The importance of infinity in the primary mathematics classroom

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Friday 21st of June 2019

University of Melbourne and MAV Primary Mathematics Conference

Infinity awakes curiosity in children already before they enter school…However, this early interest is not often met by school mathematics curriculum and not discussed in school, and infinity remains mysterious for most students throughout school years.
Pehkonen and Hannula (2006)

Infinity

- Write down your ideas abut infinity.
  - Keep what you have written to yourself, but keep it handy.

NO CONSULTING!!

Lyla: Aged 5 and a half - On Infinity

What does this video of a young girl who has just started school make you wonder about ‘infinity’ and its place in school?

Overview

- Conceptualizing ‘Infinity’
- The place of ‘infinity’ in the primary mathematics classroom
- Testing some areas of primary mathematics for ‘infinity’
    - Example 1: Geometry
    - Example 2: Decimals
    - Example 3: Patterns and Algebra
    - Example 4: Probability
- Reflection: How ‘deep’ does a child’s understanding of ‘infinity’ need to be for conceptual understanding in these areas to be appropriately developed?

What do you think of when you think of 'Infinity'?

Discuss

Here are some ideas:

- Boundlessness
- Never Ending
- On and on
- ∞
- Number?
- Geometry?
- Probability?

Do you feel that ‘infinity’ has a place in the primary curriculum?

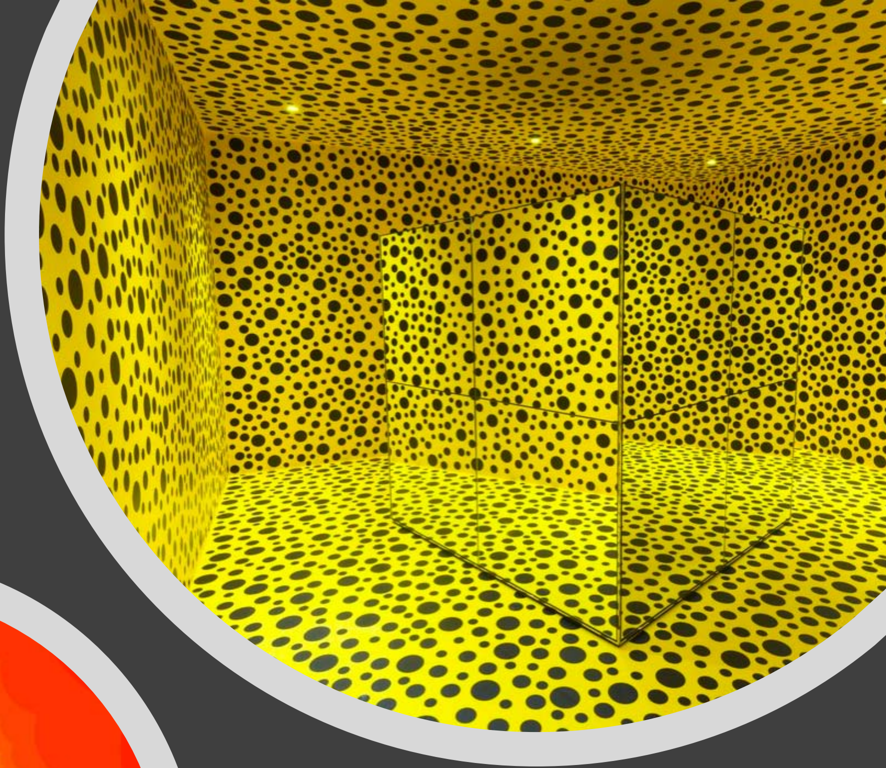
Infinity and Geometry

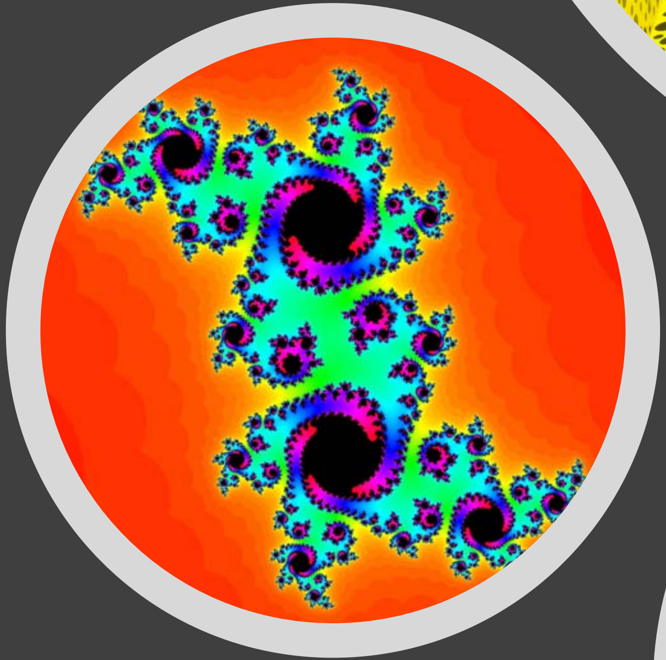
Infinity and Geometry

- Infinity arises in geometry?
- In public life:
  - Fractals
  - Football Fields
  - Roads
  - Parallel mirrors
  - Film
  - Art
- In mathematics
  - number of shapes
  - size of plane/space
  - patterns: area

Discuss: How important is it for children to know this?

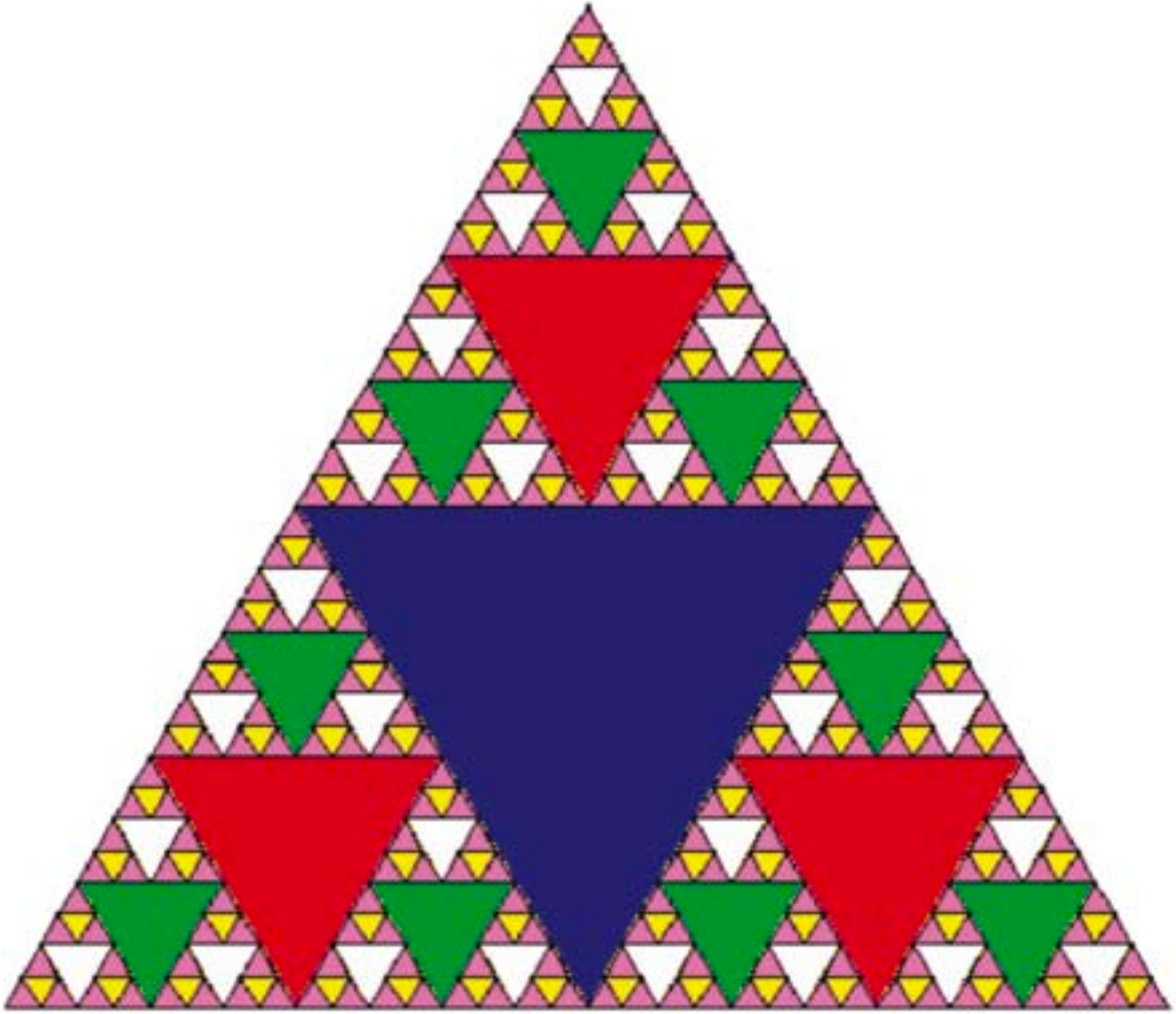
- foundation for coordinate geometry





Infinity and Geometry

- Sierpiński Triangle
  - As a ‘fun’ piece of mathematics
- Point at infinity: so infinity may be a ‘point’ and not just a number (so how can you add ∞ and ∞?)
- A point as a limit that shapes are heading for (convergence).



Coding Sierpiński Triangle – With Python

```
import turtle

def drawTriangle(points,color,myTurtle):
    myTurtle.fillcolor(color)
    myTurtle.up()
    myTurtle.goto(points[0][0],points[0][1])
    myTurtle.down()
    myTurtle.begin_fill()
    myTurtle.goto(points[1][0],points[1][1])
    myTurtle.goto(points[2][0],points[2][1])
    myTurtle.goto(points[0][0],points[0][1])
    myTurtle.end_fill()

def getMid(p1,p2):
    return ( (p1[0]+p2[0]) / 2, (p1[1] + p2[1]) / 2)

def sierpinski(points,degree,myTurtle):
    colormap = ['blue','red','green','white','yellow',
                'violet','orange']
    drawTriangle(points,colormap[degree],myTurtle)
    if degree > 0:
        sierpinski([points[0],
                        getMid(points[0], points[1]),
                        getMid(points[0], points[2])],
                   degree-1, myTurtle)
        sierpinski([points[1],
                        getMid(points[0], points[1]),
                        getMid(points[1], points[2])],
                   degree-1, myTurtle)
        sierpinski([points[2],
                        getMid(points[2], points[1]),
                        getMid(points[0], points[2])],
                   degree-1, myTurtle)

def main():
    myTurtle = turtle.Turtle()
    myWin = turtle.Screen()
    myPoints = [[-100,-50],[0,100],[100,-50]]
    sierpinski(myPoints,3,myTurtle)
    myWin.exitonclick()

main()
```

Infinity and Decimal Understanding

- Many learners experience misconceptions when developing an understanding of decimals (Steinle & Stacey, 2003)
- One of these misconceptions in particular known as (whole number thinking) is linked to an understanding of the infinite.
- Since 123 is bigger than 24, then .123 is bigger than .24.
- But 0.123 = 1/10 + 2/100 + 3/1000 while 0.24 = 2/10 + 4/100. They also need to know that 2/10 is bigger than 1/10 plus any further number of finite decimal places.
- Students need to know the basic fact that as you move each consecutive place to the right each subsequent column is a tenth of the previous column.

Numbers between Numbers

A little algorithm that is of no interest for whole numbers:

1. Start with 0 and 1
2. Keep 1 constant
3. Choose another number between the previous number and 1.
4. Repeat step 3

What happens?

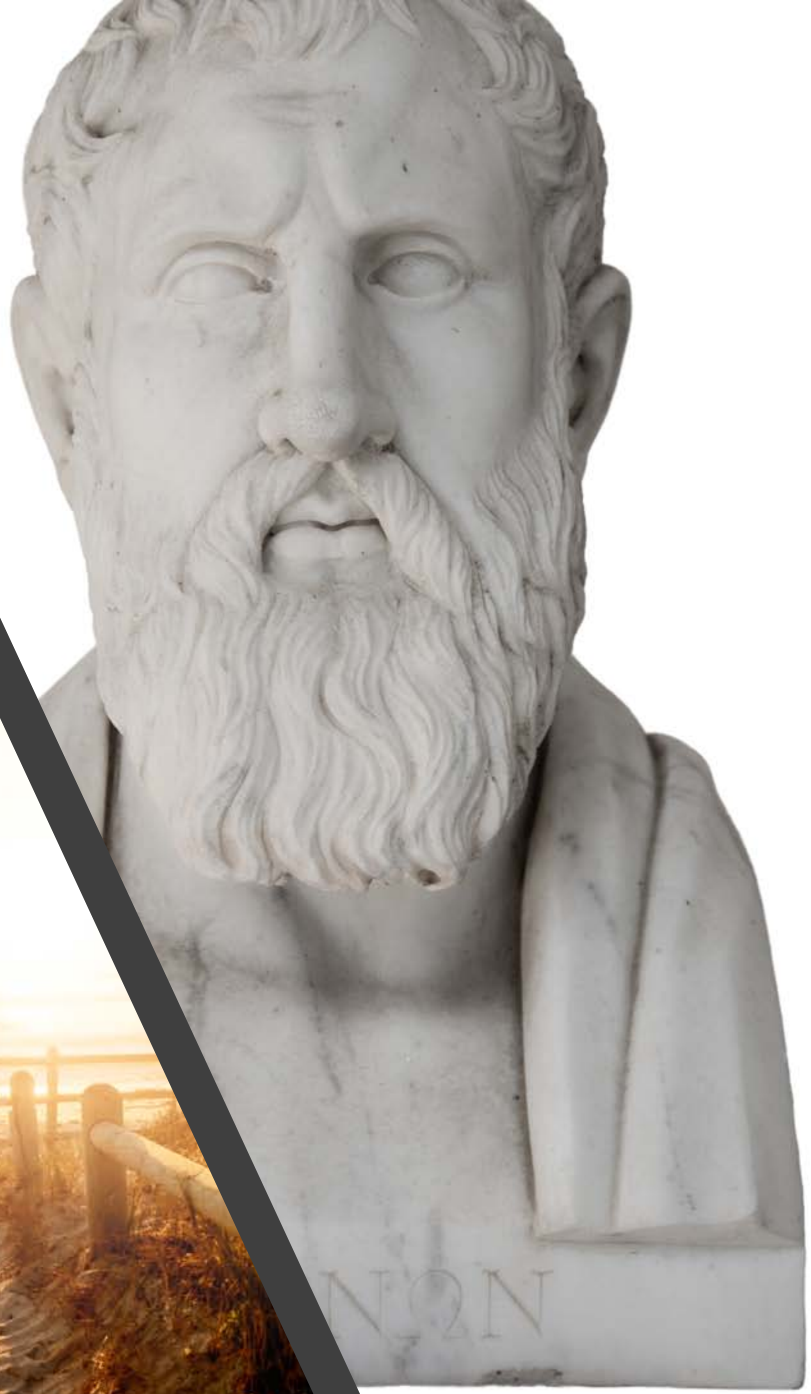
Convergence as in geometry of laptop video.
(Convergence and non-convergence is nearly everywhere and is about infinity.)

Zeno’s paradox

Suppose you want to get to a beach nearby. You will never get there. This is because you first have to walk halfway there. Then a halfway between halfway and the beach. Then a halfway between halfway and between halfway and the beach.

And so it goes on and on and on and …!

… It looks like convergence again.



Infinity and Patterns and Algebra

- We now consider the 'infinite' as the basis of algebraic thinking:

Moving students from recursive, additive approaches to multiplicative and to generalisable understanding is fundamental to the development of algebraic thinking (Kieran, 2004).

- Recursive:
  - Students could approach the allocation of even numbers by simply adding on two for each whole number.
  - The first multiple of 4 is 4, the second multiple of 4 is 4 + 4 = 8 and so on…
- This recursive approach has serious limitations.
- If, for example, we wanted to know what the 10,000th multiple of 4 is, it would take a very long time to repeatedly add 4, to get there.

Infinity and Patterns & Algebra

- However, the multiple of 4 corresponding to the multiple of any given number is four times that number.
- This is neatly represented by saying that the multiple of 4 corresponding to n is 4n.
- An infinite number of numbers is encapsulated in this simple expression.
- <u>Understanding the infinite nature of whole numbers is an essential factor in understanding how to generalise.</u>
- What's more it has a practical value over the recursive method because if tomorrow we want to find the millionth multiple of 4 we don't have to start from scratch and add some more 4s. Further it is much easier to read than saying 4 + 4 + … a million times and so it makes communication much easier – try reading Newton int the original!

4 Arm Shape

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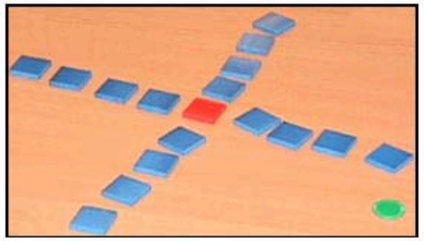
4 Arm Shapes: Investigation Guide



1. Find out the number of tiles needed to make each of these pictures. Don't forget to count *all* the tiles.

Length of one arm	1	2	3	4	...	10	...	100
Number of tiles								

2. As soon as someone tells you the length of one arm, can you tell them the total number of tiles they will need to make the shape?
Write your explanation here:

____

____

3. Now suppose someone told you the total number of tiles they used to build their four arm shape. How would you work out the length of one arm of their shape?
Try it for the numbers in the table, then write an explanation of how you did it.

Length of one arm				
Number of tiles	21	45	101	213

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4. Suppose someone told you they had used 31 tiles in total. Use the graph paper to draw a picture of their 4 arm shape.
Explain how to find the length of one arm if someone tells you *any number of tiles at all* for their total.

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Victorian Mathematics Curriculum Links?

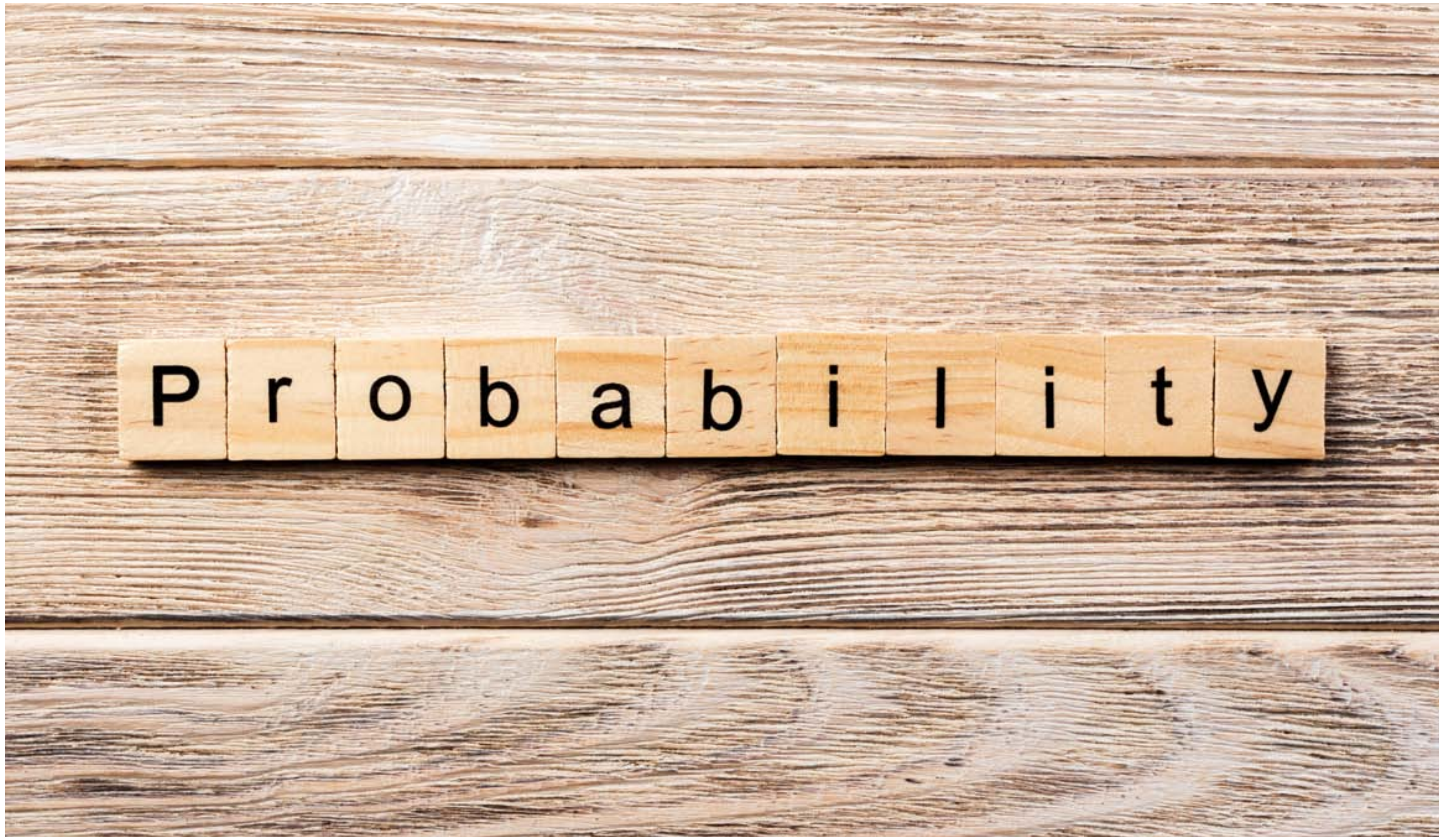
Strand: Number and Algebra

Year Level	Content Description
5	Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction (VCMNA192)
6	Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (VCMNA219)

Infinity and Probability

- In public life if we had infinite resources we could bet on the same thing – one-armed bandit – and stop when we win. (The infinite route to financial success!)

- Convergence between theoretical and experimental probability



Learning Intentions:

- 'Experimental' and 'Theoretical' probability.
  - We will consider what links them and what the differences are.

Dice Problem

Take three dice I

- Draw up a table such as the one below:

1	2	3	4	5	6

Roll all three dice
Mark the numbers that show up on the dice in the appropriate column
Do this twenty times.
Record how many 1s, 2s, 3s, 4s, 5s and 6s you get in the third row.
Pool your results with everyone at your table.

Take three dice II

- What results do we get from the whole room?
- What is the probability of getting a 6 on rolling a die?
- What has this got to do with convergence?

Victorian Mathematics Curriculum Links?

Strand: Statistics and Probability

Year Level	Content Description
5	Recognise that probabilities range from 0 to 1(VCMSP204) List outcomes of chance experiments involving equally likely outcomes and represent probabilities of those outcomes using fractions (VCMSP203)
6	Compare observed frequencies across experiments with expected frequencies (VCMSP234) Describe probabilities using fractions, decimals and percentages (VCMSP232)

Reflection/discussion

- How ‘deep’ does a child’s understanding of ‘infinity’ need to be for conceptual understanding in these areas to be appropriately developed?
- How ‘deep’ does your understanding of ‘infinity’ need to be for conceptual understanding in these areas to be appropriately developed?
- How ‘deep’ does professional statistician’s understanding of ‘infinity’ need to be for conceptual understanding in these areas to be appropriately developed?
- Can anyone know enough?

Conclusion/ Recommendations

- Children should get opportunities to play with/ experiment with/ discuss infinity across primary schools mathematics because it develops understanding and intuition and helps to see connections across the syllabus.
- Everyday life and other disciplines:
  - Linking to see what artists do in making the 2d look 3d?
  - What is happening in statistics - why aren't the predictions of election results as accurate as they have been?
  - Why/how do insurance companies and casinos make money?
  - Why do patterns of numbers and events that are aimed to cover an infinite number of cases possibly do not (early European visitors thought that all swans were white!)
- What do think of infinity now. Has it changed any?

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