# Investigation: Product of Polynomials

**Overview of the task and breakdown of components**

* **Component 1**
	+ Sketching cubic graphs of the form $y=(x-a)(x-b)(x-c)$
	+ Sketching cubic graphs of the form $y=\left(x-a\right)\left(x-b\right)^{2}$
	+ Sketching cubic graphs of the form $y=\left(x-a\right)^{3}$
	+ Recognising the connection between the values of $a, b, c$ and the $x$-interepts
	+ Recognising the connection between the degree of a factor and the behaviour of the cubic graph at the $x$-intercept(s)
	+ Summarising and generalising results/observations through comparison of examples or pattern recognition, and using mathematical notation and/or everyday language to describe generalisations
* **Component 2**
	+ Sketching graphs of the form $y=x^{m}\left(h-x\right)^{n}$ for given values of $n$ and $m$
	+ Recognising the connection between the degree of a factor and the behaviour of the polynomial graph at the $x$-intercept(s)
	+ Recognising the connection between the degree of the polynomial and the overall shape/behaviour of the graph
	+ Summarising and generalising results/observations through comparison of examples or pattern recognition, and using mathematical notation and/or everyday language to describe generalisations
* **Component 3**
	+ Sketching graphs of the form $y=\left(x^{m}-a\right)\left(x^{n}-b\right)$ for given constraints
	on $m$ and $n$
	+ Recognising the importance of fully factorising an expression to locate and identify the key features of a graph
	+ Summarising and generalising results/observations through comparison of examples or pattern recognition, and using mathematical notation and/or everyday language to describe generalisations

**Possible ways to implement such a task**

* Would typically spend maybe half a week to one week on higher-order polynomial graphs.
* Could replace textbook work with such a task (Instead of Ch6-whatever from Cambridge or Ch5-whatever from Jacaranda).
* As part of the task, it would be advised for students to summarise/comment on what they are observing/finding (e.g. what is similar/different? What do you notice?) and what they have learnt/understood. There’s often a discrepancy between what we tell/teach students and what they understand (or interpret what we say).
* Such a task could be for formative assessment purposes, with no weighting assigned to the investigation. Its primary function is a learning instrument. There could be a summative assessment (test) for the entire cubics/polynomials topic that assesses ideas explored in the investigation.

**TI Video YouTube links (Sliders and maybe some useful commands)**

# Inserting and Using Sliders | TI-Nspire CX II | Getting Started Series - Graphs Application<https://www.youtube.com/watch?v=jHY6X5Er0Ew>

# Sliders in Notes | TI-Nspire CX II CAS | Getting Started Series - Notes Application

<https://www.youtube.com/watch?v=b4pt8R_0HzM>

For additional support: <https://education.ti.com/en-au/resources/getting-started-with-ti-technology/student-course-lessons-ti-nspire-cx-ii-cas>

**Component 1**

In this component you will consider graphs of cubic polynomials.

1. Consider the family of curves of the form $y=(x-a)(x-b)(x-c)$ where $a,b,c$ are real numbers $(a,b,c\in R)$.
2. i. By selecting your own values for $a, b, c,$ where $a\ne b\ne c$, sketch 3 cubic graphs of the above form. Label your axial intercepts with coordinates. Also write the equation for each corresponding graph.
3. Comment on any similarities/differences between your graphs.
4. Discuss how $a,b,c$ affect the key features of the graph.
5. What happens to the shape and key features of the cubic graph if $a=b=c$? Investigate. Provide 3 examples to support your ideas/conjectures. Label your axial intercepts with coordinates. Also write the equation for each corresponding graph.
Comment on any similarities/differences. Try to generalise your observations.
6. What happens to the shape and key features of the cubic graph if $a=b\ne c$? Investigate. Provide 3 examples to support your ideas/conjectures. Label your axial intercepts with coordinates. Also write the equation for each corresponding graph.
Comment on any similarities/differences. Try to generalise your observations.
7. Consider the family of curves of the form $y=(x-1)(ax^{2}+bx+c)$,
where $a,b,c$ are non-zero real numbers.
Investigate how $a,b,c$ affect the shape and key features of the graph.
Provide examples to support your ideas/conjectures. Summarise/describe your observations, and try to generalise your observations.

**Component 2**

In this component you will consider graphs of the form $y=x^{m}\left(h-x\right)^{n}$, where $h$ is non-zero real number and $m,n\in \left\{0, 1, 2, 3, 4\right\}$

1. Consider the case where $h=2$.
2. Sketch the following graphs, labelling all key features.

|  |  |  |  |
| --- | --- | --- | --- |
| $$m=0, n=0$$ | $$y=x^{0}\left(2-x\right)^{0}$$ | $$m=1, n=1$$ | $$y=x(2-x)$$ |
|  |  |
| $$m=2, n=1$$ | $$y=x^{2}\left(2-x\right)$$ | $$m=1, n=2$$ | $$y=x\left(2-x\right)^{2}$$ |
|  |  |
| $$m=3, n=1$$ | $$y=x^{3}\left(2-x\right)$$ | $$m=1, n=3$$ | $$y=x\left(2-x\right)^{3}$$ |
|  |  |
| $$m=2, n=2$$ | $$y=x^{2}\left(2-x\right)^{2}$$ |  |
|  |  |

1. Comment on any similarities/differences between your graphs.
2. Discuss how $m$ and $n$ affect the behaviour/shape of the graph. Provide examples to support your ideas/conjectures (select a different value of $h$).
3. Discuss how $m$ and $n$ affect the number and nature of any turning points/point of inflection. Provide examples to support your ideas/conjectures (select a different value of $h$).
4. What would happen to the graph and key features if $h$ was negative? Investigate.
5. Create an equation of a graph of the form $y=x^{m}\left(h-x\right)^{n}$ that satisfies the following conditions:

Graph 1:

* Has an $x$-intercept and turning point at $x=3$

Graph 2:

* Has at least 2 turning points
* Has a negative $x$-intercept

Graph 3:

* Has a stationary point of inflection
* Has a negative $x$-intercept

**Component 3**

In this component you will consider graphs of the form $y=(x^{m}-a)(x^{n}-b)$,
where $a,b$ are non-zero real numbers and $m,n\in \left\{1, 2, 3\right\}$.

1. Consider the family of curves of the form $y=\left(x^{m}-1\right)\left(x^{n}-8\right)$ where $2\leq m+n\leq 4$.
Investigate how $m$ and $n$ affect:
* The location and number of axial intercepts
* The behaviour of the graph
* (The number of turning points/stationary points of inflection)
1. Consider the family of curves of the form $y=\left(x^{m}-1\right)\left(x^{n}-a\right)$ where $2\leq m+n\leq 4$ and $a$ is a non-negative real number.
Investigate how $a, m$ and $n$ affect:
* The location and number of axial intercepts
* The behaviour of the graph
* (The number of turning points/stationary points of inflection)

**End of Investigation**