Mathematical Inquiry into Authentic Problems Years 3 - 6

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MAV 2019 Primary and Early Childhood Mathematics Education Conference

Leading whole school improvement in mathematics education

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reSolve: Mathematics by Inquiry

- Free resources from www.resolve.edu.au
- Provide lesson plans, teaching advice, student materials
- Aim to assist teachers in moving towards teaching mathematics with a spirit of inquiry.
- Funded by the Australian Government Department of Education and Training; based at the Australian Academy of Science.
- Champions program run in collaboration with Australian Association of Mathematics Teachers
- MAV organised contracts for some contributed materials

re(Solve) MATHS BY INQUIRY

www.resolve.edu.au

TEACHING RESOURCES | PROFESSIONAL LEARNING | OUR CHAMPIONS | OUR VISION | OUR WORK

Promoting a Spirit of Inquiry

reSolve: Maths by Inquiry is an innovative national program that promotes relevant and engaging mathematics teaching and learning from Foundation to Year 10. It is a collaboration of the Australian Academy of Science and the Australian Association of Mathematics Teachers.

The reSolve Protocol underpins all aspects of the project and sets out a vision for teaching and learning mathematics.

The reSolve: Maths by Inquiry Champions Showcase was held at the Shine Dome, Australian Academy of Science on 15 August 2018. On the right you can watch a highlights video showcasing the ten thoughtful and inspiring presentations given by Champions from across Australia and sharing the Champions' insights.

Read more about the reSolve Champions Showcase here.

To keep updated on stories about the program, join the reSolve community.

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Teaching Resources – Primary (at June 2019)

Year		Number & Algebra		Measurement & Geometry		Statistics & Probability
		Lessons		*Lessons*		*Lessons*
F		11		2		1
1		13		1		4
2		17		3		3
3		11		4		2
4		9		13		1
5		13		15		3
6		13		8		3

* Includes classroom resources and special topics

Promoting a spirit of inquiry in school mathematics

reSolve Protocol

reSolve mathematics is purposeful

reSolve tasks are inclusive and challenging

reSolve classrooms have a knowledge-building culture

Promoting a spirit of inquiry

- A spirit of inquiry through all lessons – from one minute to a whole unit.
- Characteristics:
  - Asking questions, trying to find out, explaining what you find
  - Natural emphasis on problem solving and reasoning proficiencies
  - Fits ‘classrooms with knowledge-building culture’
- Varied styles of teaching within the teaching resources
  - Something for everyone.
- Today – sustained units of work within the teaching resources; guided inquiry adapted for mathematics

Example: Part-lesson inquiry
Reasoning with 2D Shapes (Yr 2)

<table>
<tr><td>There is only one way that two equilateral triangles can be joined together. As the sides are equal in length the shape will not change if different sides of the triangle are joined together.</td><td colspan="3"></td></tr>
<tr><td>There is only one way that three equilateral triangles can be joined together.</td><td colspan="3"></td></tr>
<tr><td>There are three ways that four triangles can be joined together.</td><td>
A triangle can be added to the top to create a larger triangle</td><td>
A triangle can be added to one of the sides to create a rhombus</td><td>
A triangle can be added to the base of the trapezium</td></tr>
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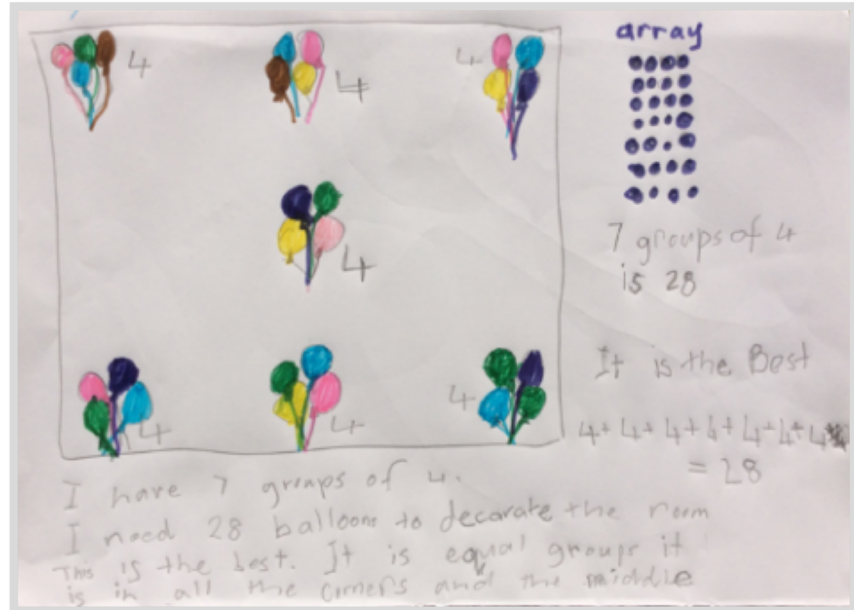
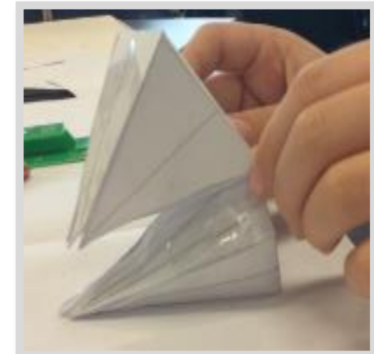


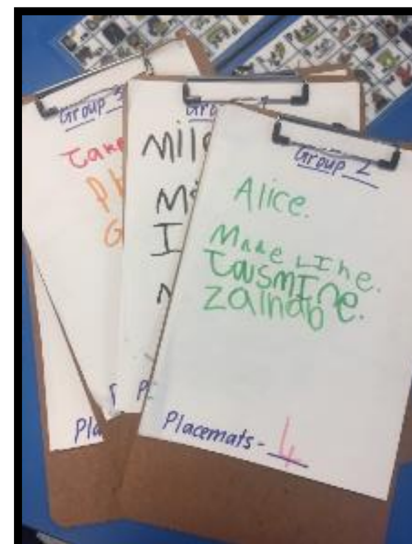
MATHEMATICAL INQUIRY INTO AUTHENTIC PROBLEMS

Created with Associate Professor Katie Makar, Dr Jill Fielding-Wells, Sue Allmond (University of Queensland)

Overview



- Aim is
  - to build students' skills in collaboratively addressing complex, non-routine problems
  - supplement the teaching of designated maths topics
  - build inquiry skills, with a focus on mathematical evidence
- 8 units, spread across F – 6



- Each unit follows the same 4D 'lesson' plan
  - Discover - Devise – Develop – Defend
  - might take longer than 4 class periods
- Engaging contexts
  - assist students to link everyday knowledge to maths
  - improve access for students at different performance levels

Ten Units F - 6

Unit Name	Year Level	Australian Curriculum Links
Tea Party	Early Foundation	ACMNA001, ACMNA002, ACMNA003, ACMNA289
Grandma's Soup	Year 1	ACMNA012, ACMNA013, ACMNA014, ACMMG019
Target Ball	Year 1	ACMNA013, ACMNA019, ACMSP262, ACMSP263
Bunches of Balloons	Year 2	ACMNA031, ACMNA032
What's for Lunch?	Year 2	ACMNA027, ACMNA028, ACMNA030, ACMNA031, ACMSP049
Bottle Flipping	Year 3	ACMNA058, ACMMG061, ACMSP068, ACMSP069, ACMSP070
10 000 centicubes	Year 4	ACMNA073, ACMNA076, ACMMG079, ACMMG084, ACMMG290
Expanded Squares	Year 4	ACMNA077, ACMMG087, ACMMG088, ACMMG091
Reaction time	Year 5	ACMNA104, ACMNA105, ACMSP118, ACMSP119, ACMSP120
Pyramids in a Box	Year 6	ACMMG140

10000 Centicubes (Year 4)

- Challenge: Always ‘authentic’ and engaging
  - Advise an educational supplier on the best container to hold 10000 centicubes (for use in classrooms)
- Main mathematical content
  - Geometric understanding e.g. the layer structure of a prism
  - place value, addition and multiplication, including for large numbers
  - decomposing numbers multiplicatively, multiplying by powers of ten, etc
  - construction skills – e.g. using a ruler,
  - geometry vocabulary – 2D and 3D
  - area and volume vocabulary and formal metric units as desired
- Inquiry skills
  - gather, organise and interpret evidence
  - draw justified inferences that combine mathematical and real-world considerations
  - explain clearly and critique explanations

Decomposing numbers multiplicatively

10 000

⇩

10 x 1000

⇩

10 x 10 x 100

10 x 40 x 25

⇩

20 x 20 x 25

10 x 20 x 50

30 x 30 would be nice

⇩

900 + 900 + 900 ……..

⇩

11 layers is very close

10000 Centicubes - Lesson Summary

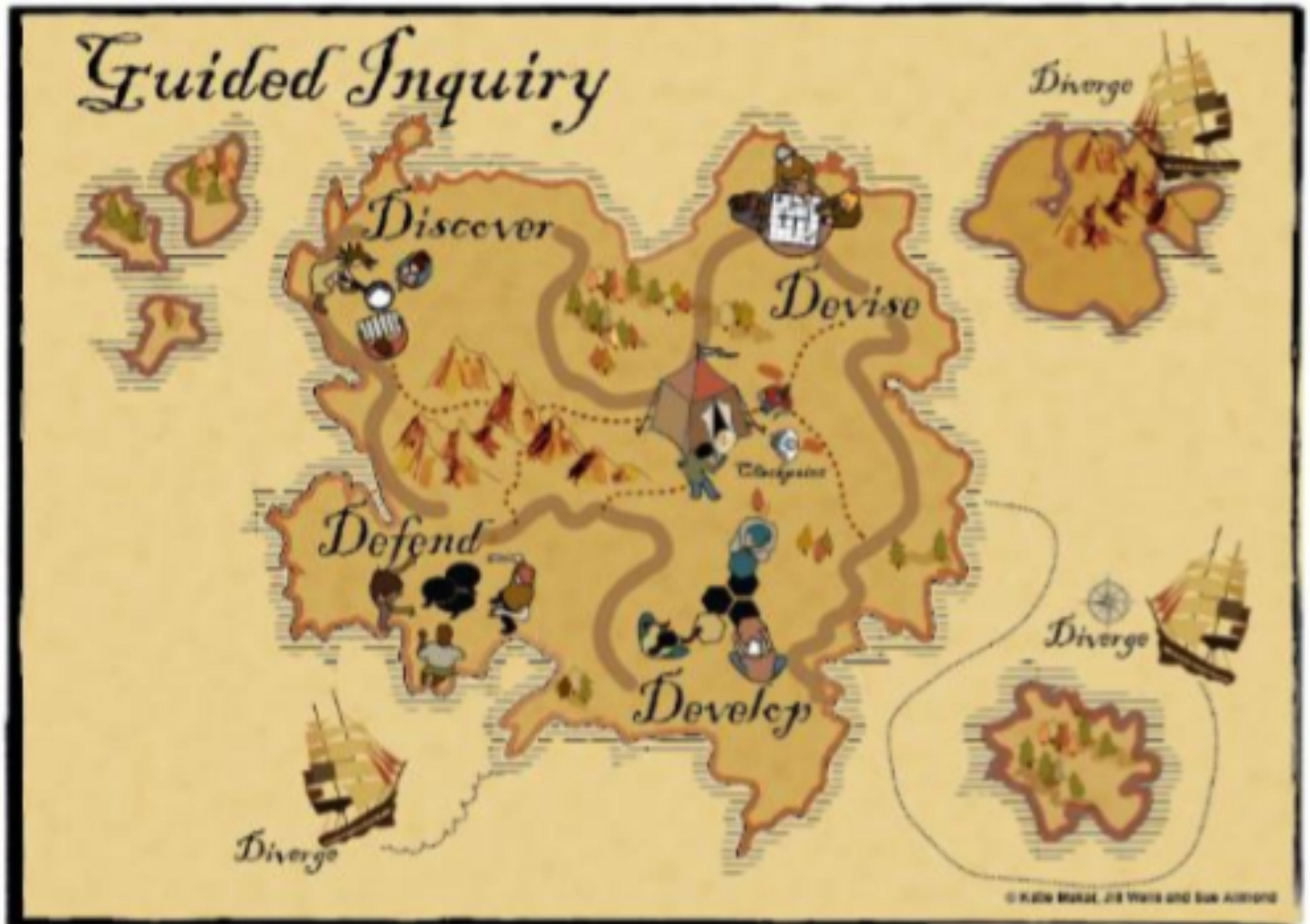
- Lesson 1: Discover Phase
  - Present challenge, review relevant vocabulary, brainstorm possibilities
  - Predict size by looking at one centicube
  - “discover’ stacking layers, multiplicative partition of 10000 to suggest some dimensions
  - Record with number sentences and pictures
- Lesson 2: Devise Phase
  - Discuss criteria for ‘best’
  - Groups plan suitable containers, record dimensions
  - Present plans with feedback from others

10000 Centicubes - Lesson Summary (cont.)

- Lesson 3: Develop Phase
  - Groups decide on ‘best’ containers and make detailed plans – with dimensions for each face
  - Swap plans to get feedback on whether mathematical evidence is sufficient for easy construction
  - Construct best containers
- Lessons 4: Defend Phase
  - Prepare and present solution
  - Gallery walk around for feedback
  - Detailed comparison with another solution – similarities and differences
  - Extensions – e.g. what if neat packing not assumed

Organised with
4D Guided Inquiry Model

Discover - Devise – Develop - Defend



Lesson 1 - Discover

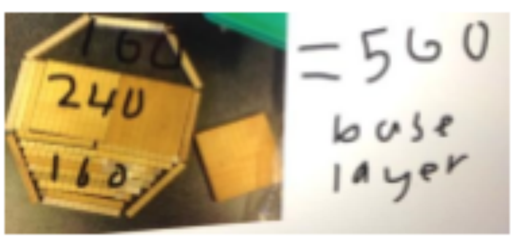
- Exploring the Mathematics in Containers
  - Explore the inquiry question
    - Focus this lesson on size, rather than shape
  - Review 3D objects and associated vocabulary
    - Provide containers of many shapes to examine, and discuss
  - Predict container size
    - Highlight suggestions that have base layers
- Layering a base to develop height
  - Salada crackers and number sentences 4 x 9 = 36, 2 x 18 = 36
  - Partition 10000 multiplicatively using doubling and halving

	Base layer has 500 (50 x 10) = 500

Lesson 2 - Devise

- What do we mean by 'best'?
  - Conclude wide discussion by settling focus on size and shape
- Planning a container
  - Thinking about length, width and height
    - Link centicubes with centimetres
  - Groups make initial plans
    - Persevere through challenges e.g. to use 15 layers, hexagonal base
- Providing and receiving feedback

Shared student work sample



Modelled Feedback

"I like the way you used MAB blocks to show how you worked out the number of centicubes on the base layer of your octagonal prism. If you had added a number sentence like 160 + 240 + 160 = 560 it would have been clearer than just telling us there were 560 on the base layer".

re(Solve) **Planning Sheet** Name: ____________

1. What shapes will you consider for your container?

2. What sizes for the base will you try?
How many layers will be required to make 10 000?

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3. How will you construct your container? *Consider size and shape of each face, materials and construction method.*

Size(dimensions) and shape of each face	Construction
Base	Materials required
Sides	Method

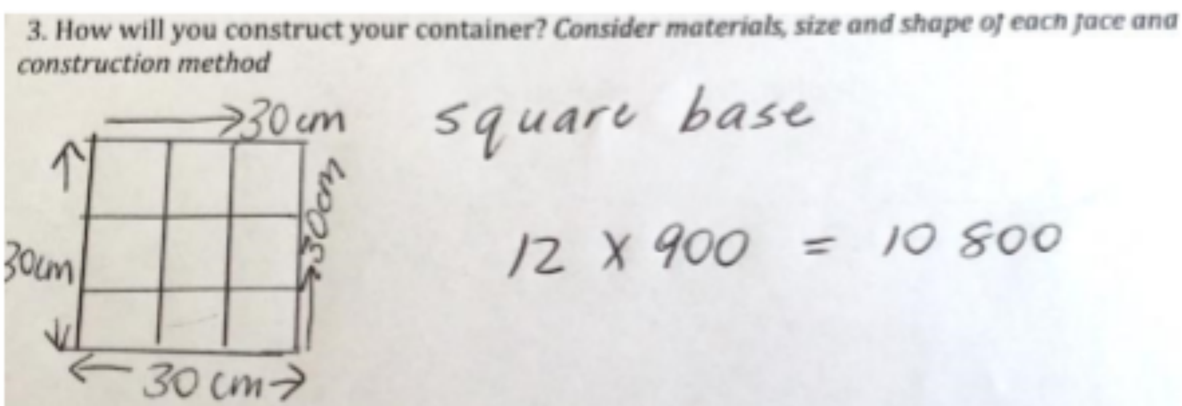
4. Feedback: *Is there enough detail provided for you to construct this container? If not, what extra detail do you need?*

Lesson 2 Handout

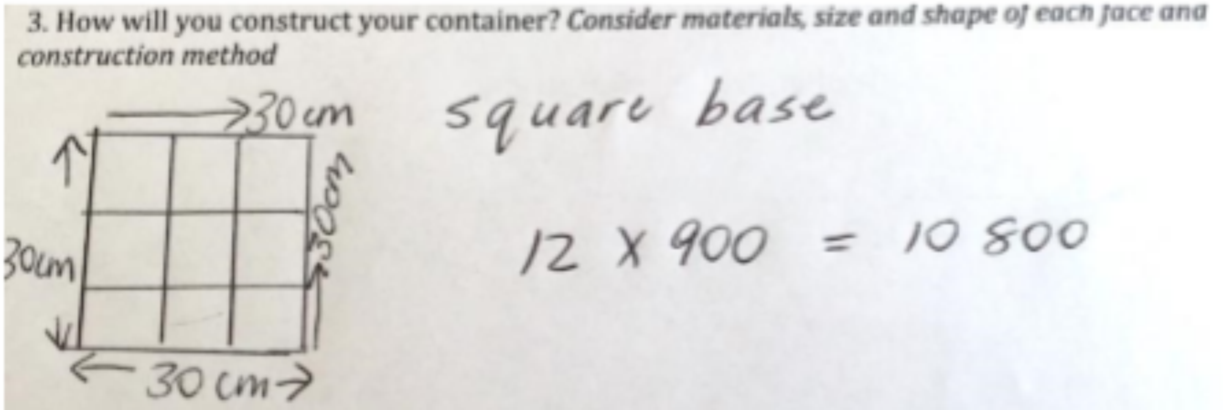
Lesson 3 - Develop

- Refine Plans
  - Group work, end with checkpoint reviewing sufficiency of plans
- Construct Physical Models of Containers
  - Provide 1 cm grid paper to make base and encourage marking in tens
  - no need for nets, tape sides together

Example container plan: Insufficient detail to construct container model



Example container plan: Insufficient detail to construct container model



Possible prompts to group to assist them to improve the plan:

T. *You have clearly indicated the shape and dimensions of your base. What do you know about the size and shape of the other faces?*

S. They will all be rectangles with a length of 30 centimetres.

T. *How wide will they be?*

S. We thought 12 centimetres.

T. *I can see from your plan that you choose 12 layers but that gave you a total number greater than 10 000. Can you explain why you choose 12 layers?*

S. When we divided 10 000 by 900, we got 11.111 which was not a whole number of layers. If we chose 11 layers the container would only hold 9 900 centicubes which would not be enough. We chose 12 layers because then the container could hold 10 000.

T. *Great. On your plan add that extra information on faces and how you worked out 12 layers was best.*

Lesson 4 - Defend

- Presenting containers, using presentation planner
- Gallery walk, constructive feedback, detailed comparison of 2

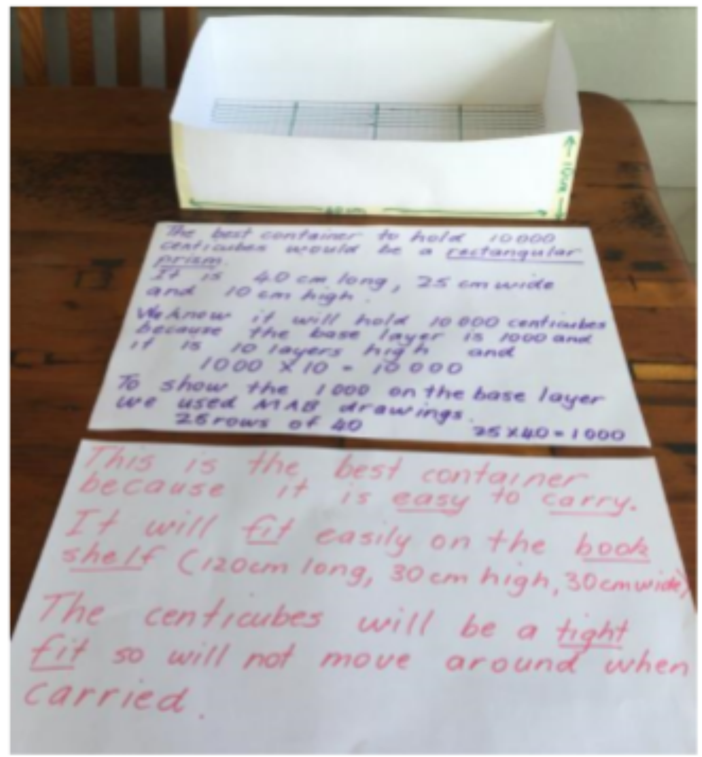
<table>
<tr><th>Presentation scaffold</th><th>Justified student solution using scaffold as a guide</th></tr>
<tr><td>

The best container to hold 10 000 centicubes is a ____________________ (name of 3D object), that is _____ centimetres long, ______ centimetres wide and ______ centimetres high.

These measurements will work because ________________ Explain the mathematical thinking you used to decide on the base and the height (number of layers)

This is the best container because____. Give more than one reason if you can.

</td><td>



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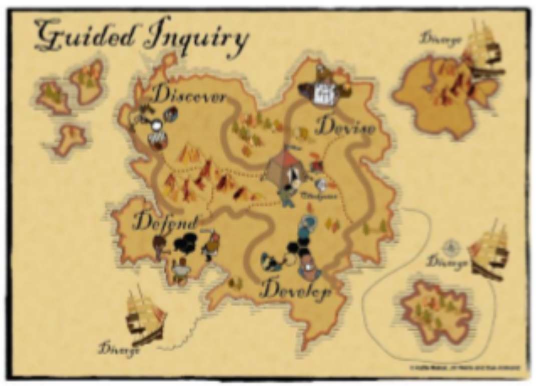
MATHEMATICAL INQUIRY INTO AUTHENTIC PROBLEMS TEACHERS' GUIDE

Contents



This guide introduces you to Special Topic 8: *Mathematical Inquiry into Authentic Problems* and provides you with general guidance on teaching the units. Inquiry covers a range of practices that incorporate open-ended problems. This Special Topic focuses on *extended* mathematical inquiry problems that: (1) are driven by an inquiry question; (2) contain ambiguities that require negotiation; and (3) require mathematical evidence to address the problem and justify a proposed solution.

The units have been developed around a 4D Guided Inquiry model with four phases—*Discover, Devise, Develop and Defend*. Each unit aims to develop content knowledge as well as assisting students to understand the process of inquiry. Mathematical inquiry is not intended to replace other teaching approaches, but rather complement them; the aim is to rebalance students' experiences so that they build procedural skills and conceptual understanding, but also learn to apply mathematics to solve and provide insight into messy, everyday problems. Teaching mathematics through inquiry can be quite different than conventional approaches to teaching mathematics. This Guide briefly provides advice for getting started and a list of resources for further development.

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

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Guided inquiry features of these units explained in Teacher's Guide

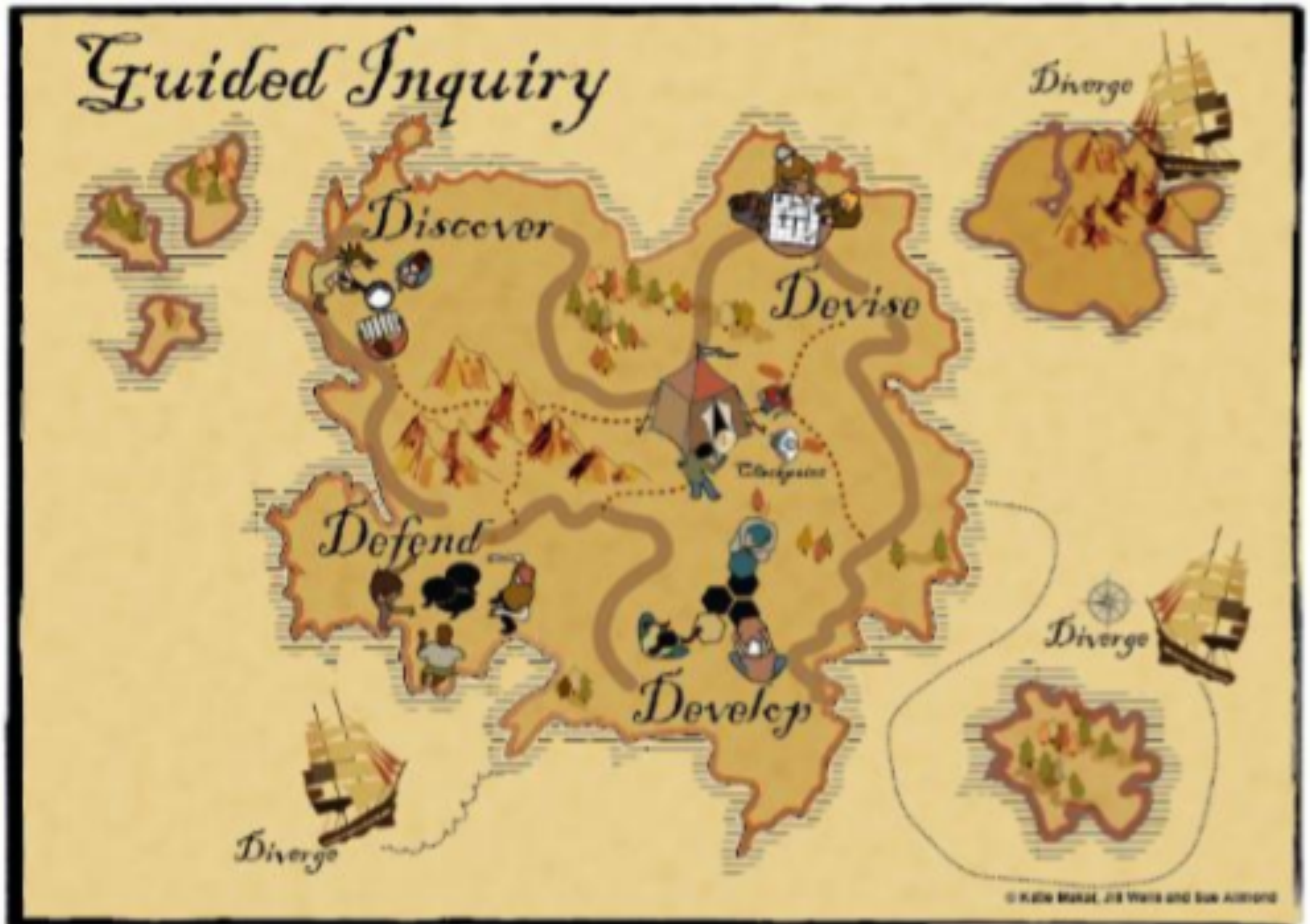
(detailed models with the lesson plans)

Guided Inquiry

- Problem characteristics
  - Problems are driven by an inquiry question to be addressed (as opposed to a task or an activity) with some authenticity
  - There are ambiguities in the inquiry question and/or method of solution that require negotiation
  - Mathematical evidence is needed to address the inquiry and persuade an audience of a proposed solution.
- Value of using inquiry approach
  - Develops general inquiry skills
  - Showing use of mathematics, in conjunction with other knowledge
  - Motivation from attractive context; negotiation promotes ownership
  - Ambiguities are common in real world uses of mathematics
- Clear mathematical goals
  - core to achieve + options for differentiation and personal interest

Organised with 4D Guided Inquiry Model

Discover - Devise – Develop - Defend



Teaching through Inquiry

- Develop classroom talk for knowledge-building culture
  - encourage active listening and explaining thinking
  - these skills need to be explicitly taught and practised (many examples given in the lesson plans)
- Build in frequent whole-class checkpoints

Checkpoints allow students to:	Checkpoints allow teachers to:
• Account for their progress by explaining what they have done and answering any requests for clarification. • See how other groups have worked on the task • Present any challenges they are facing and seek ideas to help them move forward. • Revisit the inquiry question to consider what they will need to do next. • Ensure sufficient appropriate mathematical evidence is being gathered. • Analyse other groups' ideas and provide constructive feedback, which has the potential to improve thinking and ideas.	• Highlight ideas which have the potential to improve the quality of the mathematical ideas. • Model clarifying questions and feedback that focuses on the mathematics. • Encourage students to refine plans or build further on their ideas. • Validate challenges as a normal part of problem solving. • Prompt students to consider an alternative pathway if the current one is unproductive. • Refocus the inquiry to maintain momentum.

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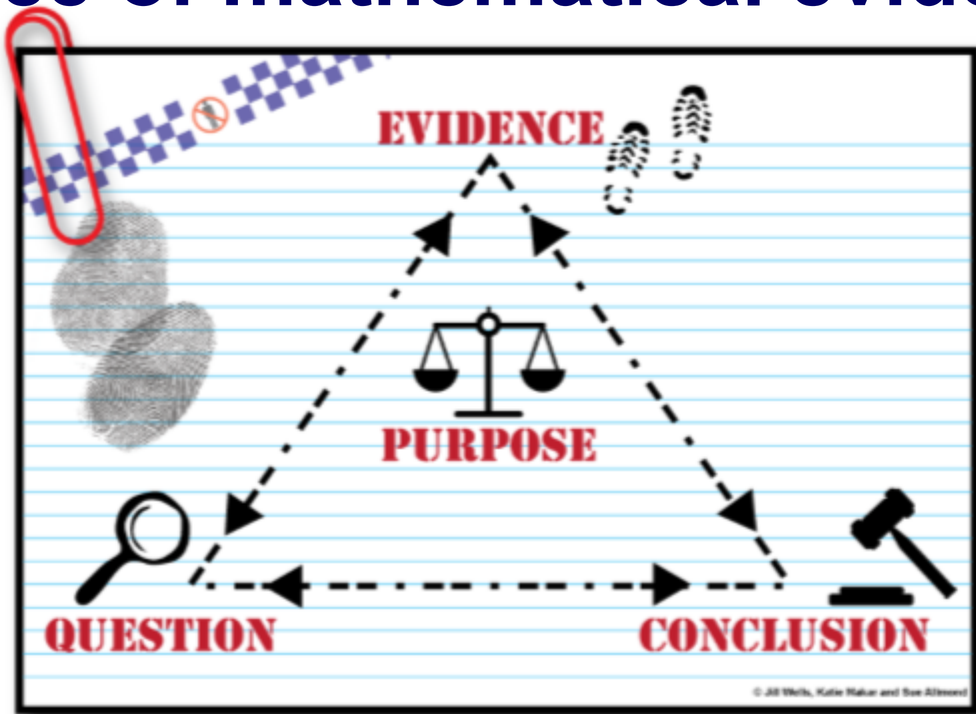
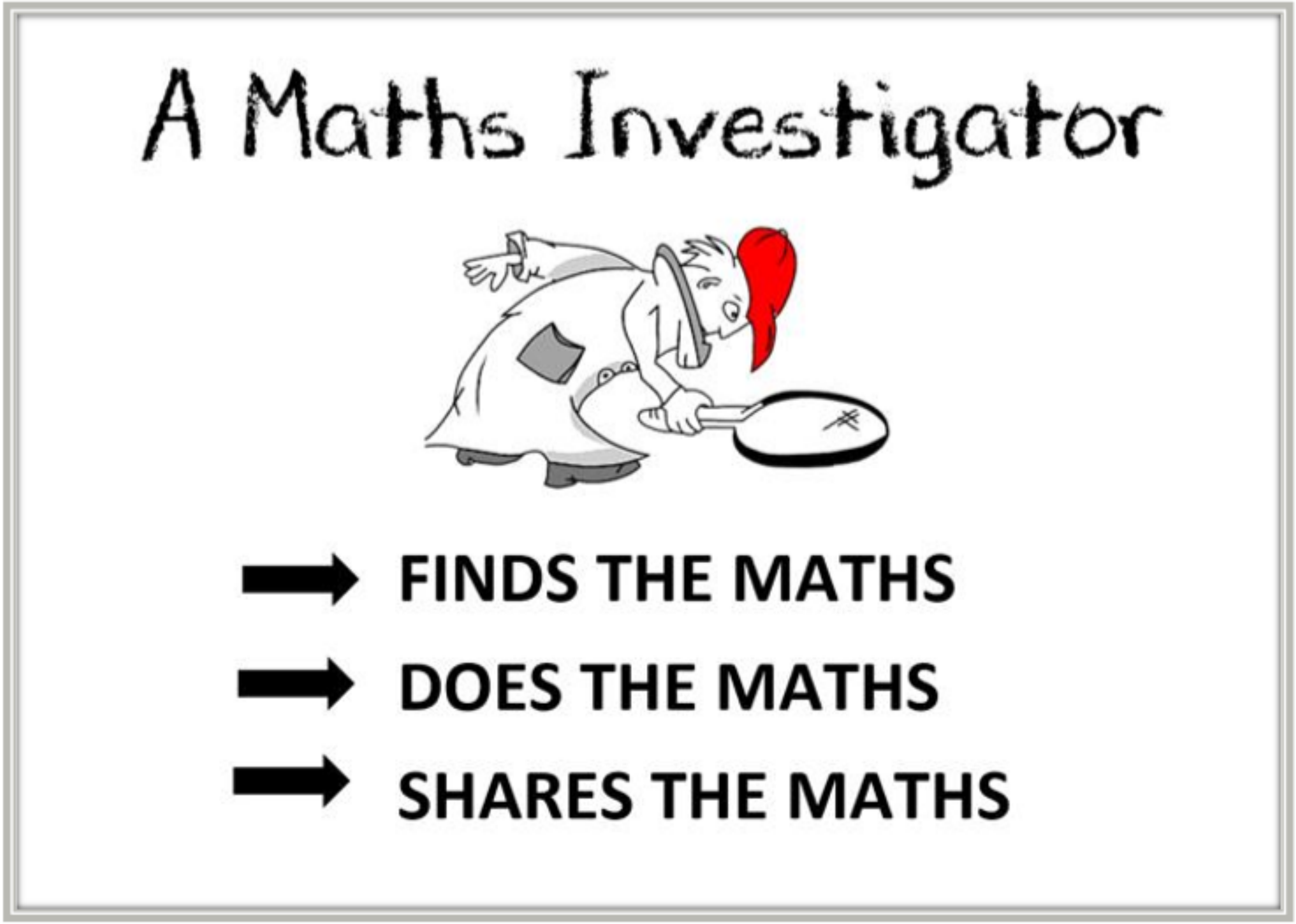
Importance of mathematical evidence



- Benefits of focus on mathematical evidence
  - thinking about the problem mathematically
  - using maths words (+ number sentences)
  - explaining thinking carefully
  - recording carefully and analysing data
  - justifying conclusions logically

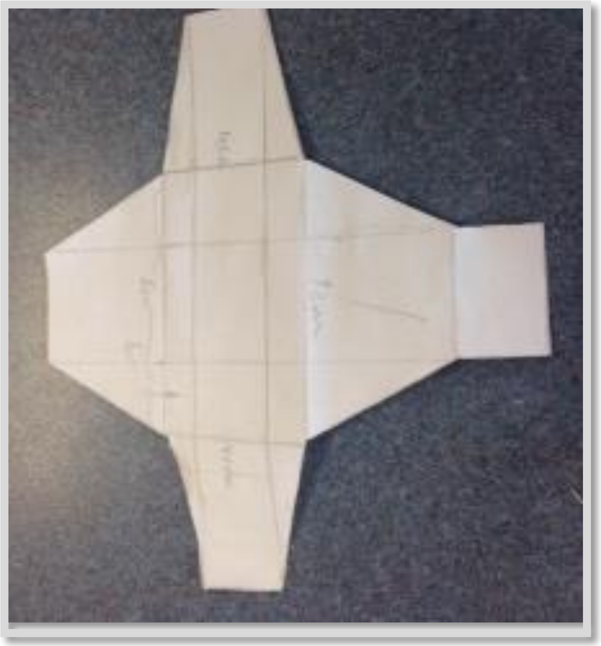


Poster for explaining to students what their role is in these inquiries.

Five Units 3 - 6

Unit Name	Year Level	Australian Curriculum Links
Bottle Flipping	Year 3	Children use unit fractions to compare the amount of water that is best for bottle flipping. They support findings with evidence using tallies from trials of flips.
10 000 centicubes	Year 4	A box to hold 10 000 centicubes is designed using smaller numbers as referents and breaking down large numbers into possible dimensions of length, width and height.
Expanded Squares	Year 4	An expanded square is an art technique that flips cut-outs of shapes inside a square to the exterior. Children estimate irregular areas to design an expanded square that flips half the area to the outside.
Reaction time	Year 5	Students work with decimals to record, compare and analyse reaction times of students in their class.
Pyramids in a Box	Year 6	Nets and features of 3D objects are explored to design and construct a box that can hold two pyramids.

Pyramids in a box (Year 6)

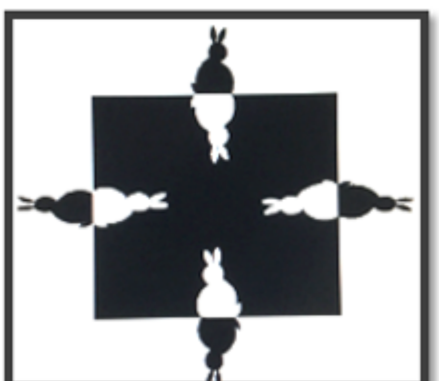


- What is the best box to hold 2 different sized items that are packaged as pyramids?
- **Focus**: prisms, pyramids, nets

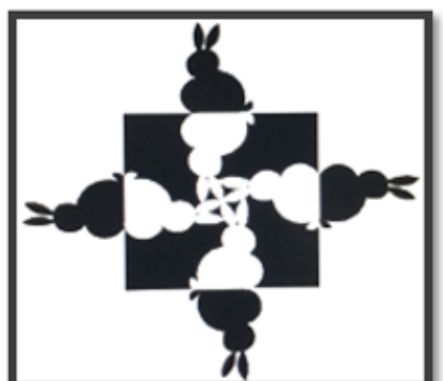
Expanded Square (Year 4)

What Fraction is White Space?

Look at the original square in the expanded squares below. What fraction of the original square is white space? You will need to estimate. Explain your estimated fraction to a partner.



Design A

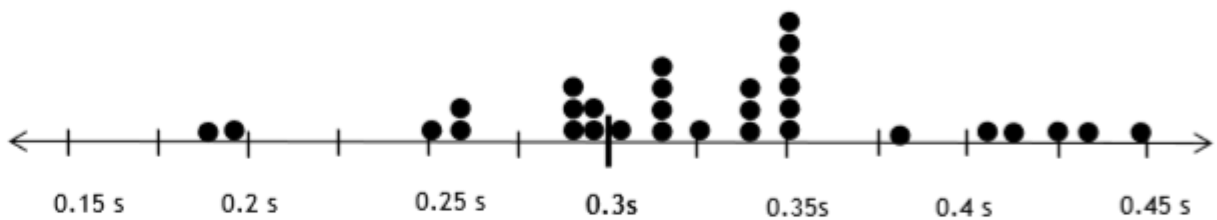


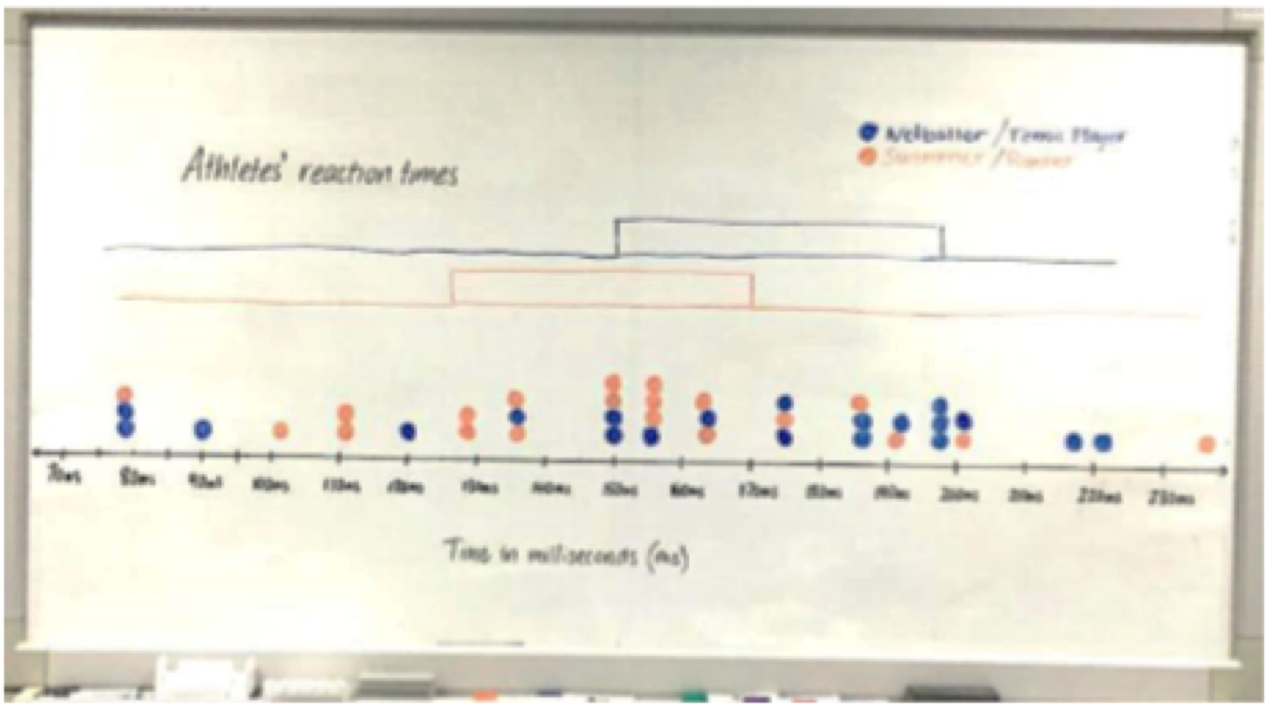
Design B

Design C

Reaction Time (Year 5)

Reaction times of students in 5F





Bottle Flipping (Year 3)

re(Solve) **The Fraction Flip** Name: ____________

- Carefully prepare the amounts of water for the bottle
- Sit on the ground for flipping
- Test each fraction of water the same number of times (e.g. 15)
- Record if the flip was successful (landed upright) or unsuccessful
- Tally the number of successes for each test and compare

Recording Sheet

FRACTION OF WATER	SUCCESSFUL	UNSUCCESSFUL
1/2		
1/3		
1/4		
1/5		
1/8		
1/10		

More free resources from these writers

- www.mathsinquiry.com
- www.resolve.edu.au
- Contact:
  Professor Katie Makar
  k.makar@uq.edu.au

Thank you

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