THE MATHEMATICAL ASSOCIATION OF VICTORIA

ACER

TEACHER NOTES

MATHEMATICS IN CAREERS

Investigation - Comparing student ability and question difficulty: exponential formula and non-linear graphs

Key career focus for this investigation: Educational researcher
Related career areas: Psychometrician

THINKING ABOUT CAREERS

- Brainstorm educational research professions you can think of where maths is frequently used. Use https://joboutlook.gov.au to explore information technology related career pathways that include use of mathematics. *How is maths used in these scenarios? What maths is used in these scenarios?*
- This task focuses on how maths is used in educational research, through investigating student ability and question difficulty on an assessment.
- Explore careers such as educational researchers, statistician, psychometrician to discover how maths is used in these. For a more extensive list of careers related to this task, with a maths / science focus, refer to the table at the end of the task and explore the maths used in these jobs.

MATHEMATICS IN EVERYDAY LIFE AND CAREERS

Mathematical focus for this investigation

- create graphs to show patterns in data
- explore the connection between algebraic and graphical representations of relations

Scientists including educational research professionals model relationships comparing different scenarios. The relationship may be linear or other. They graph data and compare the graphs, changing one variable to see the effect on the other to test hypothesis.

Linear equations use one or more variables where one variable is dependent on the other. Many people use linear equations every day, even if they do the calculations in their head without drawing a line graph.

Brainstorm and share scenarios where this mathematics may be used in educational research to solve problems.

MATHEMATICAL INVESTIGATION
STUDENT ABILITY VS QUESTION DIFFICULTY

TEACHER INFORMATION

LINKS TO VICTORIAN CURRICULUM

Mathematics links to Victorian Curriculum Level 10	Application to work and life
Patterns and algebra Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term (VCMNA333) **Real numbers** Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (VCMNA301) Explore the connection between algebraic and graphical representations of relations such as simple quadratic, reciprocal, circle and exponential, using digital technology as appropriate (VCMNA339)	Linear equations use one or more variables where one variable is dependent on the other. Almost any situation where there is an unknown quantity can be represented by a linear equation, like figuring out income over time, calculating mileage rates, or predicting profit. Many people use linear equations every day, even if they do the calculations in their head without drawing a line graph. Scientists model relationships comparing different scenarios. The relationship may be linear or non-linear. They graph data and compare the graphs, changing one variable to see the effect on the other to test hypothesis. Graphs can be used to explore and make conclusions about data. Graphs can highlight a trend that the data shows that the raw data can not easily convey. Businesses use data and graphs every day to analyse financial, social and company trends.

PROFICIENCY FOCUS: VICTORIAN CURRICULUM

This investigation focuses on: Understanding, Fluency, Problem Solving

Understanding refers to students building a robust knowledge of adaptable and transferable mathematical concepts and structures. This investigation focuses on: • Describe their mathematical thinking. • Interpret mathematical information.	**Fluency** describes students developing skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently, and appropriately, and recalling factual knowledge and concepts readily. This investigation focuses on: • Solving simple equations arising from formulas. • Re-arranging expressions to make a specified variable the subject. • Solving problems using the fact that the product of the gradients of perpendicular lines is –1 and conversely that if the product of the gradients of two lines is –1 then they are perpendicular. • Solving linear equations, including those involving one or two simple algebraic fractions, and checking solutions by substitution. • Representing word problems, including those involving fractions, as equations and solving them to answer the question.	**Problem Solving** is the ability of students to make choices, interpret, formulate, model and investigate situation, select and use technological functions and communicate solutions effectively. This investigation focuses on: • Students applying their existing strategies to seek solutions.

MATHEMATICAL INVESTIGATION
STUDENT ABILITY VS QUESTION DIFFICULTY

INVESTIGATION BACKGROUND

When analysing the results from a test question it is useful to compare the difficulty of the question with the performance by students of different ability. We expect that higher ability students will be able to get the more difficult questions correct, whereas lower ability students will not be able to get them correct. In this investigation you will explore the relationship between *student ability* and *question difficulty*, using a spreadsheet to model the given formula and to represent the results as both tables and graphs.

Exponential formula

We can never be 'certain' that a particular student will perform as expected for their ability, so the formula we use calculates the probability that a given student will get a particular question correct. The formula uses a special number as the base for the exponential terms: base e, where e is an irrational number (an infinite, non-recurring decimal, like π) with an approximate value of 2.718.

Let B represent the ability of a student.
Let D represent the difficulty of a question.
Both B and D are measured on the same scale, as shown in Figure 1 on the next page:

- –3 is a typical value for a lower ability student, or a very easy question
- most values of B and D lie between about –1 and +1
- +3 is a typical value for a high ability student, or a very difficult question

Let P represent the probability that a student with ability B successfully answers a question with difficulty D, where:

$$P = \frac{e^{B-D}}{1 + e^{B-D}}$$

This is the formula you will be exploring in this investigation.
Note that this formula only applies to questions such as true/false or multiple choice, where there is one correct answer, and no 'part marks' are awarded; these are referred to as 'dichotomous' items.

Ask yourself:

- 1. What happens when $B = D$?
  The probability is 50% when $B = D$.
- 2. What happens when $B > D$ and vice-versa? Try some values using the e^x button on your calculator.
  When $B > D$ the probability of successfully answering the question is greater than 50%.
  When $D > B$ the probability of successfully answering the question is less than 50%.

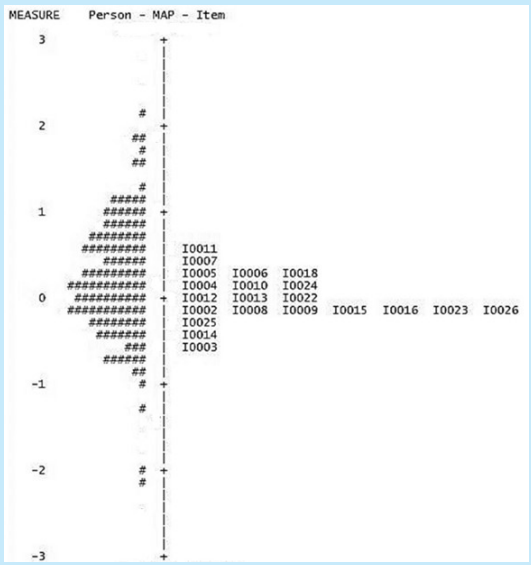


Figure 1: Map showing a typical spread of person ability and item difficulty. Adapted from http://sgo.sagepub.com/content/5/3/2158244015601717, retrieved 31/3/2016.

This map shows:

- the ability/difficulty scale from –3 to +3 at the left of the page
- each # represents a group of people with the same ability score; you can see that most of the scores are between –1 and +1
- on the right it shows the difficulty of each question (or item); item I0011 was the hardest with a difficulty of about +0.6

For these results the range of difficulty of the questions is not a good match with the range of ability of the students: the questions are too easy for the high ability students and too hard for the low ability students.

MATHEMATICAL INVESTIGATION
STUDENT ABILITY VS QUESTION DIFFICULTY

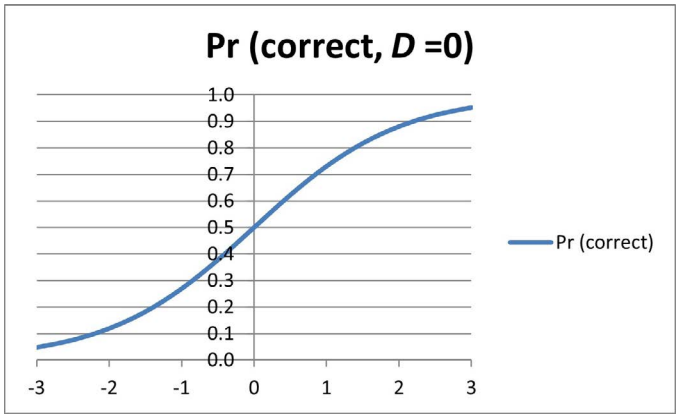
YOUR INVESTIGATION

Investigate the relationship between student ability and question difficulty for dichotomous items using the exponential formula. A possible approach is suggested below, but feel free to vary it if you wish:

PART 1

- Set up a spreadsheet with the first two columns showing values of *D* and *B*, and with the third column containing the probability formula that we are investigating, see the table on the right.
- As shown, you could start with the value of *D* = 0 and vary the values of *B* from –3 to +3.
- The third column contains the formula in spreadsheet form, where EXP means e^x

  =(EXP(B2-A2))/(1+EXP(B2-A2))
- select the values in the second column (*B*) and third column (*P*) and draw a scatterplot showing how, with *D* fixed at zero, the value of *P* changes as the value of *B* increases. The graph might look like this:



Item difficulty (*D*)	Student ability (*B*)	Pr(correct)
0	-3	0.05
0	-2.5	0.08
0	-2	0.12
0	-1.5	0.18
0	-1	0.27
0	-0.5	0.38
0	0	0.50
0	0.5	0.62
0	1	0.73
0	1.5	0.82
0	2	0.88
0	2.5	0.92
0	3	0.95

PART 2

Copy your table and graph into your report document, and comment on the trends:

- How do lower ability students perform, compared with higher ability students?
- What is the probability of success when a *B* = 0 student attempts a *D* = 0 question?
  When *B* = *D* the probability of success is 50%.

MATHEMATICAL INVESTIGATION
STUDENT ABILITY VS QUESTION DIFFICULTY

PART 3

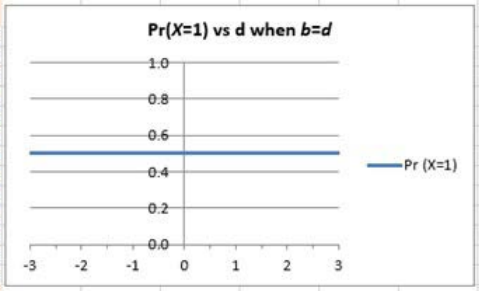
Continue your investigation systematically by using other fixed values for *D*, again comparing the trends in the table of values and in the graphs.

Graph 1: *B* = *D*

dichotomous scale (1 or 0)

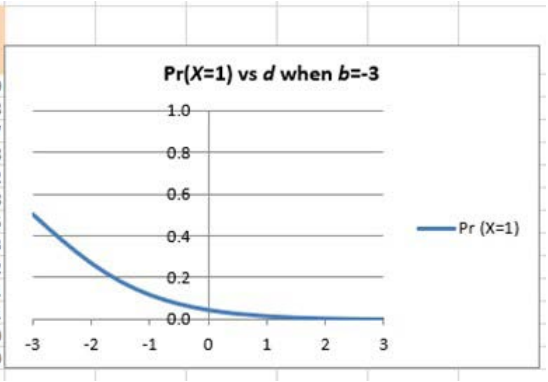
person ability (*b* logits)	item difficulty (*d* logits)	Pr (X=1)
-3	-3	0.5
-2.5	-2.5	0.5
-2	-2	0.5
-1.5	-1.5	0.5
-1	-1	0.5
-0.5	-0.5	0.5
0	0	0.5
0.5	0.5	0.5
1	1	0.5
1.5	1.5	0.5
2	2	0.5
2.5	2.5	0.5
3	3	0.5

if *b*=*d* then Pr(*X*=1) is 0.5



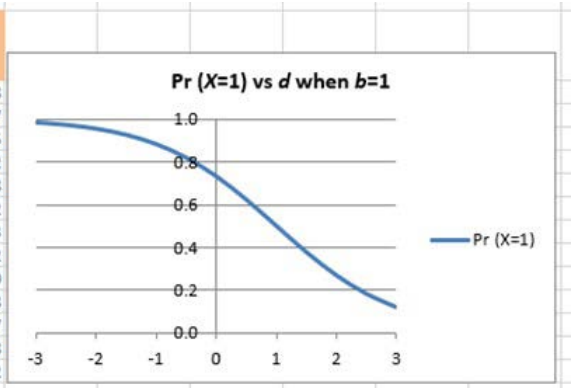
Graph 2: *B* = –3

person ability (*b* logits)	item difficulty (*d* logits)	Pr (X=1)
-3	-3	0.50
-3	-2.5	0.38
-3	-2	0.27
-3	-1.5	0.18
-3	-1	0.12
-3	-0.5	0.08
-3	0	0.05
-3	0.5	0.03
-3	1	0.02
-3	1.5	0.01
-3	2	0.01
-3	2.5	0.00
-3	3	0.00



Graph 3: *B* = 1

person ability (*b* logits)	item difficulty (*d* logits)	Pr (X=1)
1	-3	0.98
1	-2.5	0.97
1	-2	0.95
1	-1.5	0.92
1	-1	0.88
1	-0.5	0.82
1	0	0.73
1	0.5	0.62
1	1	0.50
1	1.5	0.38
1	2	0.27
1	2.5	0.18
1	3	0.12



MATHEMATICAL INVESTIGATION
STUDENT ABILITY VS QUESTION DIFFICULTY

Write a summary of what you have learnt from this investigation in terms of the mathematics used, the implications for testing, and the way you conducted the investigation.

Students should identify some trends (for example)

- when $B = D$ the chance of getting the question correct is 50% (as shown in graph 1).
- when a student has a low ability ($B = -3$) then the best chance they have of successfully answering a question is 50% (when $D = -3$ also). As D increases the chance of success decreases and eventually reaches 0% when $D = 3$ (shown in graph 2).
- when a student has an ability $B = 1$ the chance of successfully answering a question when the difficulty = -3 ($D = -3$) is very high at 98%. As D increases the probability of success decreases slowly at first. When $B = D = 1$ the chance of success is 50%. As D continues to increase then the chance of success decreases more quickly until reaching 12% when $D = 3$ (shown in graph 3).

CAREERS RELATED TO THIS INVESTIGATION

Refer to the student investigation, it provides:

- An extensive table of careers related to this investigation
- Further career references

CAREERS ACTIVITIES

Refer to the student investigation, it provides:

- A table of the top 10 rated jobs of 2021. This data comes from careercast.com. Have students investigate the jobs specific to this investigation.

INDUSTRY PARTNER

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