Seventh Squadron Gantugstan Air Force

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Contact

- I am happy to be contacted by email (dholton@unimelb.edu.au)
- It may take me a day or two to answer email as I don’t go to university every day.
- I am also happy to meet individuals or groups at some later time, but not today
- I’m happy to provide copies of these slides

Aims

- My aims are to play with:
  Addition/subtraction
  Higher level thinking
- Provide a problem that might be useful in your classroom.
- Try to show what research mathematicians (as well as the rest of us) do and its relevance to the classroom.

Then what is it mathematicians do?

- Maths and this ‘higher’ type of thinking:
  - involves: experiments, conjectures, proofs, extensions, generalisations
  - possible to do even in the lowest grades of school
  - and it’s done by artists, actors, musicians, animals
- There is **nothing special about mathematics** (proof?)

The Seventh Squadron, GAF

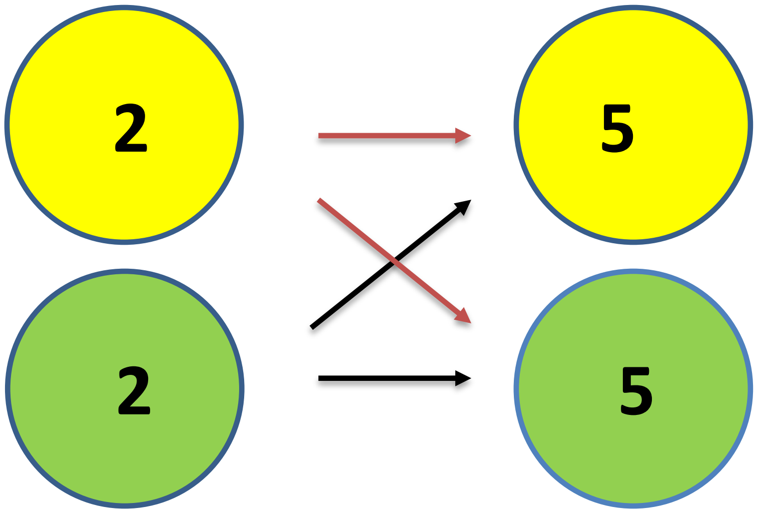
Seventh Squadron has new planes.

The service section is painting numbers on the wings.

Can every number on one wing when separately added to every number on the other wing give 7?

Example

The circles represent the wings with the yellow being the top of the wings and the green the bottom. The left yellow and green are the left wing.



More examples?

Are there any more examples?

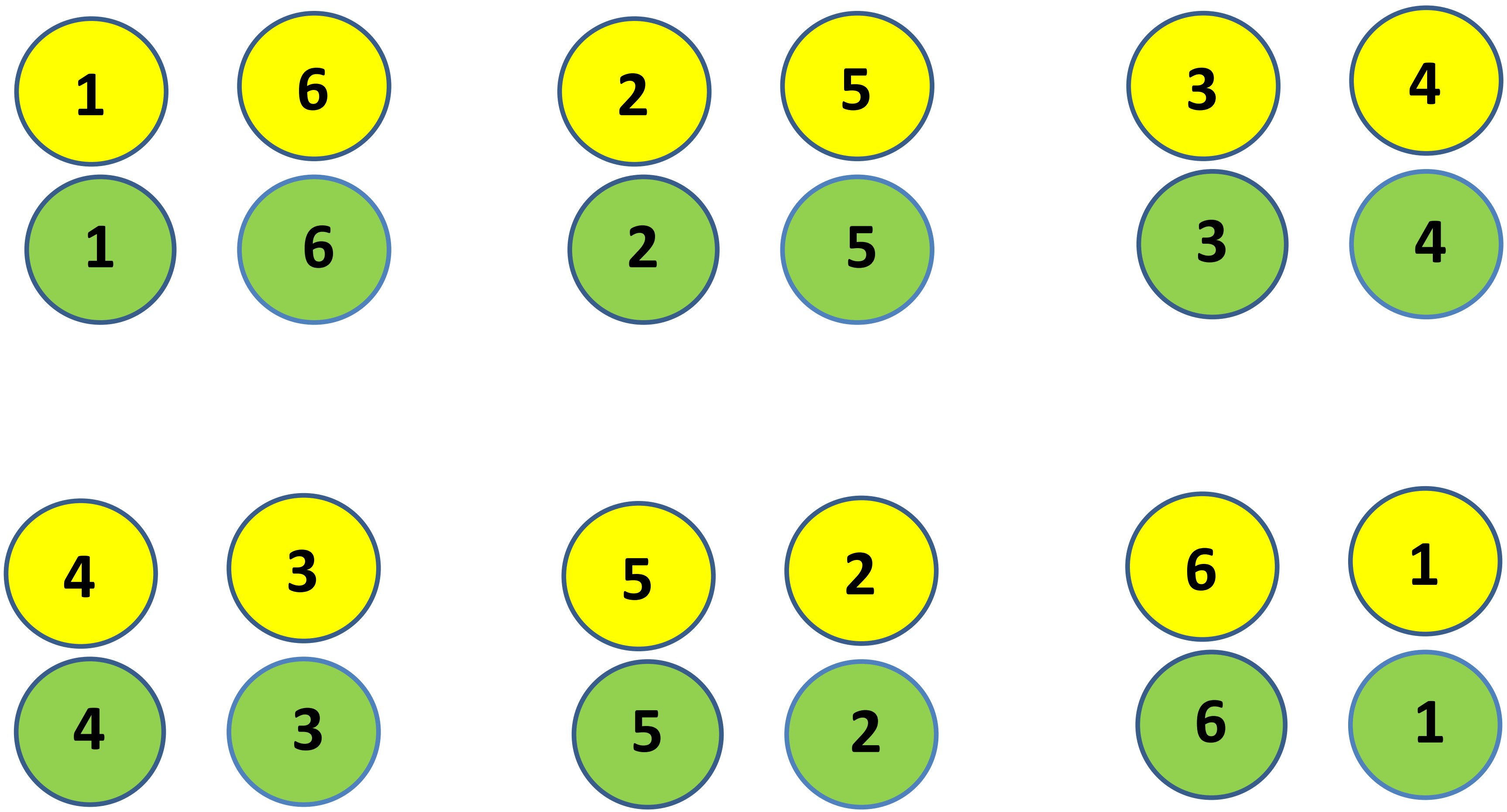
If so, how many?

How do you know that you have them all?

Wildcatting versus being systematic

Write your answers on the board

Possibilities



MISS

Is

Making

It

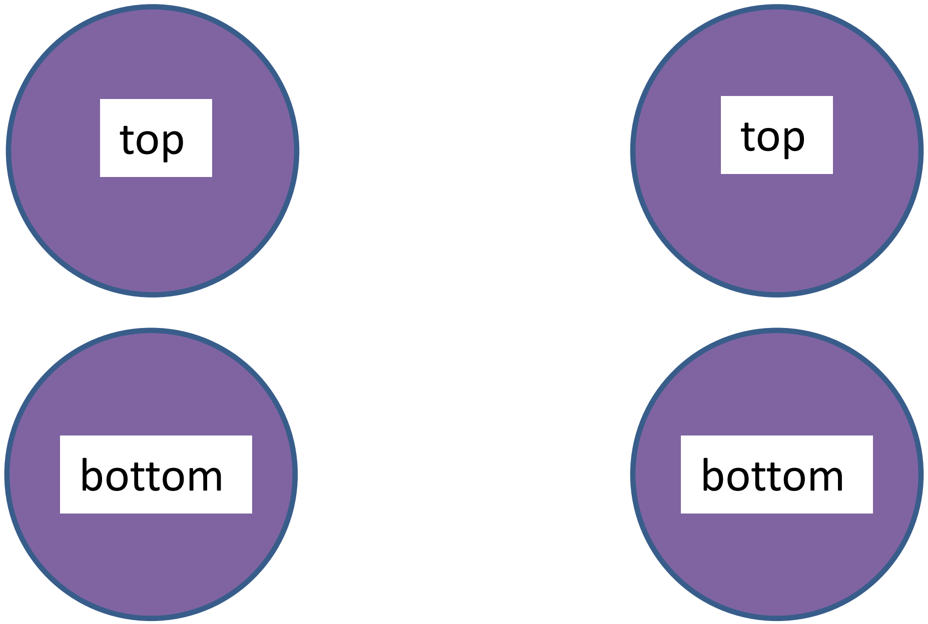
Simpler,

Stupid

or a good idea?

An Idea

Is it simpler or stupid to do this problem with two ***discs?***



disc 1 disc 2

How many pairs of discs?

Less work?

Less being systematic?

Can we convert discs into wings?

An extension or two

What about the Eighth Squadron?

Can they have the Seventh Squadron's treatment?

How many planes can be painted?

What about the Ninth and Tenth Squadrons? Anybody want to do other squadrons?

Use the board for results please. ***Make a list of these.***

How and what?

How did you do it?

With wings or pairs of discs?

How many different paintings did you get?

Same result using wings and discs?

Are discs really a good idea?

Extension and/or generalisation?

Extension:

same basic problem

little difference

Generalisation:

one basic extension that covers an infinite number of cases

looking for patterns

Patterns I

What pattern(s) were there for all the wings with the 7 sum?

What patterns are there for ***every*** number as sum?

Conjecture/guess?

Proof?

Patterns II

Give power!

Increase in knowledge

Something to use next time

Thinking I

How many ways to make a sum?

- number strand
- fluency
  - knowing what maths to use
  - converting addition facts to suit problem

Extending/generalising

- fluency
  - knowing concepts and their use

Meta-maths I

Understanding?

what did question ask?

Mathematics?

symmetry

be systematic

fluency

Reasoning?

based on the maths

Exit point 1

Depending on your students and why you are doing this, you might want to stop here.

Exit and entrance points will be omitted from here.

What’s next?

Are there any patterns
in the previous wings?

Extensions/generalisations
Examples

Ideas for extending the problem?
List

Exit point 2

Depending on your students you might want to stop here.

What will they have got from the problem by going this far?

A generalisation or two

What patterns/generalisations can you see from these examples?

These things are called Conjectures.

Loved by mathematicians!

1/100

Exit point 3

Depending on your students you might want to stop here.

What will they have got from the problem so far?

A proof?

How could you justify/prove these?

What ideas might be needed?

Do you have to use algebra? Yuk!

Meta-maths II

Understanding?

where are we going?

Mathematics?

symmetry

be systematic

checking

fluency

patterns

proof

Reasoning?

extending/generalising

Exit point 4

Depending on your students you might want to stop here.

What will they have got from the problem so far?

Another extension

To get ***just*** 7 and 8 by summing the numbers on the wings of a plane.

What would you do?

How did it work out?

But first …

Is

1 6	6 1	1 7	7 1
1 7	7 1	1 6	6 1

four planes or one?

One might be easier to deal with.

Reformulation

Turn painting into discing.

Multiply by 4 (and we know how).

Disc results

What do all discs with the sums 7 and 8 look like?

What do all the discs with sums 9 and 10 or 23 and 24 look like?

Conjecture!

Proof?

Exit point 5

Depending on your students you might want to stop here.

What will they have got from the problem so far?

Four sums

Can you get four sums?

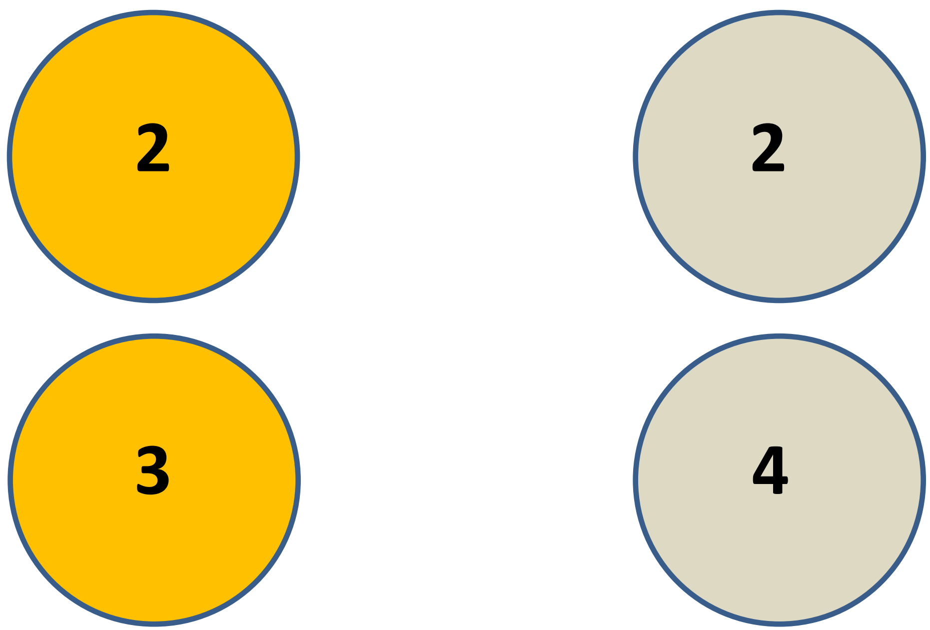
Can you get any four sums?

Can you get every consecutive sums?

Can you get any conjectures and proofs?

Add four numbers

Write a number on each side of each disc to get 4, 5, 6, 7?



Yet another extension

Can you get the sums 7, 8, 9, 10 with the discs?

If so, how many planes can we paint to give sums of 7, 8, 9, 10.

Answers and Justification (Proof)

You have ***experimented***.

- Could you put your answers on the board, please.

- Is that all there are? How do you know?

Can you make a ***conjecture*** or two?

Exit point 6

Depending on your students you might want to stop here.

What will they have got from the problem so far?

SO?

Any difficulties?

Anything hard?

Anything interesting?

Could you use anything we’ve done? How/why?

More generalisations

What patterns might you be seeing for consecutive sums?

What patterns might there be for any sums?

What patterns are there for numbers that give no sums?

What experiments might be needed?

Can you prove anything?

Question 2

If we had

- put a non-negative whole number on each side of the discs;
- tossed the discs like coins; and
- added the sums of the numbers that were face up,

could we have found the sums 6, 7, 8 and 9?

Answers

1:	0	6	2:	0	6	3:	1	5	4:	1	5	
	1	8		2	7		2	7		3	6	
5:	2	4	6:	2	4	7:	3	3	~~8:~~	3	3	
	3	6		4	5		4	5		5	4	
Please check these												

Seven ways to get sums of 6, 7, 8 and 9.

Be systematic when producing lists of possibilities.

Proving stuff

Are these all there are?

How can we justify this?

Proof by Exhaustion!!

Question 2 (**<u>Extension</u>** 1)

What if the numbers you needed to get as sums were 8, 9, 10, 11?

Is it possible?

If not, why not?

If so, in how many ways?

More extensions?

Do you have any more extensions?

Think, talk and report back.

Question 2 (Extension 2)

Can we get:

123, 124, 125, 126

using two discs?

Gut feeling vote: yes, no, not sure.

‘Yes’ – you have to show how

‘No’ – you have to give a reason why not

‘Not sure’ – you need to experiment

Question 2 (**Generalisation** 1)

What would a generalisation look like here?

Guess/conjecture?

Justify/prove

More Extending

What problems can you think of that are based on discs plus numbers plus tossing?

These are extensions of the original two wing problem that might be generalisable.

Towards More Extensions

Can we test potential sums to see if they are possible.

Are all of the following four sets of numbers possible?

2, 3, 6, 7;

2, 4, 8, 10;

2, 3, 6, 9?

Why?   Why not?

Towards a Generalisation

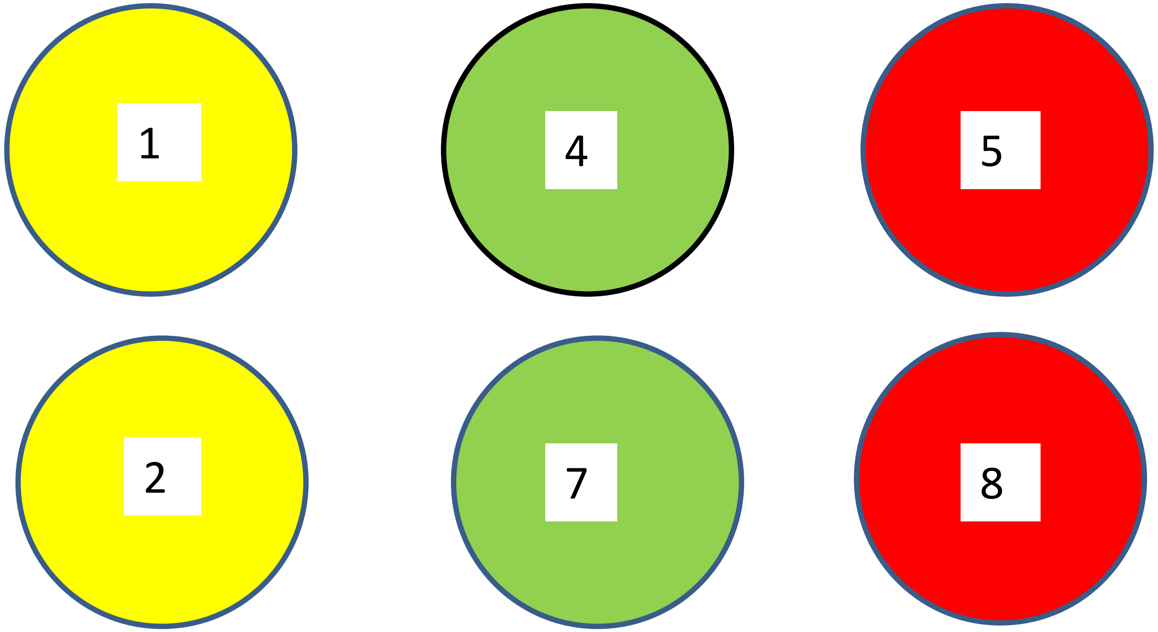
How many ways are there to get the sums:

2, 4, 5, 7;
3, 8, 11, 16;
7, 108, 299, 400?

Can you make a conjecture for ***any*** four numbers numbers? How about **proving** it?

Extension 2

Consider the problem with three discs. So



These gives sums of 1 + 4 + 5, 1 + 4 + 8, 1 + 7 + 5, 1 + 7 + 8, 2 + 4 + 5, 2 + 4 + 8, 2 + 7 + 5 and 2 + 7 + 8.

Even further

What questions can you ask about three (or more!) discs?

What conjectures can you make?

What conjectures can you prove?

What use would these questions be in the classroom?

Why use in class?

Genuine proficiency strand activity

problem solving especially

communication – working on board and in groups; writing proofs

revision/use in context of 2-digit addition/subtraction – fluency

understanding through justification

Use in your class

- Could you use this idea in your class?
- If you did would you use all of it or not?
- What you would hope to get out of the (never-ending) problem?

Exits and entries

Re-entries are always possible when you leave before an proof or something that is too hard.

On-line references: problem solving

- https://www.resolve.edu.au/
- http://www.nzmaths.co.nz/
- http://www.maths300.esa.edu.au/
- http://nrich.maths.org/frontpage