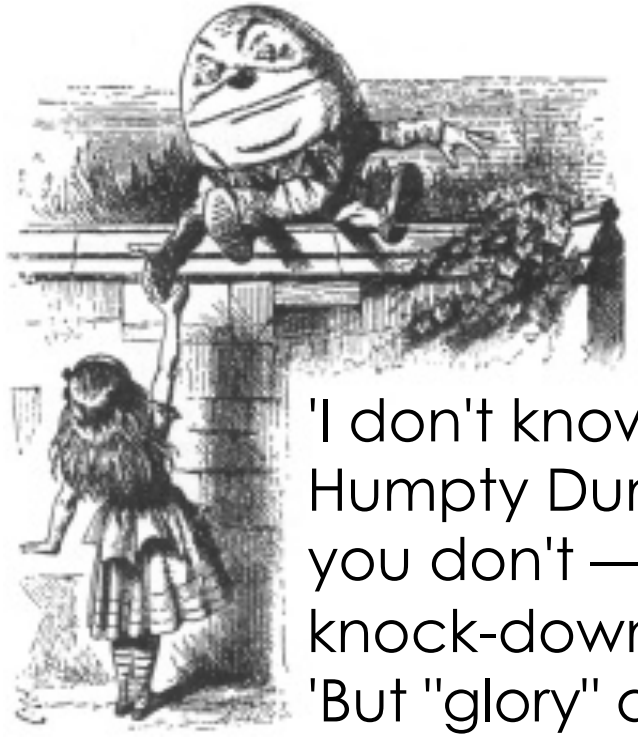
Revisiting problem solving: The matter of language

Philip Clarkson
Melbourne University / ACU

Philip.Clarkson@acu.edu.au
@PhilipCClarkson



'I don't know what you mean by "glory",' Alice said.
Humpty Dumpty smiled contemptuously. 'Of course you don't — till I tell you. I meant "there's a nice knock-down argument for you!"'
'But "glory" doesn't mean "a nice knock-down argument",' Alice objected.
'When I use a word,' Humpty Dumpty said, in rather a scornful tone, 'it means just what I choose it to mean — neither more nor less.'
'The question is,' said Alice, 'whether you can make words mean so many different things.'
'The question is,' said Humpty Dumpty, 'which is to be master — that's all.'

Clarkson, 2019

- When you have set your kids to work on a problem solving exercise;
  - What is it you are looking for as you interact with them?
  - What are they looking for from each other?
  - What are they looking for from you?
  - Has any of this got to do with language?

The Mathematics curriculum focuses on developing increasingly sophisticated and refined;

- mathematical understanding,
- fluency,
- reasoning,
- modeling and problem-solving.

These capabilities enable students to respond to familiar and unfamiliar situations by employing mathematics to make informed decisions and solve problems efficiently.

That is hugely different from 'just' quickly solving various number bonds.

Clarkson, 2019

- I am about to give you a problem to solve in groups.
- What are the behavious you should be noticing as you observe each other?
  - [What is it you are looking for as you interact with members of your group?
  - What are they looking for from other members of the group?
  - What are they looking for from you?]
- IMPORTANTLY: Which of these deal with language issues?

- How far did Little Red Riding Hood go?
  - Use the A3 sheets to devise your group solution

Clarkson, 2019

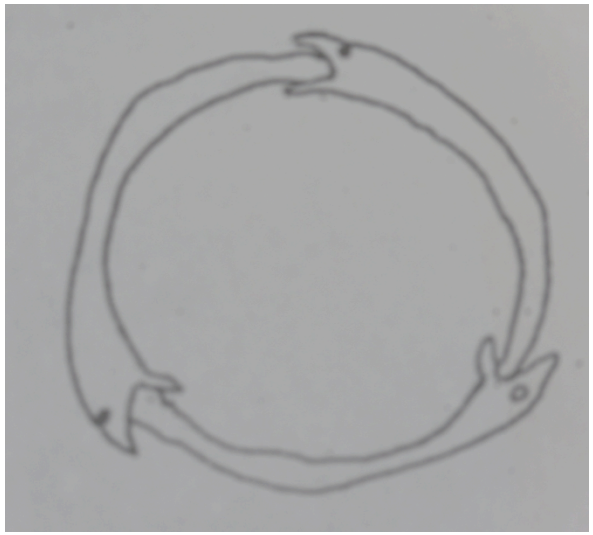
- So what did you observe about your colleagues' language during this problem solving exercise?
- Was the language used in this question very mathematical?
- The next problem I want you to attempt individually.
- BUT be thinking about what is the process(es)that you are employing.

Clarkson, 2019

Three snakes all the same size form a circle.

The diameter of the circle is 4 meters.

Each snake begins to swallow its neighbor at the same moment and at the same speed.

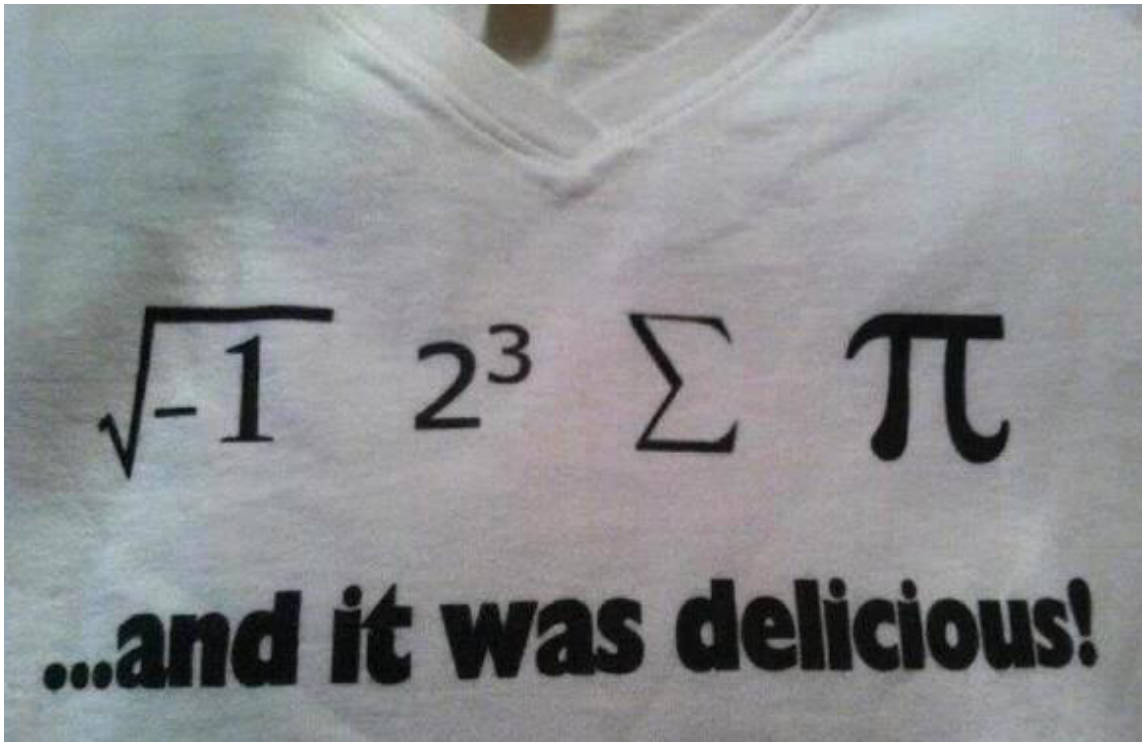


- When each snake has swallowed ¾ of its neighbor's length,

  (i) What is the diameter of the circle?

  (ii) What fraction of the length of each snake will be inside itself?

- What did you do to begin the problem?
- What then?



I ate some pie

Clarkson, 2019

- "We broke the problem into eight different stages and at that time, the view in the scientific community was the chances of getting through all eight stages were near to impossible. … Over a period of 10 years we systematically solved those eight challenges. … It all culminated in creating the first single atom transistor in 2012."
  - Prof Michelle Simmons; UNSW
    - 2018 Australian of the Year

Clarkson, 2019

- We broke the problem into eight different stages

- We often have a solution routine composed of various sequences of steps or procedures we follow in solving problems (an algorithm).
- At least some of the steps are language based.
- Errors may be made at any step

Some ‘real life’ mathematics

What does ‘real life’ and/or ‘authentic’ mean in a maths context?

Clarkson, 2019

- Natalie paddled 402km of the Murray River in her canoe over 6 days. She paddle the same distance each day. Now how far did Natalie paddle each day?

- Show video

- What context would make this 'real life'?
- What changes could we make to help make this problem more 'authentic'?
- Should we?

1. Read the question to me. [Reading]
2. (a) What is it you are trying to find out?

   [general comprehension]

   (b) What does this … mean? (pointing to the word, phrase or symbol; don't say the word or phrase)
   [specific comprehension]
3. How will go about solving the problem? (Maybe add, 'Is there anything else you have done that will help here?' Let them think about reference to their own notes, teacher's material, etc.) [Transformation]
4. Solve the problem? [Processing]
5. Have you finalised the solution? (Write it out.)
   [Encoding]


How is my family the same as and different from my parents and grandparents?
What is important to my family?
Clarkson, 2019

Clarkson, 2019

Error Types		% initial errors	
		6th low	6th normal
1. Reading	Hierarchical ↓	13	5
2. Comprehension		22	8
3. Transformation		12	25
4. Process Skills		26	32
5. Encoding		2	2
6. Unknown		25	28
7. Motivation		0	0
8. Question Form		0	0

Clarkson, 2019

- Language errors start early and continue.
- Problems progress beyond one step types and hence errors become more complex to isolate.
- Loops get built into the solving process, although the overall pattern of the Newman Model is still evident.
- Language errors are important and need to be addressed in our teaching of mathematics.

Clarkson, 2019

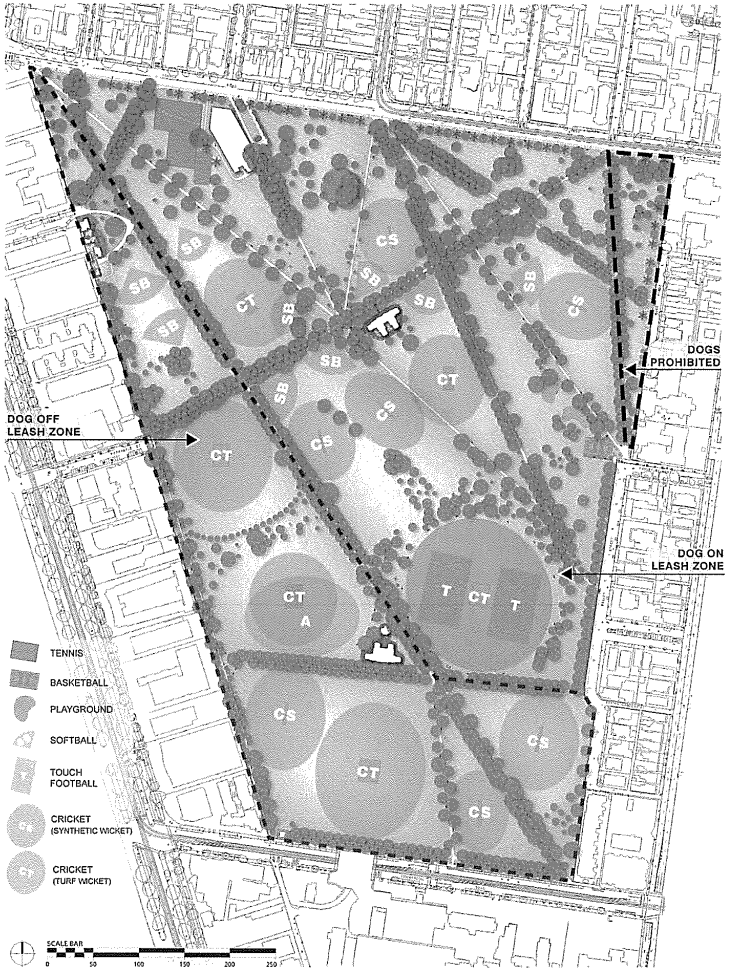
- Indeed this language is part and parcel of ‘doing mathematics’.

- These skills / abilities are not something separate from mathematics that some other teachers need to take care of.

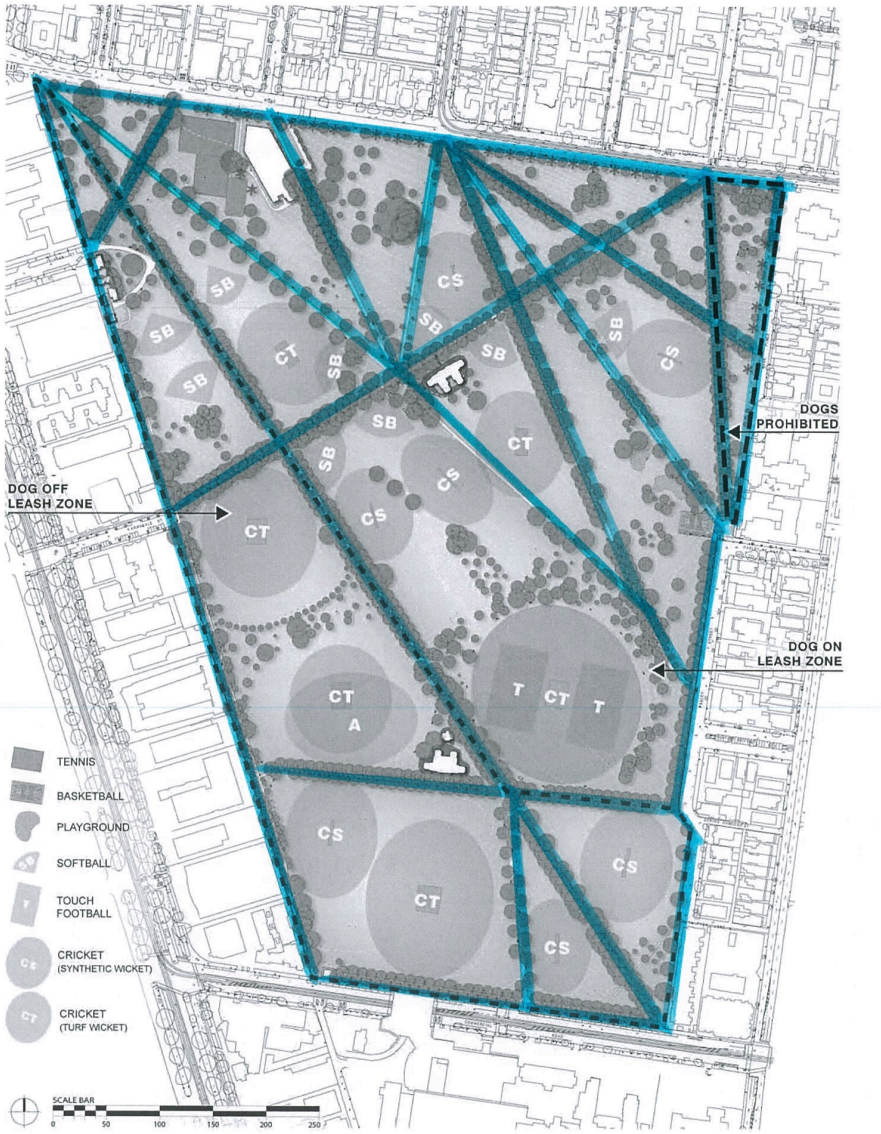
Clarkson, 2019

- Here's another problem to try.
- Unlike the last two problems, but like the first one, you will need to break it into parts.
- Again, in small group's attempt this problem, but pay attention to each other's language use.
- Use the A3 sheet to develop your joint solution.

A walk in Fawkner Park



Clarkson, 2019



Clarkson, 2019

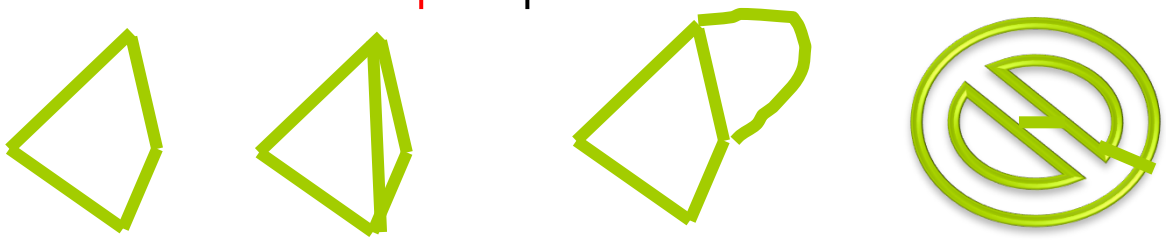
- *You can only walk down a pathway (or part of a pathway) once.*
- *Directly crossing over a pathway is not the same as walking down it.*
- *Walking 'all' the parts of a pathway at different times will count as walking down the whole pathway.*
- ***Can you find a route so that you walk just once down every pathway?***
- ***If you cannot find such a route, can you;***
  - ***Delete a path or section of a path that will allow this, or***
  - ***Add a path that will allow this?***

Clarkson, 2019

- Did you break this problem into parts?
- Was it a 'real life' problem for your students?
- Was it closed or open ended?
- Was there multiple solutions?
- So what did you observe about your colleagues' language during this problem solving exercise?

Clarkson, 2019

- Start with simple problems of close networks:



- The problem is all to do with the number of odd and even vertices.
- Which networks:
  - can / can't be travelled,
  - how many odd vertices,
  - how many even vertices,
  - which networks can be travelled but start and finish at the same vertex?
  - [PATTERNS / TABLES]
- You need to investigate EULER trails and circuits, and the Koenigsberg bridge problem.

- Problems are best if:
  - Unfamiliar to your students
  - Do not have a set solution
  - Are not exemplars of recently taught ideas
- Should have a mixture of problems that are:
  - Straight forward and closed
  - Multi step
  - Open ended
  - Multiple solutions

Clarkson, 2019

Polya

{Newman}

- Understand the problem
  - reading & comprehension
- Devise a plan
  - transformation
- Carry out the plan
  - process
- Look back
  - encode & check

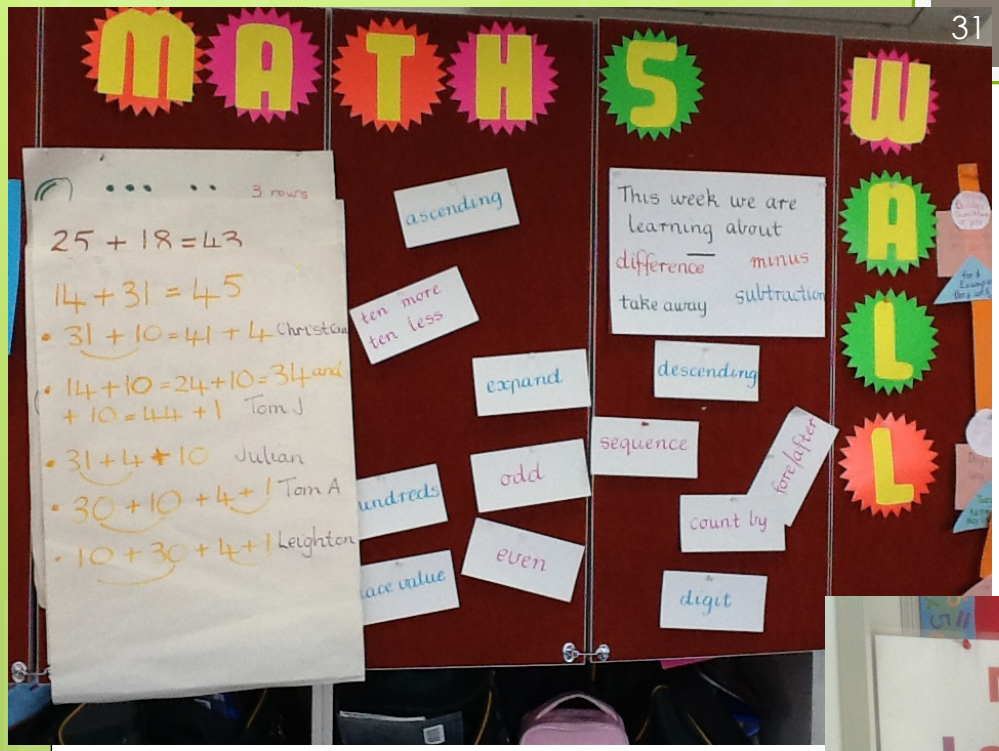
Clarkson, 2019

Polya: Devise a plan

[Newman: transformation stage]

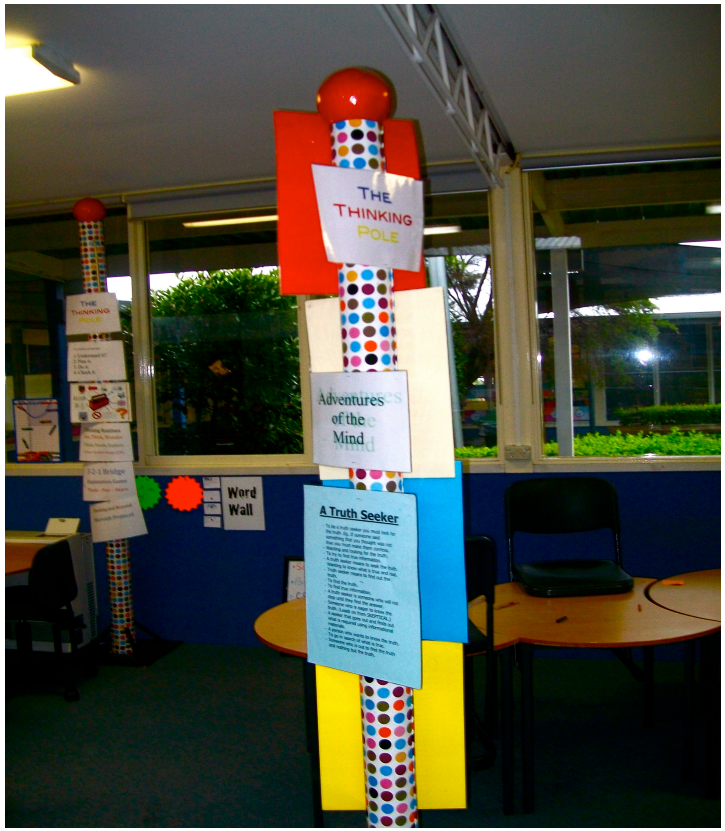
- Organize data systematically
- Look for a pattern
- Check your working regularly
- Use the pattern
- Make an orderly list
- Draw a picture
- Eliminate possibilities
- Solve a simpler problem
- Use symmetry
- Use a model
- Consider special cases
- Work backwards
- Look for a general rule
- Be ingenious

Clarkson, 2019



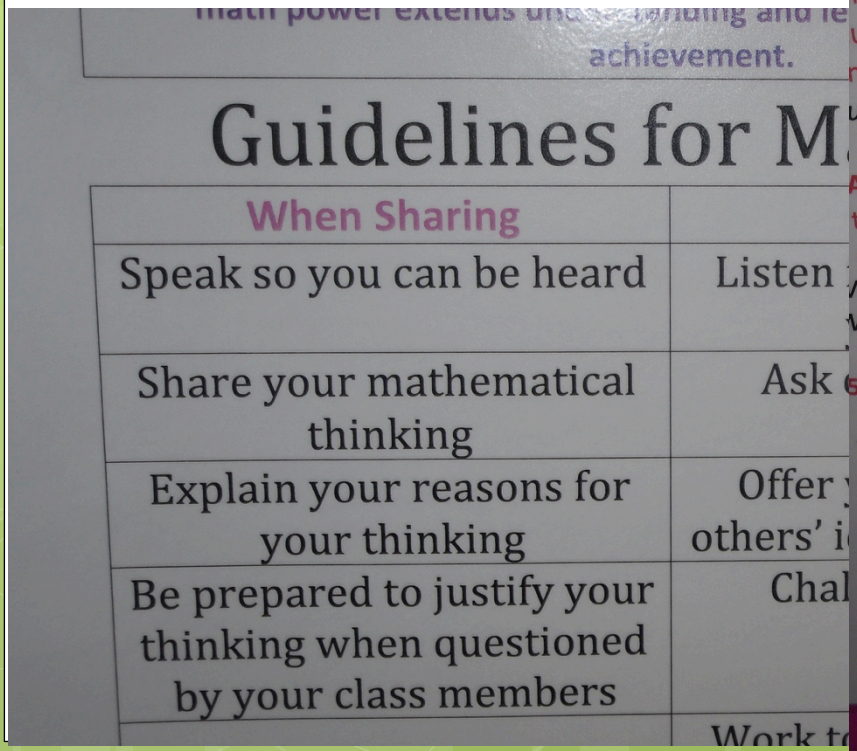
Math walls are good

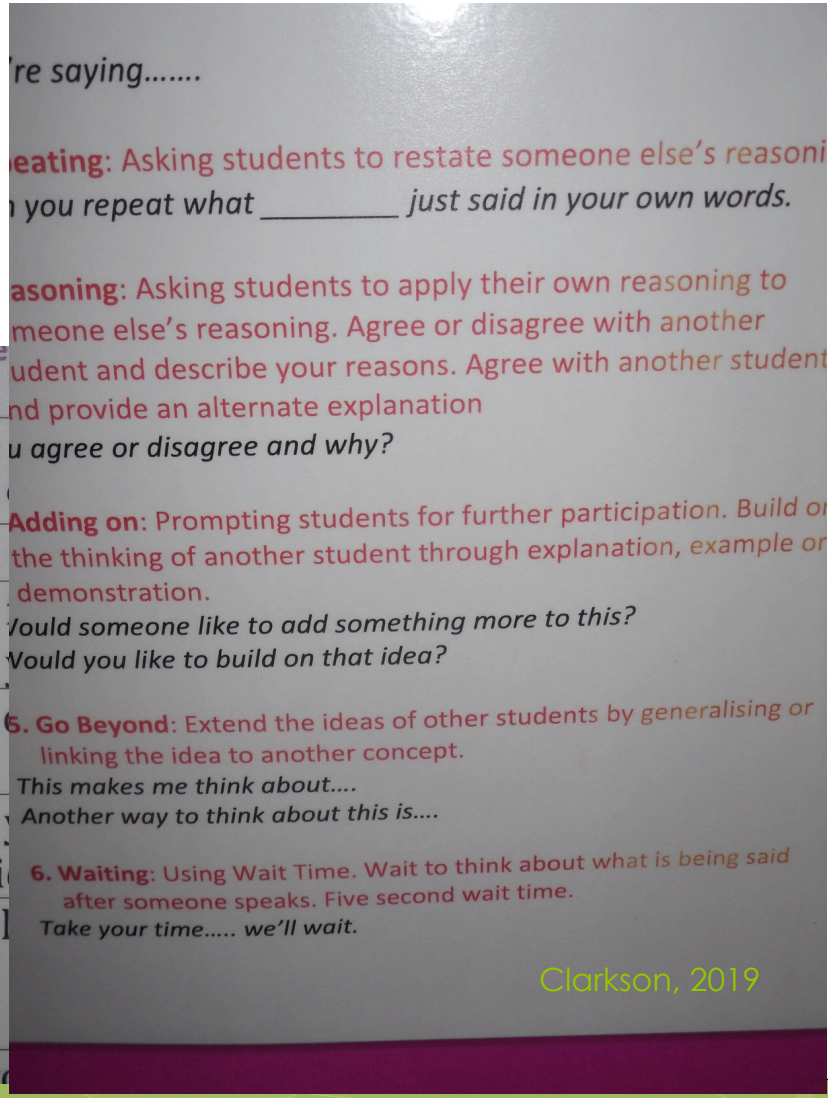
Movable math pillars are even more accessible



Clarkson, 2019

Don't overlook reminders in the staff room



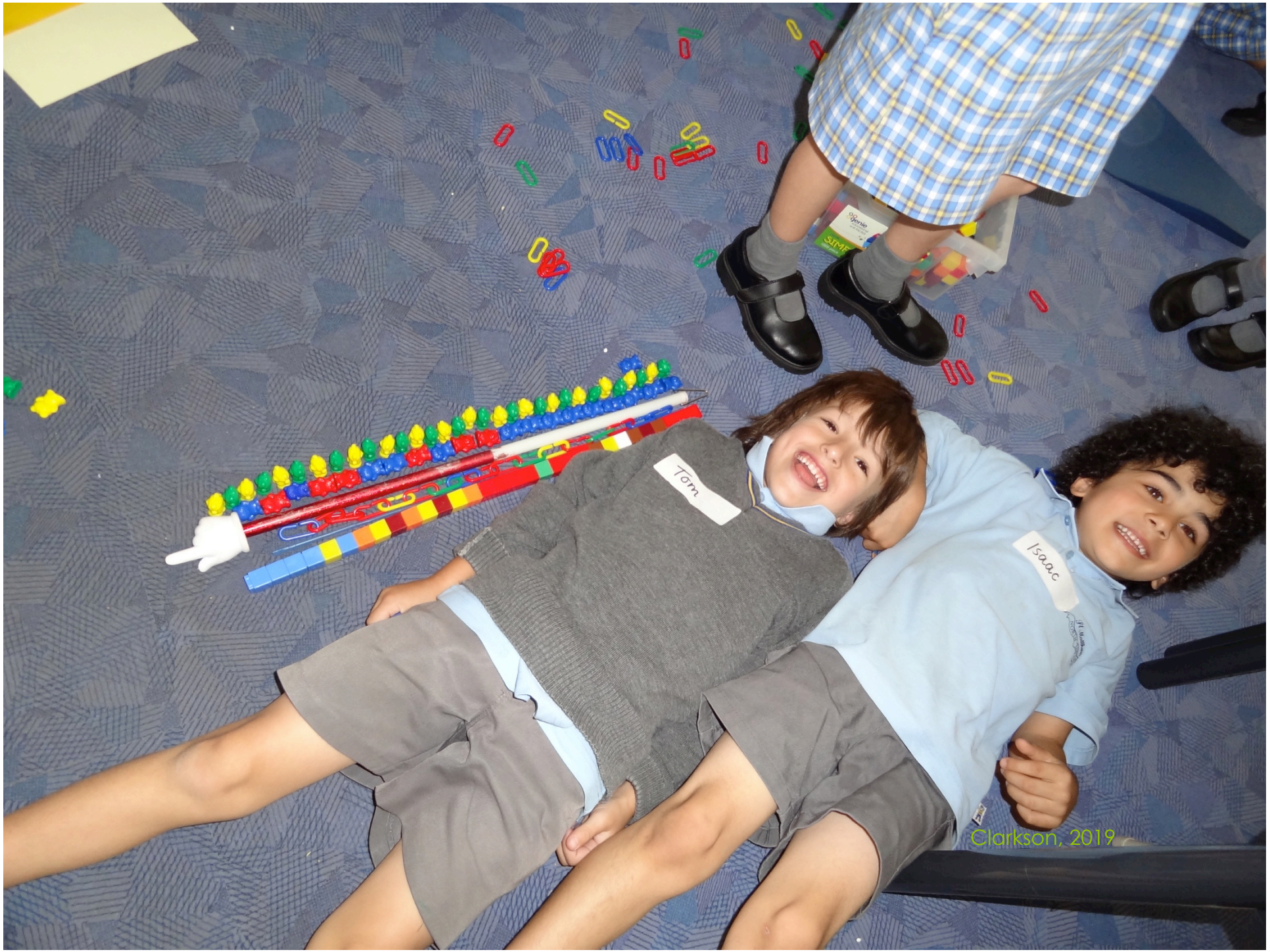


- ***A grid of mathematical words:***
  - ***The possibilities are endless***

- ***Logical connectives:***
  - ***Often forgotten,***
  - ***Vitally important***

- Kids will make up their own ways of talking about mathematics.
- We need to listen and learn.
- Then decide whether their language aligns with the community's way of discussing mathematics.
- Their language can be a very powerful bridge in solving the problems they are in tune with.

Clarkson, 2019

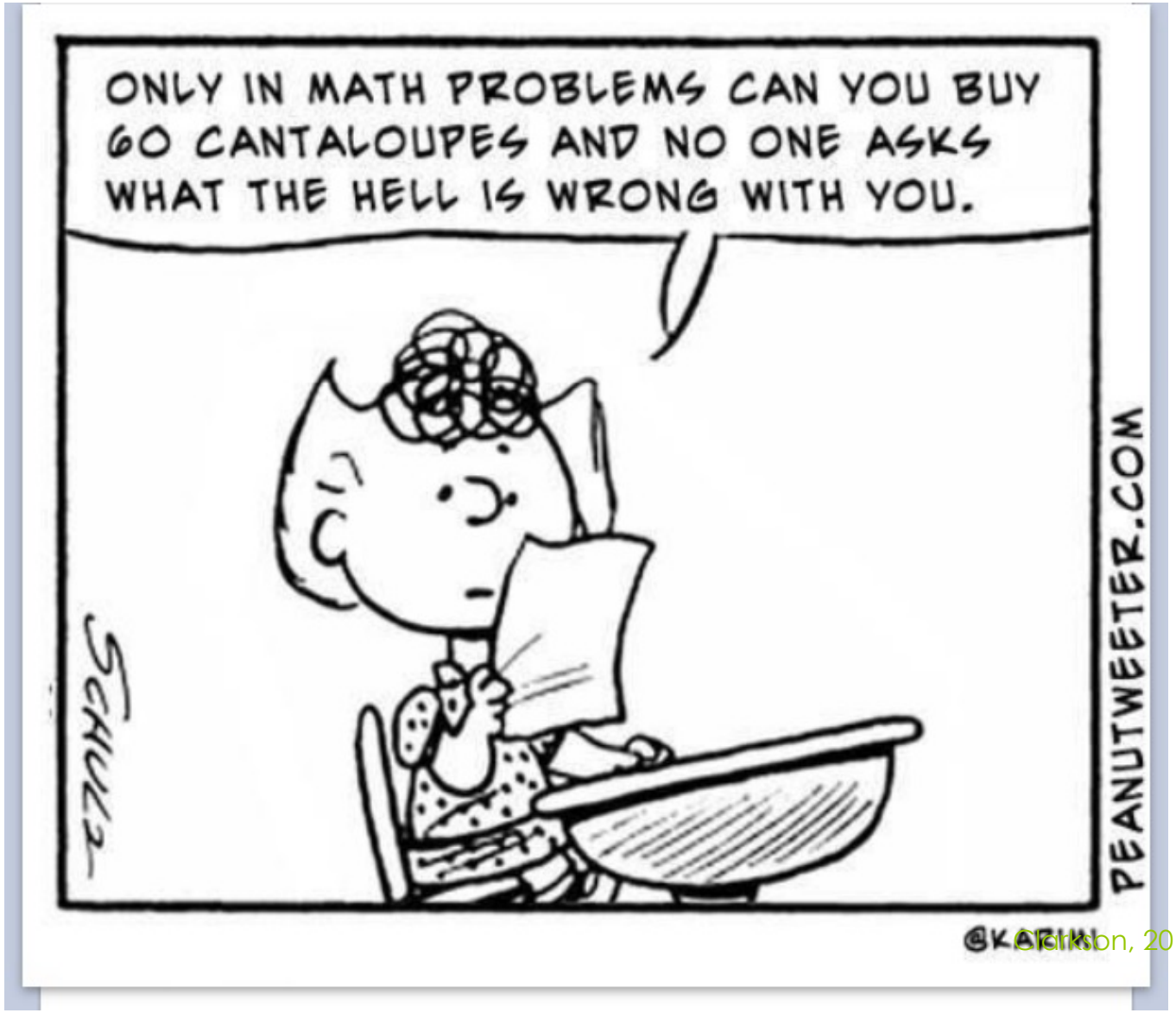

Tom
Isaac
Clarkson, 2019

Silence is just as important

Clarkson, 2019

- Novel situations can provide great mathematical problem situations to explore
- BUT
  - Are they interested in; How long Rapunzel's hair was? Does the knight weigh too much? Is that of concern?
  - Does it matter to them: How long is the basketball court that we play on at lunchtime?
  - What is the population growth of the world? When will they care about that?

Clarkson, 2019



Clarkson, 2019

- So what problems should be solved in mathematics?
- Is there a difference between numeracy and mathematics?
- Is it still a useful distinction to make?
- Why do we do mathematics then?

Clarkson, 2019

- Moving between forms (verbal, written, graph, diagram and symbol) is good.
- Do we alert students to key words (good)?
  - But also to other language forms?
- Do we use maths language often, precisely and consistently?
  - Do our students?
- Do we use questions that expect justification, explanation?
  - Do our students?
- To whom do we ask questions?
- Do our text books just state facts or do they relate ideas to the underlying fabric of maths?

Clarkson, 2019

- ‘When I use a word,’ Humpty Dumpty said in rather a scornful tone, ‘it means just what I choose it to mean – neither more nor less.’

- Well yes
- But
- The key is, what does he mean by ‘mean’?
- Language is a social construct
- Therefore ‘meaning’ must be agreed

Clarkson, 2019

‘The question is,’ said Alice, ‘whether you can make words mean different things – that’s all.’

‘The question is,’ said Humpty Dumpty, ‘which is to be master–that’s all’

- Humphrey asks, ‘are we to master language, or is language to master us?’
- First, language is a human creation: we didn’t find it lying around, ready-made.
- But second, each of us is born into a linguistic world and a linguistic community. Whether we like it or not, that provides us with our basic conceptual categories, and shapes the way we perceive the world.

Clarkson, 2019

- And that means
- We need to enculturate our kids into the mathematical world which throughout their lives they will be touching on, whether they like it or not, and whether they realize it or not.
- BUT knowing the language of mathematics will certainly help.
- Help in all sorts of ways, including solving real problems in their world.

We don't want them to be isolates because they cannot communicate in appropriate language

Clarkson, 2019

But much of their chatter will be in their own everyday language, as will be their thinking. They will swap between that and maths language.

Clarkson, 2019

Thanks for participating

Philip.Clarkson@acu.edu.au
@PhilipCClarkson

Find publications on
Google Scholar; Academia; Researchgate

Clarkson, 2019