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Shining a Spotlight on Polya's Problem Solving Model

Taryn Volpe & Nikki D'Antonio

Workshop Overview

- This session will focus on the impact of George Polya's Problem Solving Model and the pivotal role that Professional Learning Communities (PLCs) and teacher collaboration played in the whole school implementation of this model in a Primary setting.
  Nikki & Taryn will unpack what each phase of the model looks like in practice, shining the spotlight on how each of the mathematical proficiencies are embedded into the model. They will share real examples and student work samples along the way, providing authentic and practical way that teachers can analyse and assess their student' problem solving skills. You will walk away with a bank of resources and strategies designed to support leaders and teachers to implement the model into their own classroom.

Key Takeaways

- Understand the role PLCs can play in the implementation of Polya's Problem Solving Model.
- Identify how the mathematical proficiencies are embedded within each phase of the model.
- Be exposed to practical examples of how student problem solving tasks can be analysed and assessed.

Welcome to …

Shining a Spotlight on Polya's Problem Solving Model.

This is Us.

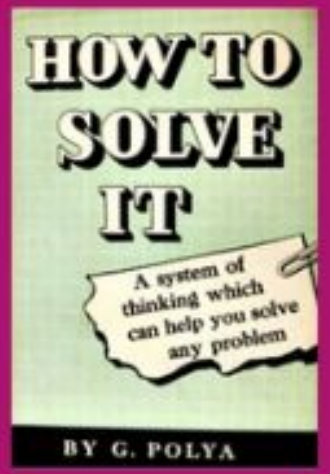
- Taryn has 10 years experience across Junior, Middle & Senior school (primary)
- First Learning Specialist role in 2019.
- Recently had first child, Harvey.
- Nikki has 15 years experience, predominantly taught in Year 3- 6.
- Taken time to have a family and has 3 children - Year 8, Year 6, Year 4
- Taryn is currently on Family leave and Nikki recently resigned from her ongoing position at Mill Park Heights P.S.
- Limitless Maths keeps us very busy with tutoring, workshops, incursions and consulting in schools.

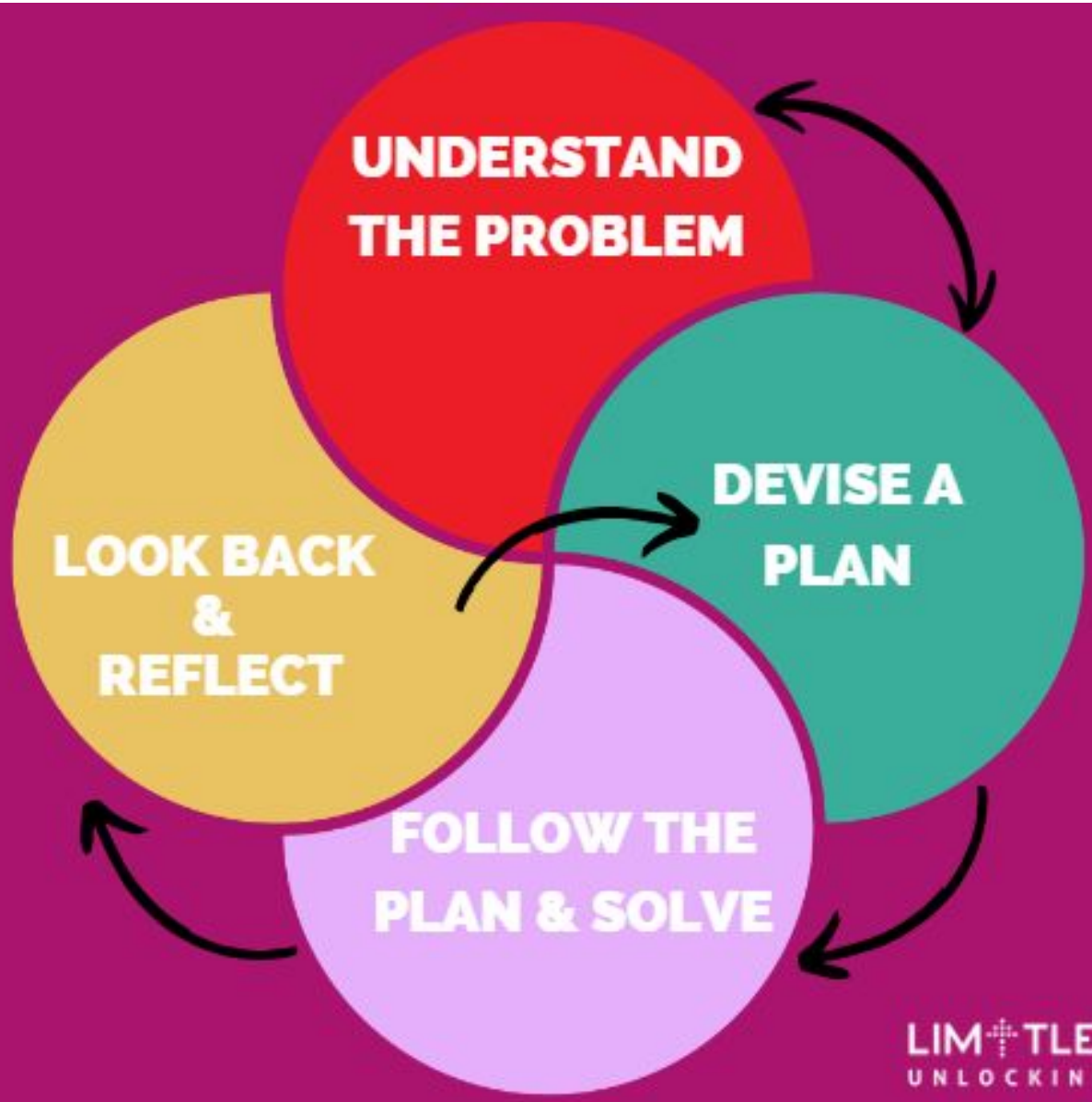
LIMITLESS MATHS
UNLOCKING MINDSETS

What is Polya's Model?

POLYA'S MODEL

George Polya was a famous math mathmatician and in 1945 he published the book "How To Solve It" which quickly became his most prized publication. It sold over one million copies and has been translated into 17 languages. His book identifies four basic principles of problem solving.





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What is Polya's Model?

George Pólya's model, widely known as **Pólya's Problem-Solving Process**, is a four-step approach to solving mathematical problems effectively. It provides a structured method for tackling problems systematically.

1. Understand the Problem

- Identify what is being asked.
- Determine the known information and any conditions or constraints.
- Rephrase the problem in your own words for clarity.
- Ask: "What do I know? What do I need to find out?"

2. Devise a Plan

- Think of strategies to solve the problem, such as drawing a diagram, breaking it into smaller parts, using a formula, or guessing and checking.
- Relate the problem to similar ones you've solved before.
- Consider multiple approaches to find the most efficient one.

3. Carry Out the Plan

- Execute your chosen strategy step-by-step.
- Be meticulous and check each step for accuracy.
- Keep working persistently but be ready to revise if you notice an error.

4. Look Back

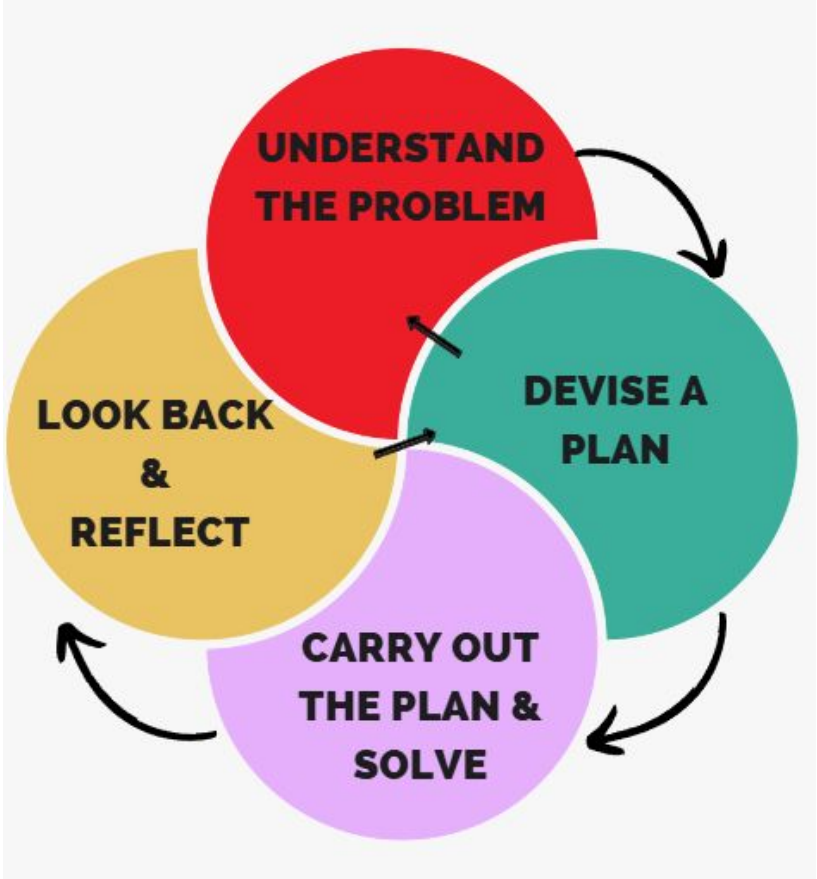
- Check the solution to ensure it answers the original question.
- Verify your calculations and reasoning.
- Reflect on what worked and how this approach could help solve similar problems in the future.

POLYA'S MODEL OVERVIEW

UNDERSTAND THE PROBLEM →	DEVISE A PLAN →	FOLLOW THE PLAN & SOLVE →	LOOK BACK & REFLECT
Do you understand all the words used in stating the problem? What are you asked to find or show? Can you restate the problem in your own words? Can you think of a picture or diagram that might help you understand the problem? Is there enough information to enable you to find a solution?	Have you seen it before? Or have you seen the same problem in a slightly different form? Could you restate the problem? Can you use a problem solving strategy you are familiar with? Can you research a new problem solving strategy and apply it to the problem? "If you begin to devise a plan but realise you need to understand the problem in further detail, then you can go back to the first phase.	This step is about having a go and trialing your plan. You will learn from this attempt even if it doesn't lead you to the solution this time. How are you going to attack the problem? If you follow your plan and reaslise it isn't going to be successful, reflect on the strategy and work out why. Then you can abandon the strategy and go back to 'Devise a Plan'.	Did you answer the question? Is your result reasonable? Can you check your answer by using a different method than what you used to solve the problem? Was it an effective strategy? Why? What changes would you now make to your strategy on reflection? " If you have successfully solved the problem then it is important to reflect on these questions and articulate what you learnt. If you haven't solved the problem, use these prompts to think of a new strategy to use to solve the problem.

Why Polya's Model?

- Research-based (Created by Mathematician, George Polya in 1947)
- Systematic approach to problem solving
- Promotes deep and flexible thinking
- Can be explicitly taught to students
- Can be applied to a range of different problem solving scenarios and across all areas of mathematics.
- Suitable for all ability levels and year levels, allowing for a consistent school-wide approach to problem solving that can be built on each year.



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PLCs @ Mill Park Heights Primary School

The implementation of PLCs at Mill Park Height P.S began in 2021 when a team completed the Training in 2021.

In 2022, PLC leaders were allocated to each team and us as the Learning Specialists met with the PLC leaders once a week to ensure consistent practices and documentation was occurring across the school. Throughout the year the PLC leaders team:

- Developed shared norms and language around PLCs across the school.
- Consistent agendas and PLC protocols.
- Created a PLC Timeline outlining the process week by week.
- Introduced new protocols each term e.g. 4 Cs Protocol.
- Emphasis on consistent professional learning across the school.

In 2023, we wanted to build on the consistent practices that were currently in place and further strengthen the PLC process by placing a stronger emphasis on the professional learning within the PLC cycle. This also coincided with the school's focus on Numeracy.

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Why Polya's Model at MPH?

- Teachers were teaching problem solving in different ways across the school, which meant explicit teaching around problem solving differed for students from year to year (inconsistent whole-wide approach to problem solving)
- Students lacked problem solving skills and strategies across F-6 - students in the Senior year levels still only had basic problem solving skills and struggled to interpret and solve age appropriate problems.
- Problem solving was often de-prioritised and the first thing to 'go' if teachers ran out of time across the week.
- Teachers weren't allowing students to problem solve as they didn't have a true understanding what problem solving is - they would jump in as soon as they saw students 'struggling'.
- Professional learning was available to support teachers to build their understanding of this model (through PLCs and whole-school PL).
- The model was supported by resources/PL we'd already started exploring as a school e.g. Resolve.

PLCs @ Mill Park Heights Primary School

- One of our 2023 Leadership goals was to further strengthen the PLC process by placing a stronger emphasis on the professional learning within the PLC cycle.
- We made the decision to have a whole school PLC focus, which became the 'cake'.
- In Term 1, the 'cake' was 'Productive Learning Dispositions (aligned with our whole-school professional learning focus) and in Term 2, our whole school 'cake' changed to the Mathematical Proficiencies.
- As some teams were using Polya's Problem Solving Model for the first time, they chose to focus on the 'Problem Solving' Proficiency.
- Teams explored the phases of the model on a deeper level (based on their student needs) and we provided each team with relevant professional learning to complete as a PLC, allowing PL around Polya's Model to happen in a more organic and purposeful way.
- As a whole-school, we had termly meetings dedicated to celebrating and sharing the learning that occurred through PLCs, allowing all staff members to see the consistency of practice across the school which included their learning about Polya's Model.
- Polya's model continued to be focussed through PLCs throughout the year, strengthening the professional learning of staff as well as the development of a clear problem solving model across the school.

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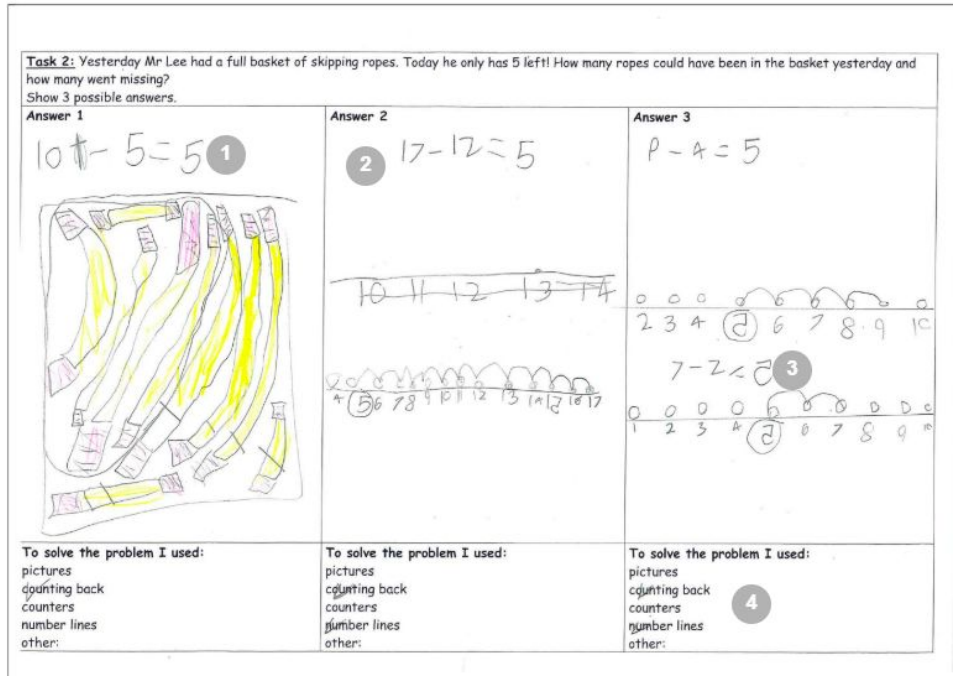
PLC Example - YEAR 3



- Team had already discussed focussing on Problem Solving for our PLC focus based on previous observations.
- We lead a whole-school PL dedicated to using the proficiency portfolio to further explore the proficiencies and chose a pre-test for PLCs.
- All students completed the Pre-test - team members bought 'interesting' student work samples to our PLC to discuss.

Proficiency Portfolio - Pre & Post Test

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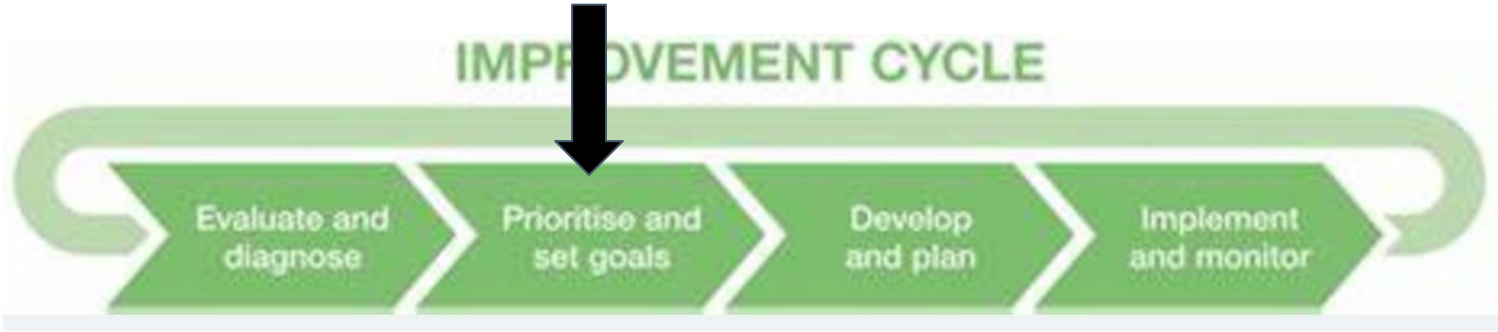
PLC Example - YEAR 3



Year 3 Tuesday 3&4	**Slice:** Problem Solving **Crumb**:To choose (plan) an appropriate strategy for problem solving before solving the problem.	Professional Reading - NRICH/DET POLYA Problem Solving Model - Professional Reading POLYA Problem Solving Model - Slides for Students

Learning Goal: To formulate/plan-out an effective problem solving strategy before solving a problem.

PRETEST - CONTINUUM

Does not attempt to formulate/plan out an effective strategy for problem solving or show their answers/thinking.	Attempts to choose a strategy for problem solving and show their thinking, however answers are incorrect and thinking is unclear.	Can choose a strategy for problem solving and come up with a solution/answer to a problem, however strategies are ineffective and thinking is unclear.	Can choose an effective strategy for problem solving, however their answers/thinking are not set out clearly and coherently.	Can formulate/plan-out effective strategies for problem-solving and set out their answers/ solutions/thinking clearly and coherently.
Tye	Chase, Jordan	Jaelynn, Mason, Sofia	Emma, Sam	

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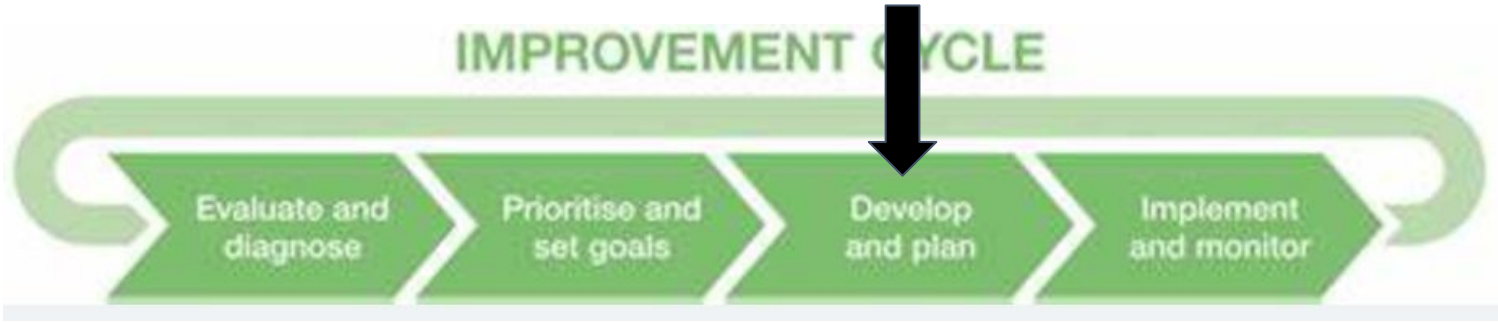
PLC Example - YEAR 3



Problem-solving skill	Examples of NRICH activities
Pattern spotting	Domino Patterns and Consecutive Numbers
Working systematically	Same Length Trains and Beads and Bags
Using diagrams and pictorial information	Fraction Match and Build It Up
Working backwards	Doing and Undoing and Andy's Marbles
Trial and improvement	Find the Difference and Fifteen Cards
Visualising	Happy Halving and How Would We Count?
Conjecturing and generalising	Break it Up! and Take Three Numbers
Reasoning logically	I Like "¦ and Sealed Solution

https://nrich.maths.org/articles/whats-problem-problem-solving

- Team planned all problem solving lessons together
- Team agreed to prioritising weekly problem solving sessions (Fridays)
- Planned to team teach lessons where possible
- Problems from NRICH - exposed students to the same type of problem for several weeks in a row
- Explicitly modelled problem solving strategies
- Planned to observe target students and collect weekly work samples.

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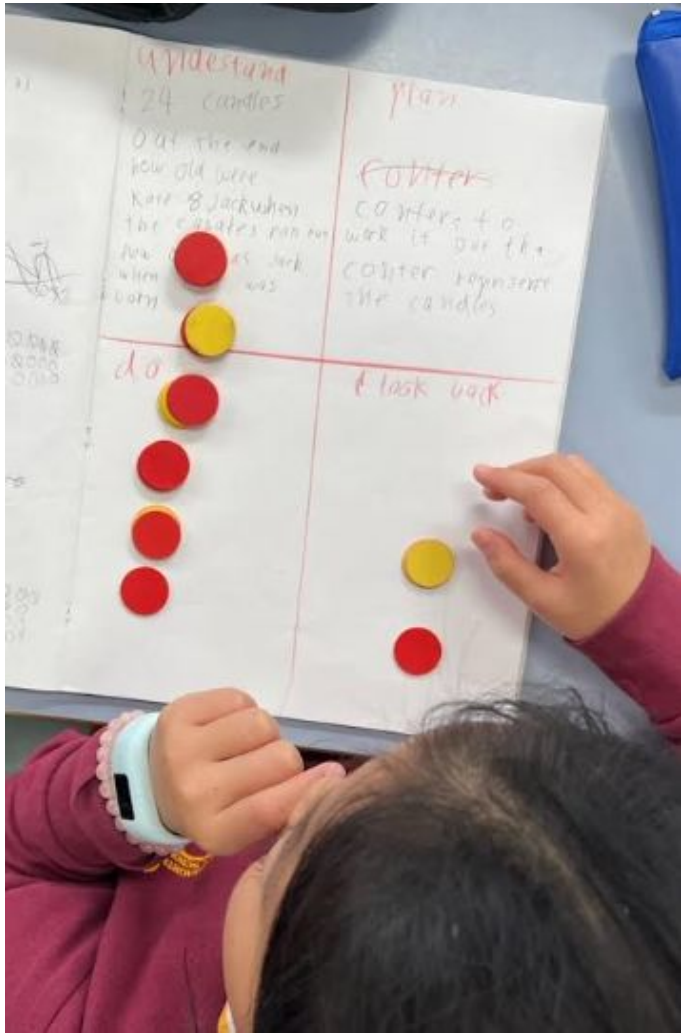
PLC Example - YEAR 3



Learning Goal: To formulate/plan-out an effective problem solving strategy before solving a problem.

POST TEST - CONTINUUM

Does not attempt to formulate/-plan out an effective strategy for problem solving or show their answers/thinking.	Attempts to choose a strategy for problem solving and show their thinking, however answers are incorrect and thinking is unclear.	Can choose a strategy for problem solving and come up with a solution/answer to a problem, however strategies are ineffective and thinking is unclear.	Can choose an effective strategy for problem solving, however their answers/thinking are not set out clearly and coherently.	Can formulate/plan-out effective strategies for problem-solving and set out their answers/ solutions/thinking clearly and coherently.
Tye			Chase, Mason, Chase	Emma, Jaelynn, Sam, Sofia

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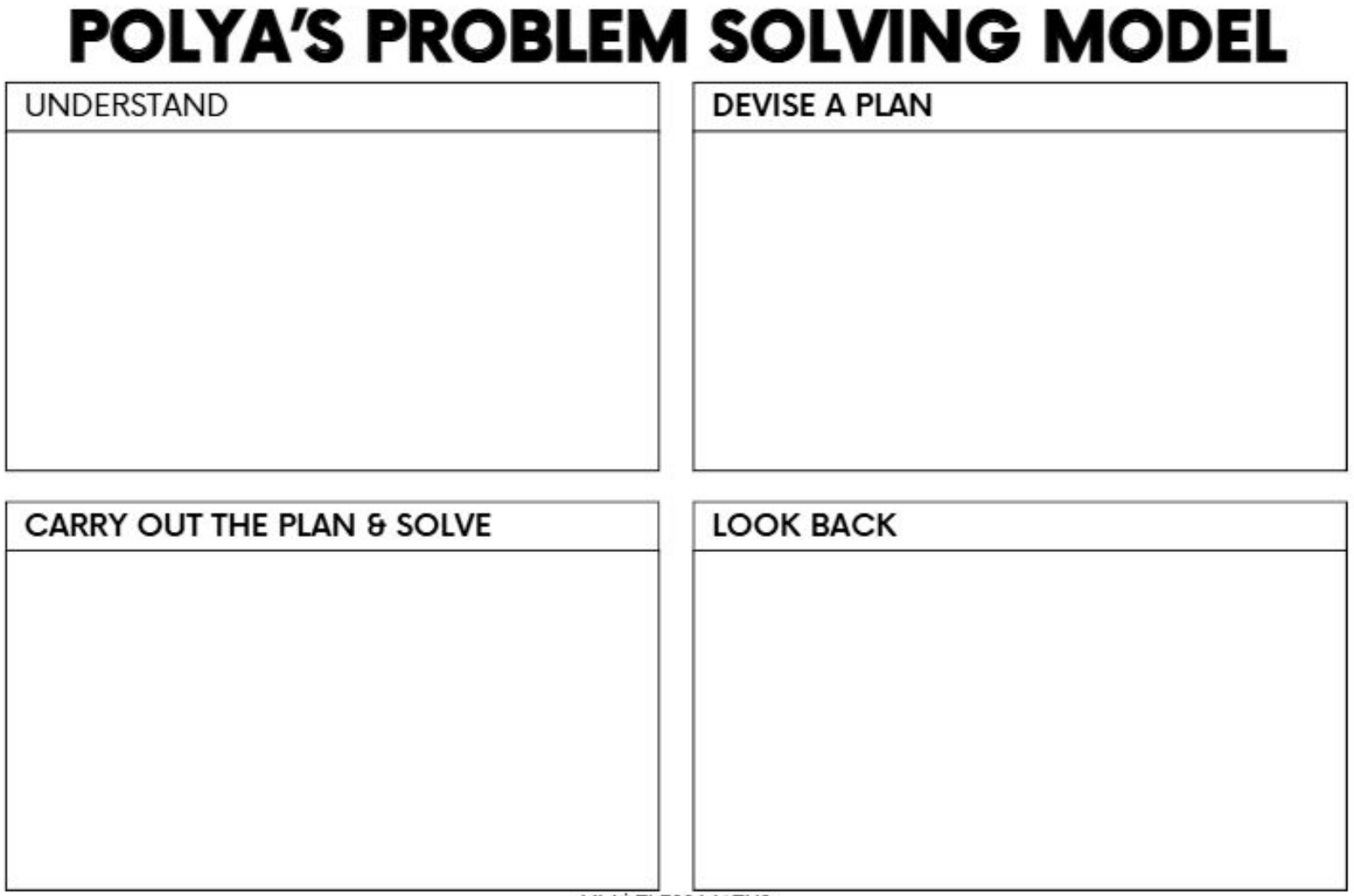
POLYA'S MODEL IN ACTION

Draw:



It’s your turn!

The Problem

Can you find every number between 1 and 20 using only four 4’s and any operation?

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Polya's 1st Principle:
Understand the problem

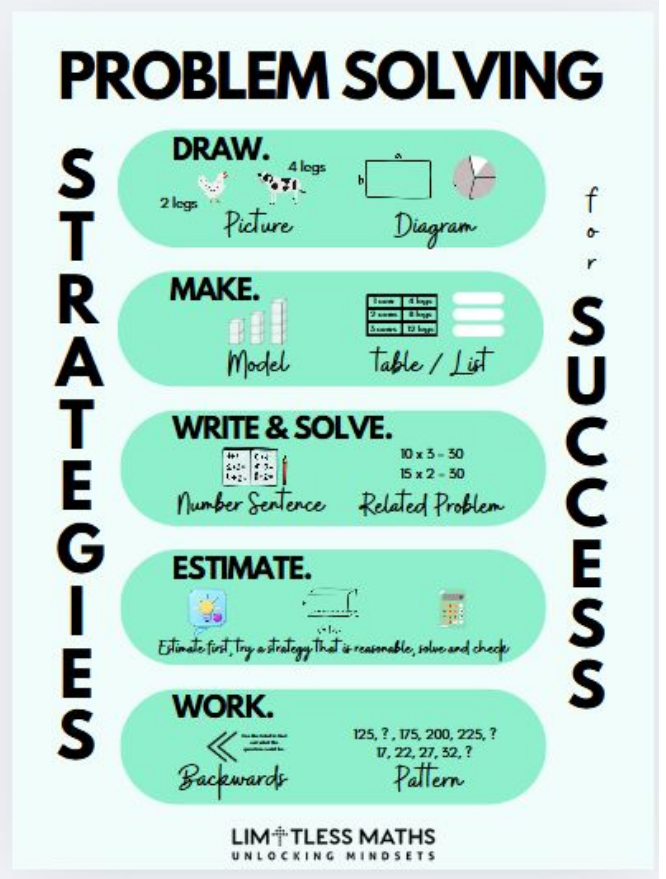
Understanding the Problem Prompts:

What is known?
What is not known or missing?
What do you need to find out?
What type of answer is required?
Is the problem similar to other problems you've seen?
Are there any important terms for which you should look up definitions?
Do you understand all the words used in stating the problem?
What are you asked to find or show?
Can you restate the problem in your own words?
Can you think of a picture or diagram that might help you understand the problem?
Is there enough information to enable you to find a solution?
Can you explain the problem to someone who doesn't understand the problem?

Polya's 2nd Principle:
Devise a Plan

Devise a Plan Prompts

Have you seen it before?
Or have you seen the same problem in a slightly different form?
Could you restate the problem?
What problem solving strategies do you know that would best suit this problem.
Can you research a new problem solving strategy and apply it to the problem?
What prior knowledge of problem solving strategies to your team members have?
What strategy are other teams using?



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Polya's 3rd Principle:
Carry out the Plan and Solve

Carry out the Plan and Solve Prompts:

How are you going to attack the problem?
How are you checking each step?
Can you see clearly that each step is correct?
Can you prove that it is correct?

This step is about having a go and trialing your plan. You will learn from this attempt even if it doesn't lead you to the solution this time.

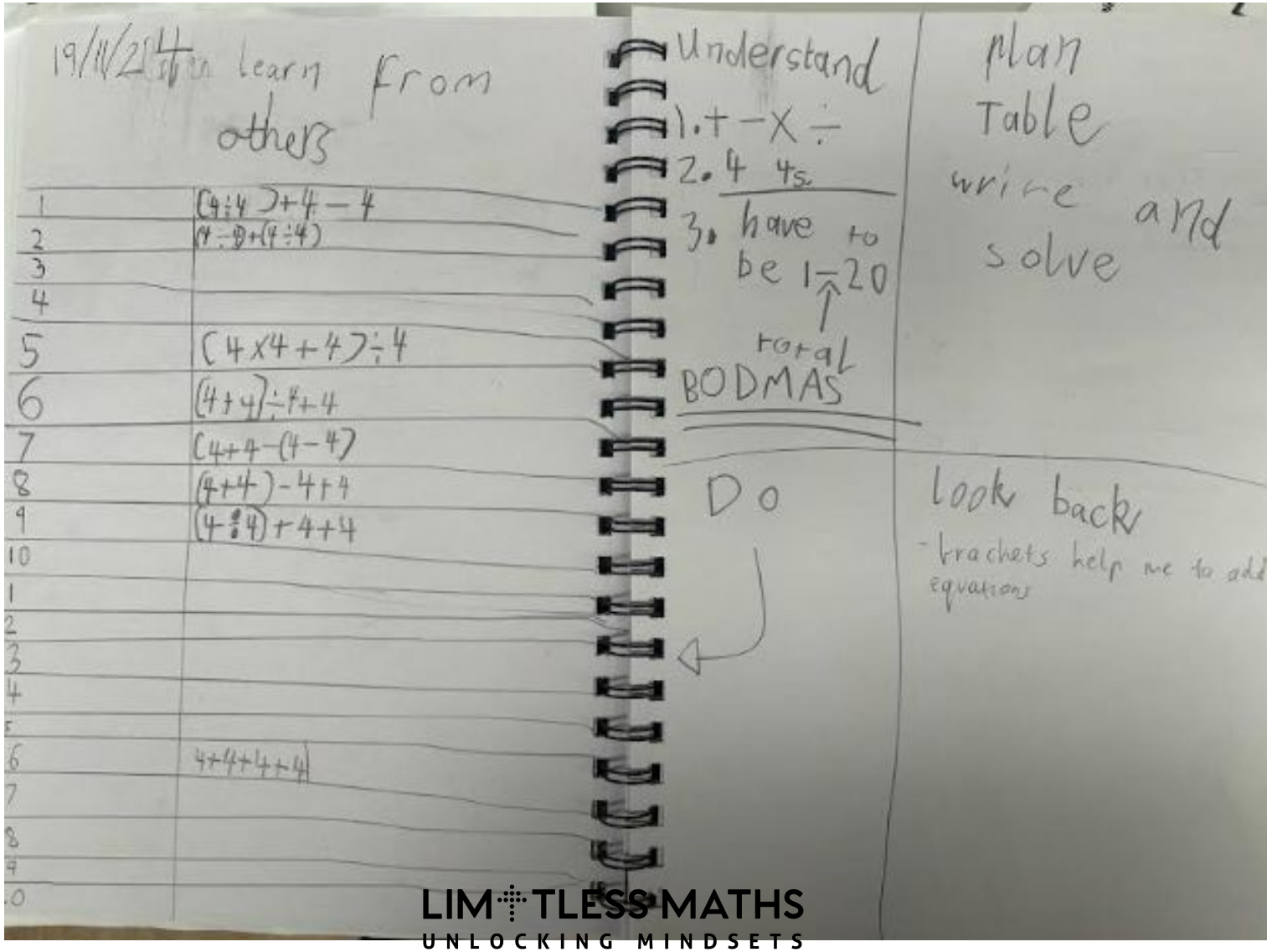
Polya's 4th Principle:
Look Back

Look Back & Reflect Prompts:

Did you answer the question?
Is your result reasonable?
Can you check your answer by using a different method than what you used to solve the problem?
Was it an effective strategy? Why?
What changes would you now make to your strategy on reflection?

Tutoring Example - Year 2 Extension

Can you find every number between 1 and 20 using only four 4's and any operation?



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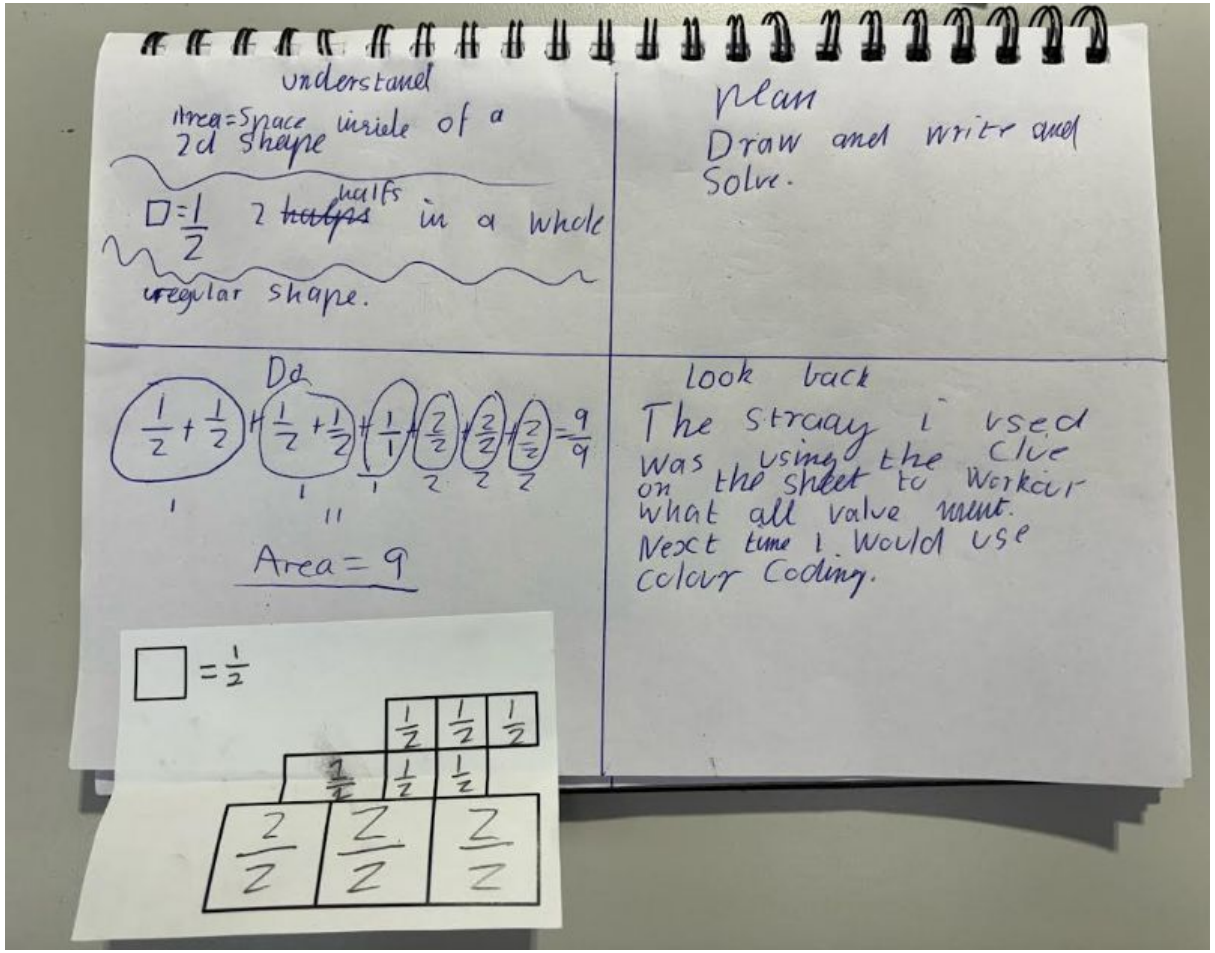
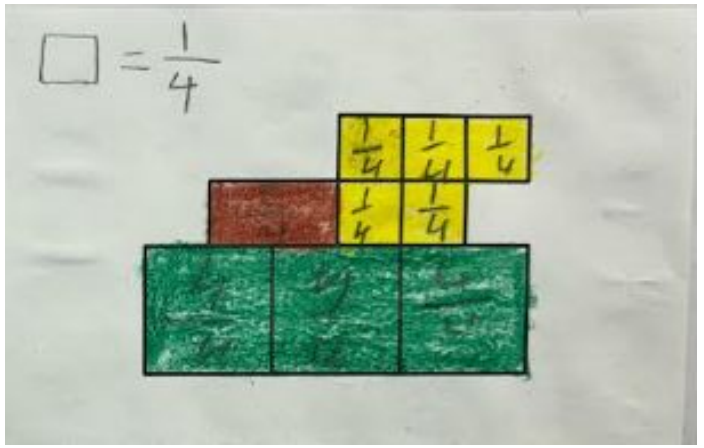
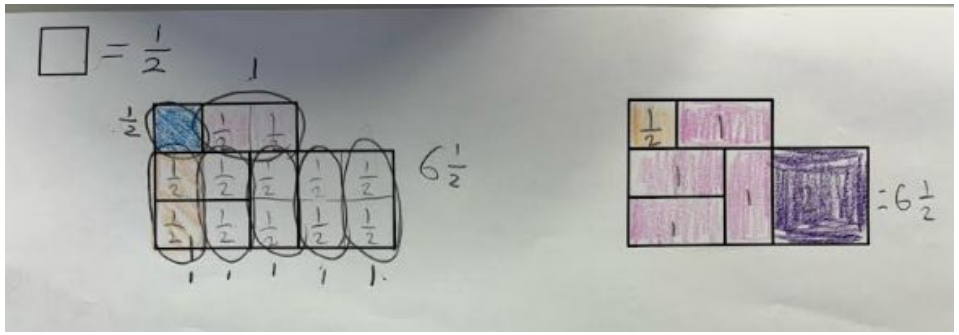
ANOTHER TUTORING EXAMPLE

Can you find the area of this shape?







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The Mathematical Proficiencies

The **mathematical proficiencies** (understanding, fluency, problem-solving, and reasoning) are deeply embedded in **Polya's Problem-Solving Model**. By embedding these proficiencies into each stage, Polya's model not only supports students in solving individual problems but also equips them with the skills to approach a wide variety of mathematical challenges effectively.

1. Understanding the Problem (Understanding and Reasoning)

- **Understanding:** Students identify what the problem is asking by clarifying key information, recognising patterns, and making connections to prior knowledge. This builds conceptual understanding of mathematical ideas.
- **Reasoning:** Students analyse the problem to determine relationships, assumptions, and constraints, using logical thinking to identify relevant details.

2. Devising a Plan (Reasoning and Problem-Solving)

- **Reasoning:** Students explore possible strategies (e.g., working backwards, making a table, drawing a diagram) and justify why certain approaches may work. This requires logical and systematic thinking.
- **Problem-Solving:** Creativity and strategy come into play as students decide on the best approach to tackle the problem.

3. Carrying Out the Plan (Solve) (Fluency and Problem-Solving)

- **Fluency:** Students apply mathematical procedures accurately and efficiently while executing their chosen plan. This stage reinforces skills like computation and algorithmic processes.
- **Problem-Solving:** As they work through their plan, students must adapt and persist if challenges arise, developing resilience and strategic thinking.

4. Looking Back (Reasoning and Understanding)

- **Reasoning:** Students evaluate the solution to determine if it is logical and consistent with the problem. They may also explore alternative methods, fostering deeper reasoning skills.
- **Understanding:** Reflecting on the process helps solidify their grasp of mathematical concepts and how they interconnect. This metacognitive step strengthens overall comprehension.

Formative Assessment Questions

1. Understanding the Problem

Comprehension of the problem:
Can the student…

- Explain the key mathematical words used in the problem?
- Restate the problem in their own words?
- Explain the problem to a peer?
- Decide if there is enough information to solve the problem?

Connections and patterns:
Can the student…

- Think of a similar problem they have solved before?
- Share relationships, patterns or connections from the information provided?
- Make assumptions about the problem?

2. Devising a Plan

Exploring strategies:
Can the student…

- Brainstorm multiple strategies they could be used to solve this problem?
- Identify which strategy may be most efficient for solving the problem?
- Justify their choice/s of problem solving strategy.

Planning steps:
Can the student…

- Decide what they will do first and explain why?
- Decide on the tools (e.g., diagrams, tables, equations) that will use to help them solve this problem?

3. Carrying Out the Plan and Solve

Can the student…

Execution:

- Follow the steps in their plan?
- Work independently/collaboratively to solve the problem?
- Explain how they know their calculations or reasoning are accurate?
- Work effectively to find a reasonable solution?

Adjustments:

- Overcome obstacles and challenges? How do they revise and change their plan?

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4. Looking Back

Verification:
Can the student…

- Justify why their solution makes sense in the context of the problem?
- Check their work for any errors?
- Check their answer by using a different method?
- Prove and convince others that their solution is correct?

Reflection:
Can the student?

- Identify if there is a simpler or more efficient way to solve this problem?
- Explain what they learnt from solving this problem?
- Identify how they may approach the problem differently next time?

FORMATIVE ASSESSMENT EXAMPLE…

Using any four consecutive numbers and any of the four operations as well as brackets, in any order and using all of them, can you make 19?

NINETEEN

using any four consecutive numbers (positive, to start with) in any order and using all of them

using any of the four operations: add, subtract, multiply, divide and brackets

can you make **19** ?

some examples

5(6 – 3) + 4

9(10 – 7) – 8

9 × 11 – 8 × 10

https://donsteward.blogspot.com/2014/02/making-19.html

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Understand

Understand

→ consecutive, in order
eg: 1,2,3,4,5

→ four operations, add, minus, multiply, divide

→ 9(10-7) - 8
= 19

Carry out the plan (Solve)

Do

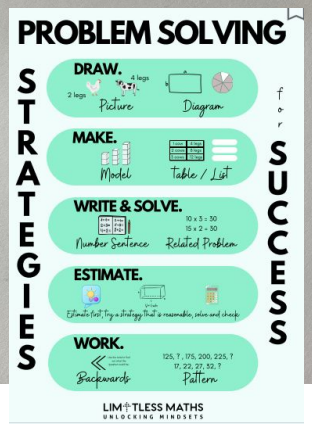
Example
1,2,3,4
4 (2+3) -1

Example
2(3+4)+5

Devise a Plan

Plan

Strategy = Write and solve.



Look Back

Student verbally discussed the Look Back phase. She realised that she was also using Estimate and Trial and Error because she said she could see the link between the operations and how that was making her closer or further away to 19.

Next time, she would be more systematic with the numbers she choose and use all operation before moving on to new consecutive numbers.

Understand

Understand

→ consecutive, in order
eg: 1,2,3,4,5

→ four operations, add, minus, multiply, d

→ 9(10-7) - 8
= 19

1

Understands/explains the maths terminology used in the problem

Could identify a similar problem that they've solved before (Four 4s)

Could restate the problem in their own words.

Carry out the plan (Solve)

Do

Example 1
1,2,3,4
4 (2+3) - 1

Example
2(3+4)+5

Worked independently to find solutions (solutions listed here, working out in book)

Used a combination of estimation/trial and error and write and solve.

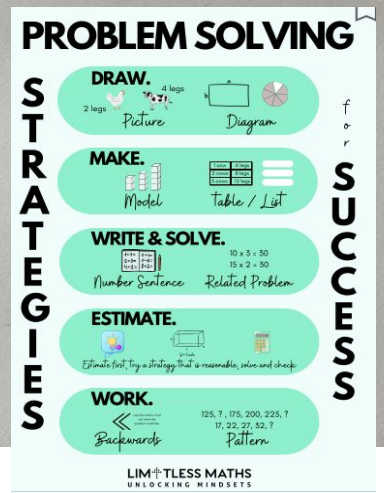
Came up with 2 different solutions.

Devise a Plan

Plan

Strategy = Write and solve.

Equations



2

Selected an appropriate strategy - write and solve.

Could not identify another strategy that may be suitable for a problem like this.

Didn't have a plan in regards to 'steps' that would be taken - strategy only.

Look Back

Student verbally discussed the Look Back phase. She realised that she was also using Estimate and Trial and Error because she said she could see the link between the operations and how that was making her closer or further away to 19.

Next time, she would be more systematic with the numbers she choose and use all operation before moving on to new consecutive numbers.

Could reflect on their choice of strategy and how they would approach the same task differently next time.

FORMATIVE ASSESSMENT EXAMPLE ANNOTATED

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FORMATIVE ASSESSMENT EXAMPLE…

Understands/explains the maths terminology used in the problem

Could identify a similar problem that they've solved before (Four 4s)

Could restate the problem in their own words.

NINETEEN

using any four consecutive numbers (positive, to start with) in any order
and using all of them

using any of the four operations: add, subtract, multiply, divide and brackets

can you make
19 ?

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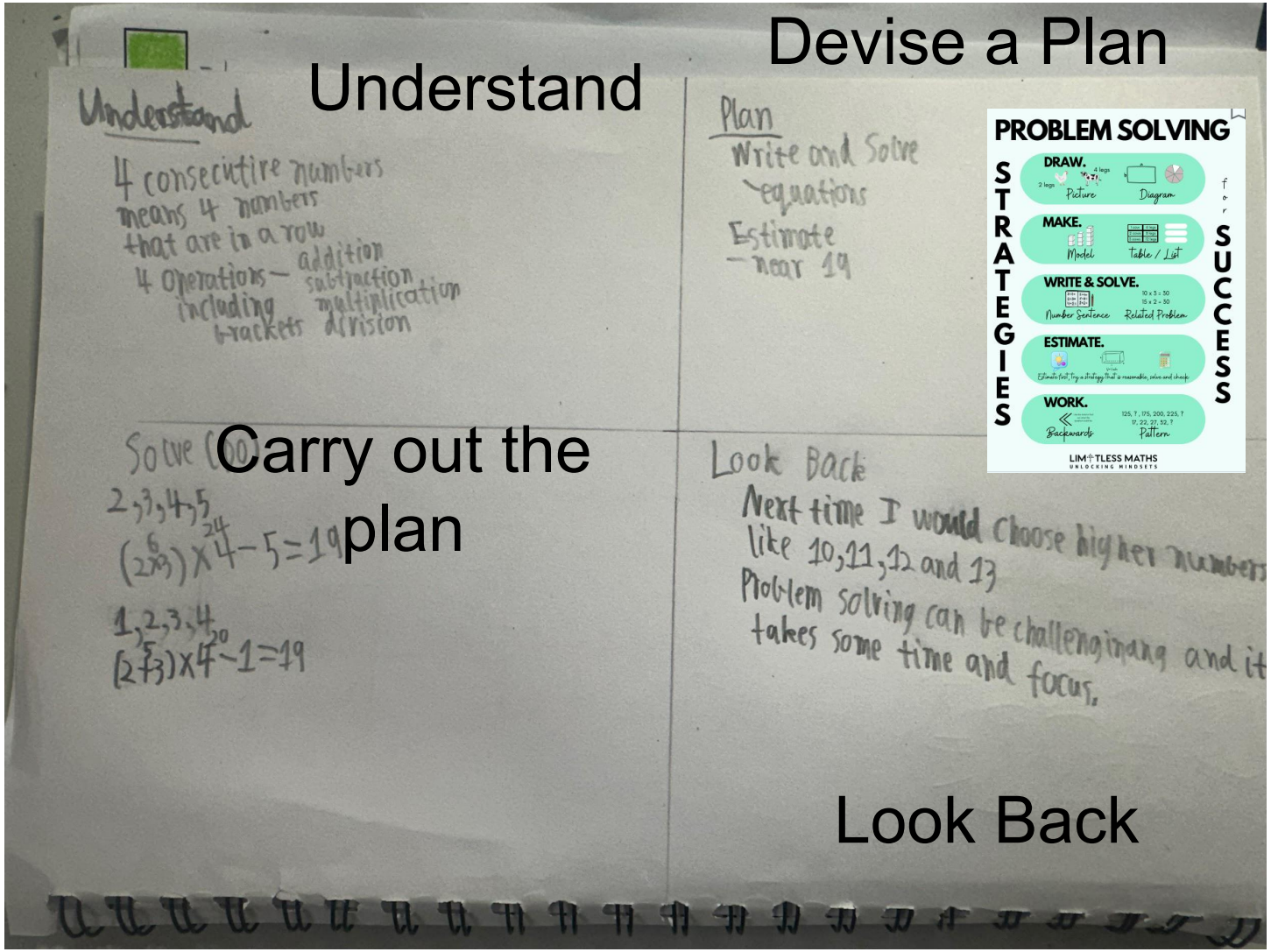
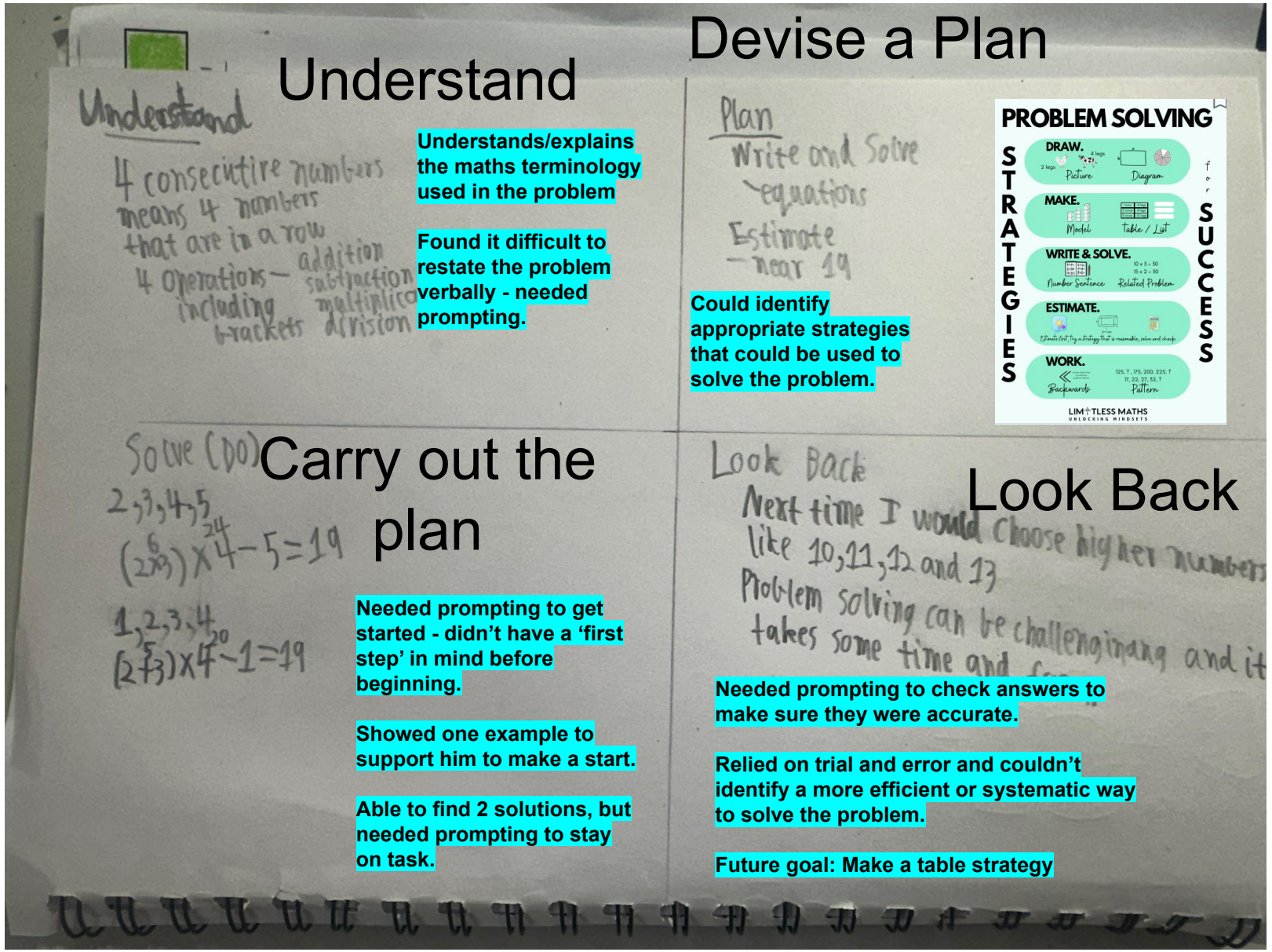


FORMATIVE ASSESSMENT EXAMPLE ANNOTATED



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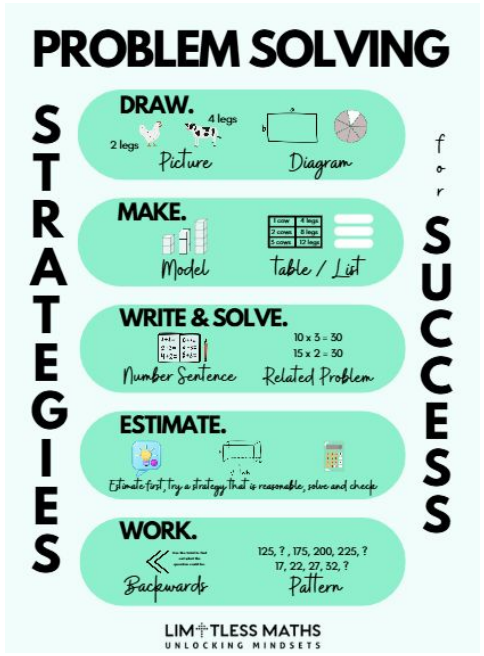
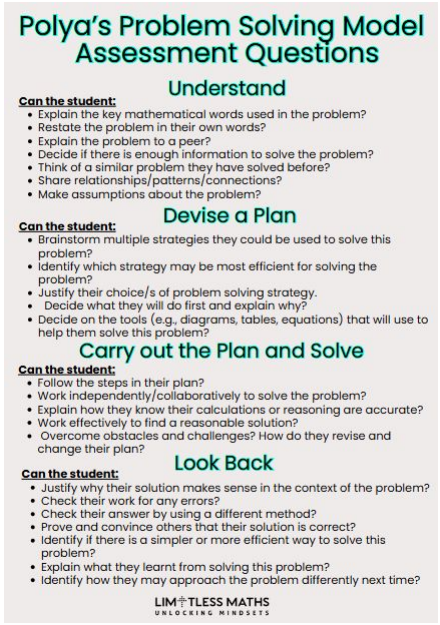
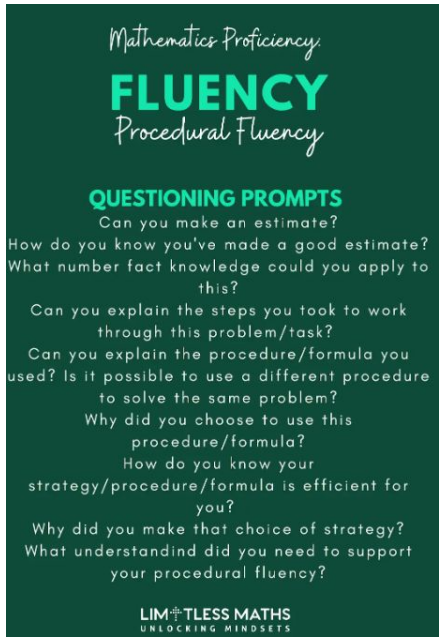
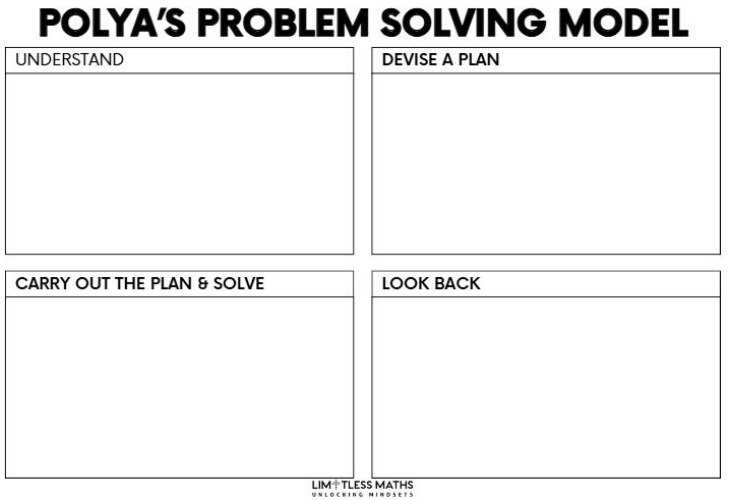
Limitless Maths Resources









Please visit our 'Teacher Resource' page on our website to download the resources above.

www.limitlessmaths.com.au

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UNLOCKING MINDSETS

Resources readily available

NSW Government

https://app.pre.education.nsw.gov.au/learning-tools-selector/LearningActivity/Card/595

No Source

https://drive.google.com/file/d/1JYhson-vs9T1KTK5Vktu6iS1xsR3dcXH/view?usp=sharing

https://www.competitivekids.org/blog/problem-solving-strategies

https://drive.google.com/file/d/1OKMZN3GOyQ_bKrPElK95g95OFUjgk9ae/view?usp=drive_link

Nagoya University

https://drive.google.com/file/d/1JvM-FYVyCkknTqNp0aO8BQcuUlQts-XG/view?usp=drive_link

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