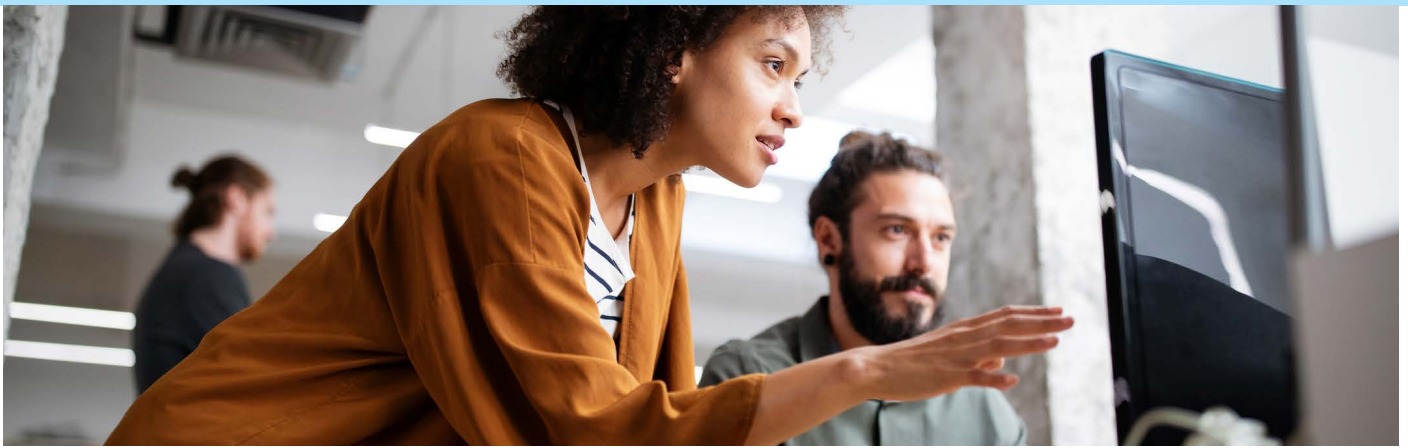# THE MATHEMATICAL ASSOCIATION OF VICTORIA

ACER

MATHEMATICS IN CAREERS

Investigation - Comparing student ability and question difficulty: exponential formula and non-linear graphs

Key career focus for this investigation: Educational researcher
Related career areas: Psychometrician

THINKING ABOUT CAREERS

- Brainstorm educational research professions you can think of where maths is frequently used.
  Use https://joboutlook.gov.au to explore information technology related career pathways that include use of mathematics. *How is maths used in these scenarios? What maths is used in these scenarios?*

- This task focuses on how maths is used in educational research, through investigating student ability and question difficulty on an assessment.

- Explore careers such as educational researchers, statistician, psychometrician to discover how maths is used in these. For a more extensive list of careers related to this task, with a maths / science focus, refer to the table at the end of the task and explore the maths used in these jobs.

MATHEMATICS IN EVERYDAY LIFE AND CAREERS

Mathematical focus for this investigation

- create graphs to show patterns in data
- explore the connection between algebraic and graphical representations of relations

Scientists including educational research professionals model relationships comparing different scenarios. The relationship may be linear or other. They graph data and compare the graphs, changing one variable to see the effect on the other to test hypothesis.

Linear equations use one or more variables where one variable is dependent on the other. Many people use linear equations every day, even if they do the calculations in their head without drawing a line graph.

Brainstorm and share scenarios where this mathematics may be used in educational research to solve problems.

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MATHEMATICAL INVESTIGATION
STUDENT ABILITY VS QUESTION DIFFICULTY

INVESTIGATION BACKGROUND

When analysing the results from a test question it is useful to compare the difficulty of the question with the performance by students of different ability. We expect that higher ability students will be able to get the more difficult questions correct, whereas lower ability students will not be able to get them correct. In this investigation you will explore the relationship between *student ability* and *question difficulty*, using a spreadsheet to model the given formula and to represent the results as both tables and graphs.

Exponential formula

We can never be 'certain' that a particular student will perform as expected for their ability, so the formula we use calculates the probability that a given student will get a particular question correct. The formula uses a special number as the base for the exponential terms: base e, where e is an irrational number (an infinite, non-recurring decimal, like π) with an approximate value of 2.718.

Let B represent the ability of a student.
Let D represent the difficulty of a question.
Both B and D are measured on the same scale, as shown in Figure 1 on the next page:
- −3 is a typical value for a lower ability student, or a very easy question
- most values of B and D lie between about −1 and +1
- +3 is a typical value for a high ability student, or a very difficult question

Let P represent the probability that a student with ability B successfully answers a question with difficulty D, where:

$$P = \frac{e^{B-D}}{1 + e^{B-D}}$$

This is the formula you will be exploring in this investigation. Note that this formula only applies to questions such as true/false or multiple choice, where there is one correct answer, and no 'part marks' are awarded; these are referred to as 'dichotomous' items.

Ask yourself:
- 1. What happens when $B = D$?
  The probability is 50% when $B = D$.
- 2. What happens when $B > D$ and vice-versa? Try some values using the e^x button on your calculator.
  When $B > D$ the probability of successfully answering the question is greater than 50%.
  When $D > B$ the probability of successfully answering the question is less than 50%.

This map shows:
- the ability/difficulty scale from −3 to +3 at the left of the page
- each # represents a group of people with the same ability score; you can see that most of the scores are between −1 and +1
- on the right it shows the difficulty of each question (or item); item I0011 was the hardest with a difficulty of about +0.6
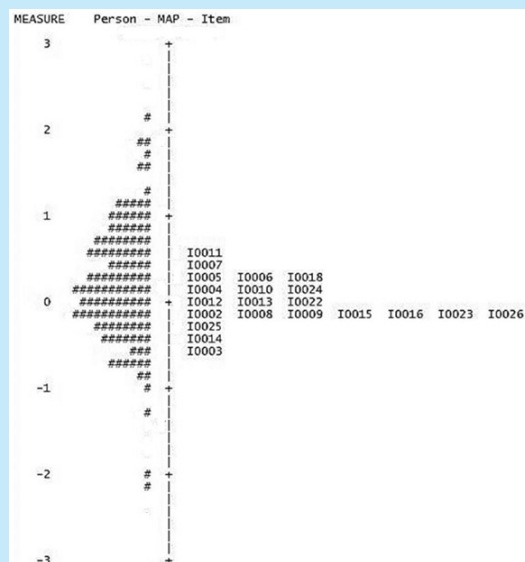
For these results the range of difficulty of the questions is not a good match with the range of ability of the students: the questions are too easy for the high ability students and too hard for the low ability students.

Figure 1: Map showing a typical spread of person ability and item difficulty. Adapted from http://sgo.sagepub.com/content/5/3/2158244015601717, retrieved 31/3/2016.

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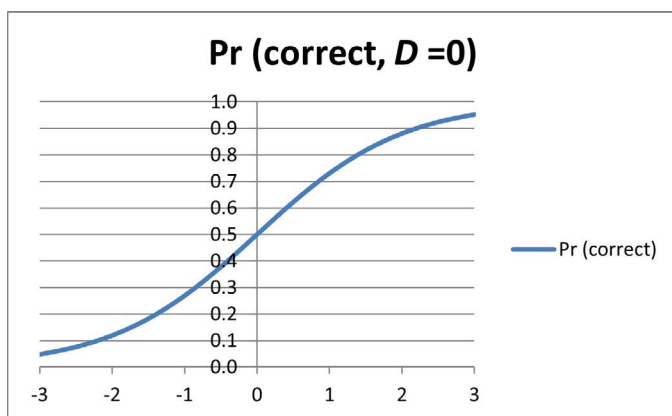
YOUR INVESTIGATION

Investigate the relationship between student ability and question difficulty for dichotomous items using the exponential formula. A possible approach is suggested below, but feel free to vary it if you wish:

PART 1

- Set up a spreadsheet with the first two columns showing values of D and B, and with the third column containing the probability formula that we are investigating, see the table on the right.
- As shown, you could start with the value of $D = 0$ and vary the values of B from –3 to +3.
- The third column contains the formula in spreadsheet form, where EXP means e^x

  =(EXP(B2-A2))/(1+EXP(B2-A2))

- select the values in the second column (B) and third column (P) and draw a scatterplot showing how, with D fixed at zero, the value of P changes as the value of B increases. The graph might look like this:

Item difficulty (D)	Student ability (B)	Pr(correct)
0	-3	0.05
0	-2.5	0.08
0	-2	0.12
0	-1.5	0.18
0	-1	0.27
0	-0.5	0.38
0	0	0.50
0	0.5	0.62
0	1	0.73
0	1.5	0.82
0	2	0.88
0	2.5	0.92
0	3	0.95

Pr (correct, *D* =0)

PART 2

Copy your table and graph into your report document, and comment on the trends:
- How do lower ability students perform, compared with higher ability students?
- What is the probability of success when a $B = 0$ student attempts a $D = 0$ question?

PART 3

Continue your investigation systematically by using other fixed values for D, again comparing the trends in the table of values and in the graphs.

Write a summary of what you have learnt from this investigation in terms of the mathematics used, the implications for testing, and the way you conducted the investigation.

MATHEMATICAL INVESTIGATION STUDENT ABILITY VS QUESTION DIFFICULTY

CAREERS RELATED TO THIS INVESTIGATION

EDUCATIONAL RESEARCHER/RESEARCH DIRECTOR

Career description	Key skills required	More information
Educational research is an area that creates and conducts studies that yield information regarding aspects of education. This research can provide great insight into current issues within the field of education, from preschool through to university.	• Competent in data collection and analysis • Reasoning and problem solving • Teamwork and communication • Scientific method and quantitative skills • Flexibility and adaptability	https://www.abadegreeprograms.net/faq/what-is-an-educational-researcher/

EDITOR/EDITORIAL ASSISTANT

Career description	Key skills required	More information
Editors are responsible for checking facts, spelling, grammar, and punctuation. They are also responsible for ensuring that a written piece corresponds with in-house style guides and feels polished and refined when done. An editorial assistant supports the editor in writing, editing, and commissioning copy and operating the online content management system for industry magazines, online publications. Provides production, marketing and administrative support to the editor and the publication's digital sales specialist.	• Source and create content in all formats (articles, infographics, videos and podcasts). • Team work and communication • Operate the online content management system under the guidance of the editor and digital sales specialist to support editorial and advertising. • Ensure text and illustrative material conforms to current copyright law, including permissions clearance.	https://acer.bamboohr.com/jobs/view.php?id=124

PSYCHOMETRICIAN

Career description	Key skills required	More information
• Psychometrics is the field of science associated with the development of tool that measure knowledge, skills and attributes. A psychometrician develops assessments such as examinations for educational, employment or professional credentialing purposes.	• Experience in quantitative measurement and statistics. • A good eye for detail and sharp analytic skills. • Teamwork and communication • Research experience. • The ability to effectively explain complicated ideas. • The ability to conduct high-level interaction with a range of corporate team members, consultants, clients etc.	https://careersinpsychology.org/become-a-psychometrician/

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MATHEMATICAL INVESTIGATION STUDENT ABILITY VS QUESTION DIFFICULTY

CAREERS RELATED TO THIS INVESTIGATION

OTHER RELATED CAREERS TO EXPLORE

- Statistician
- Data analyst
- Survey researcher
- Curriculum designer
- Educational consultant

CAREERS ACTIVITIES

THE 10 BEST RATED JOBS OF 2021			
Rank	Career	Median salary	Projected growth
1	Data Scientist	$98 230	33%
2	Genetic Counsellor	$85 700	21%
3	Statistician	$92 270	35%
4	Medical Services Manager	$104 280	32%
5	Mathematician	$110 860	33%
6	University Professor	$80 790	9%
7	Operations Research Analyst	$86 200	25%
8	Information Security Analyst	$99 730	31%
9	Actuary	$111 030	18%
10	Software Engineer	$110 140	22%

Data from Careercast.com.

Select one of the careers, from either of the tables above, that interests you and find out:

- What are the tasks involved in this career? What may a typical day look like?
- What level of education or qualifications do you need to do this career?
- What are some other similar or related careers?
- What mathematics skills would be used in this career?
- Where does the career you have selected, to investigate, rank according to careercast.com?
- How many people in Australia are currently employed in this career (or field)?

INDUSTRY PARTNER

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FURTHER CAREER REFERENCES

Australian Jobs Report 2021

www.nationalskillscommission.gov.au/australian-jobs-report
An overview of trends in the Australian labour market to support job seekers and employment service providers, career advisers, those considering future training and people interested in labour market issues.

Business Victoria Future Industries

https://business.vic.gov.au/grants-and-programs/future-industries
Future Industries is about supporting investment in high-growth industries through industry excellence and development projects, including establishing collaborative networks and building supply chain readiness capabilities.

Career Education

www.education.vic.gov.au/school/teachers/teachingresources/careers/Pages/default.aspx
Career Education teaching resources to help teach students to make informed career decisions and equip themselves for the world of work.

CEAV Online Learning Resources

https://ceav.vic.edu.au/media/250615/careers-in-the-construction-technology-industries-student-resource.pdf
Designed to enable students to attend a virtual Industry Immersion Experience, these online resources will help students discover more about Victoria's priority growth industries and give them the opportunity to reflect on their skills, interests and undertake career planning and exploration.

Jobs Victoria

www.jobs.vic.gov.au

JobOutlook

www.joboutlook.gov.au
Relevant and current labour market trends and career information.

MyFuture

www.myfuture.edu.au
A database of over 600 careers.

National Careers Institute

www.dese.gov.au/nci
The National Careers Institute (NCI) ensures Australians have access to reliable and accurate careers information, resources, and support.

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