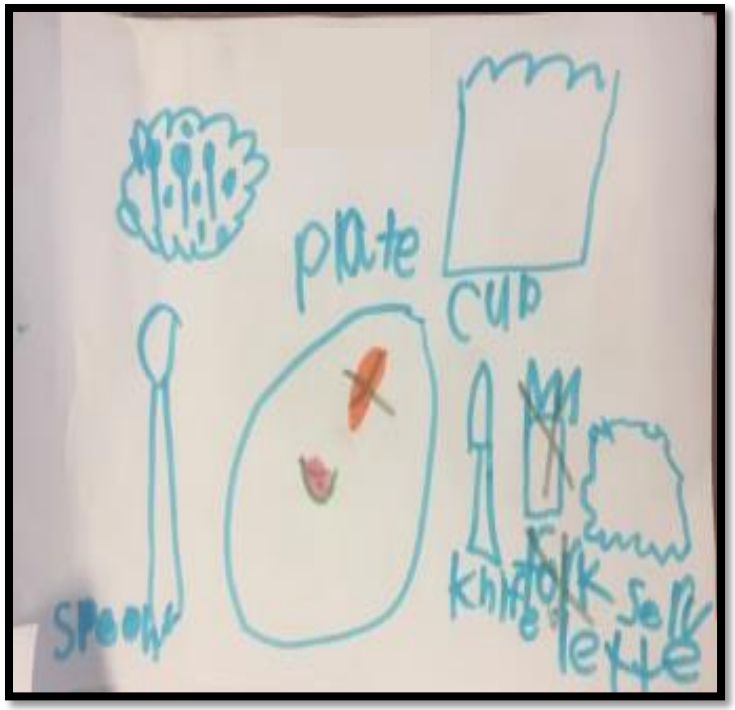
Teacher: *How did you calculate your term total?*

Student representation (Elizabeth)



Foundation – Tea Party

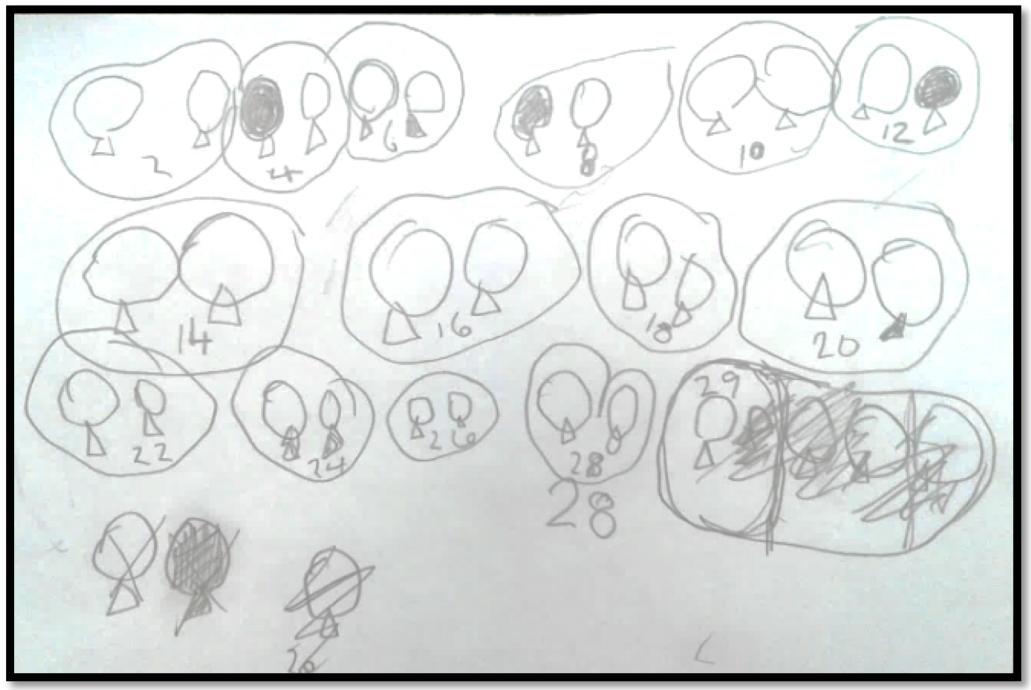
- **Foundation:** How can we plan a tea party for our friends?
- **Focus**: counting, one-to-one correspondence



Foundation children record their count.

Year 2 - Bunches of Balloons

- **Year 2**: What is the best way to decorate our room with bunches of balloons? (Packet of 29)
- **Focus**: arrays, division into equal groups



More free resources from these writers

- www.mathsinquiry.com
- www.resolve.edu.au
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  Professor Katie Makar
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Thank you

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