Scaffolding mathematical reasoning for junior secondary

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BitMaths

firefly education
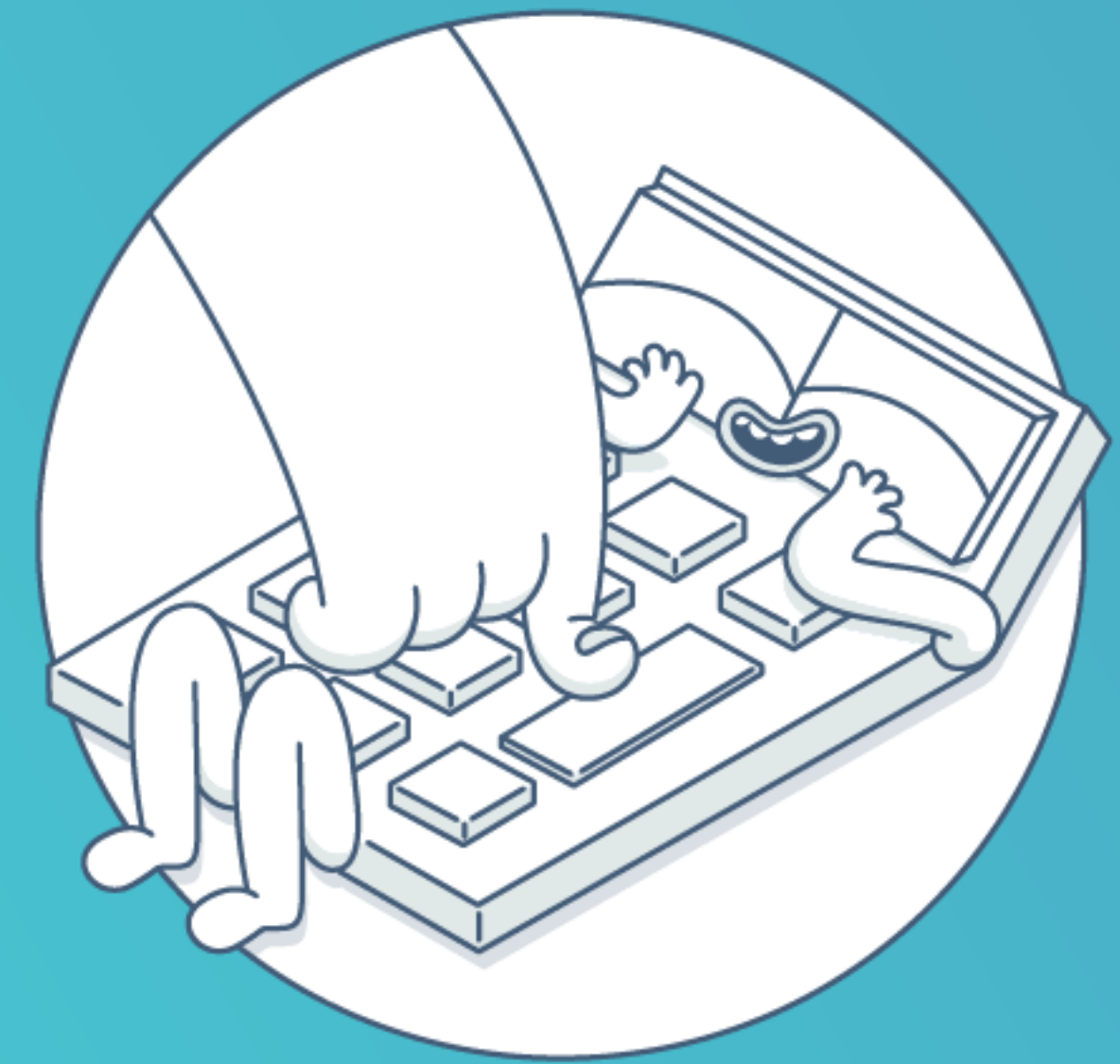
- Why we need to teach reasoning skills

- Problem solving

- How to write a reasoning task using cognitive verbs

- The Reasoning Wizard – scaffolded tasks to develop reasoning skills

To develop reasoning skills, students need to:

1. understand what the problem is asking and the reasoning required to solve it

2. know how to write a well-reasoned answer.
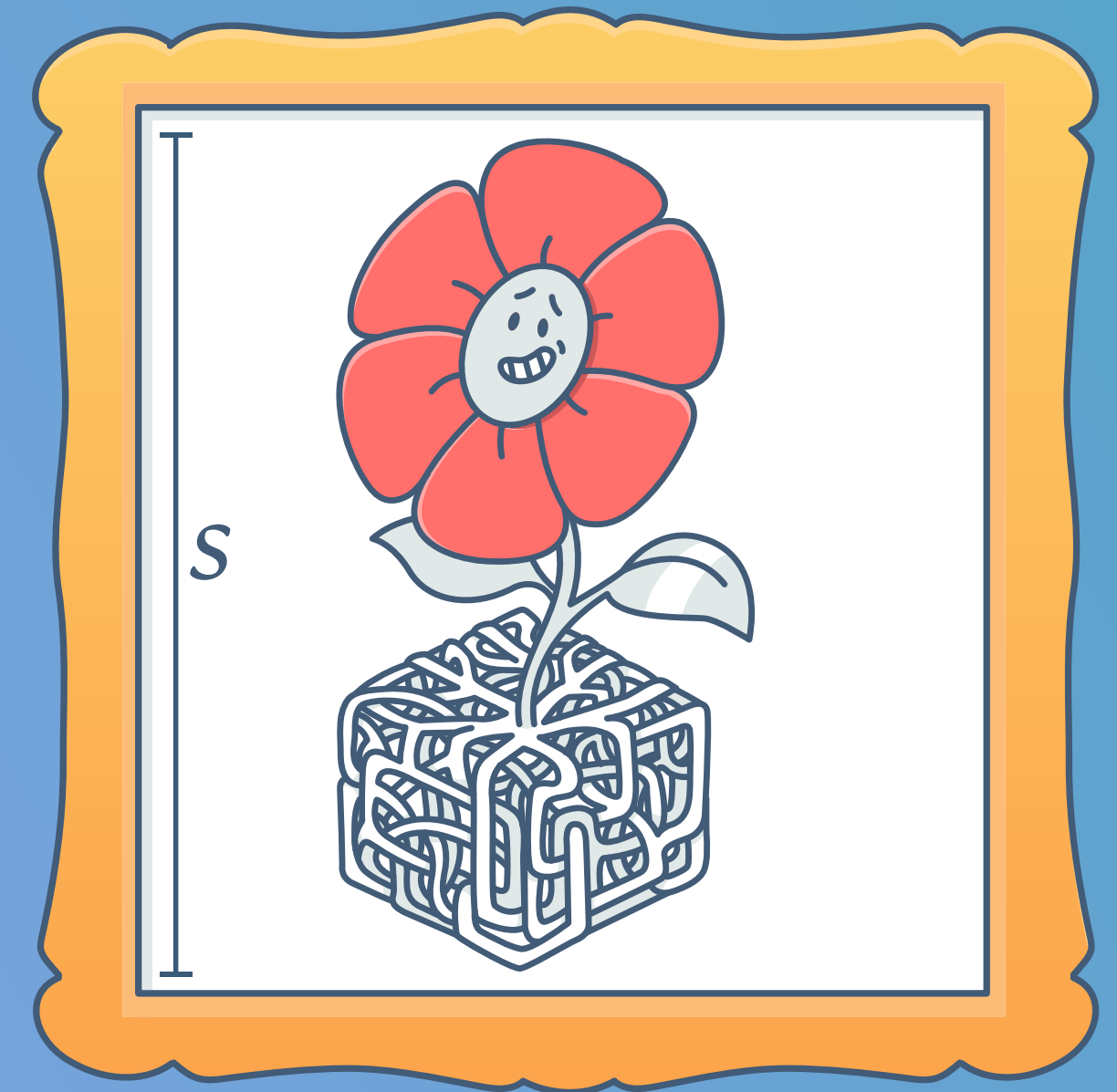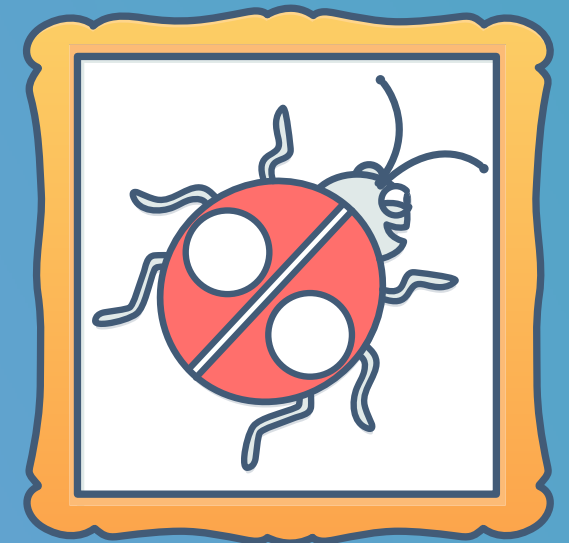
As teachers, we need to model both.

Before we look at reasoning ...

... let's look at problem solving

Problem Solving Task

I hung two artworks on my bedroom wall. Both artworks were square. The side length of the first artwork was 11 cm and the side length of the second was double that.

What is the total area of both artworks?

What might students do to solve this problem?

1. Identify what the problem is asking them to do

   Find the total area

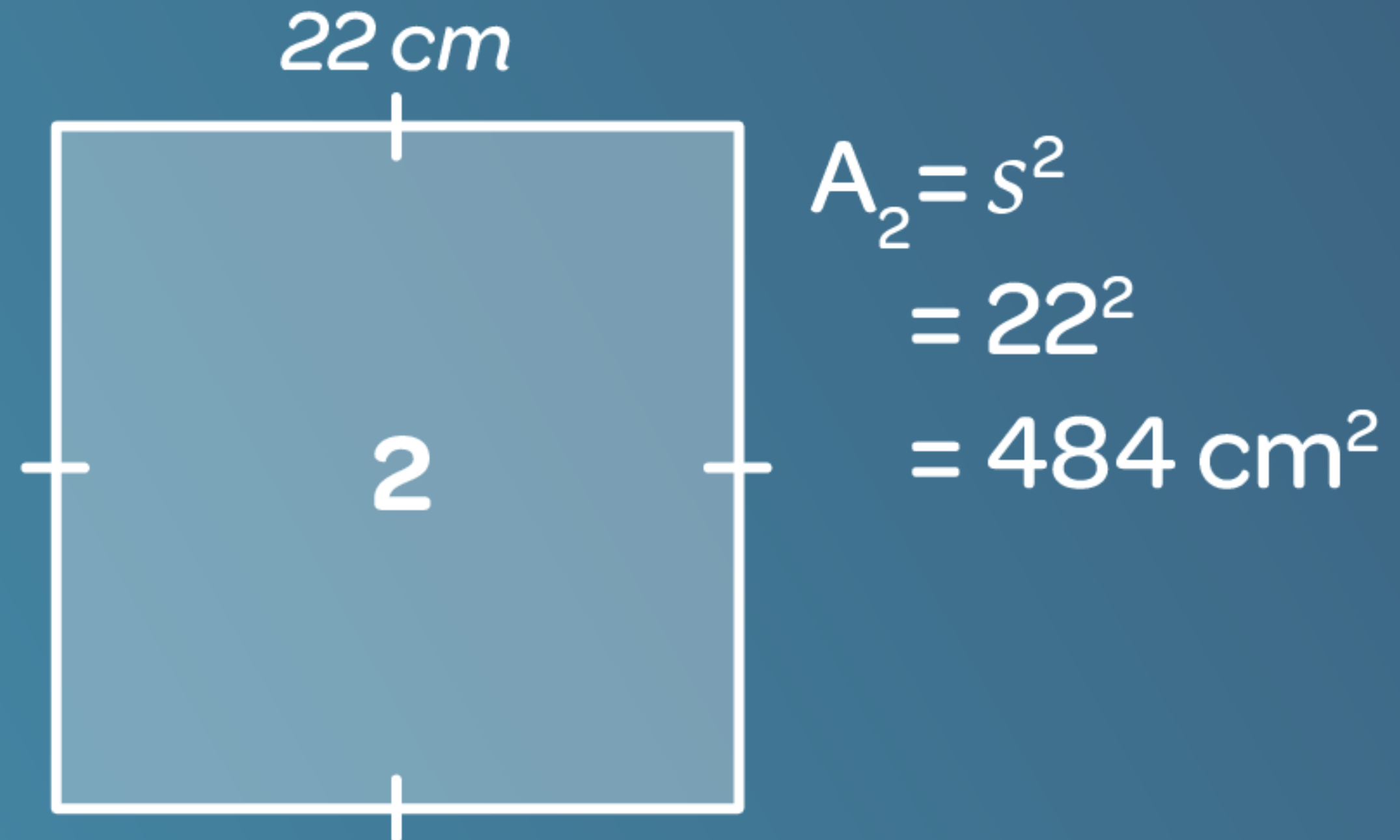2. Identify the important information

   The shape and side lengths of both artworks

3. Determine what a good solution could include

   Diagrams and area calculations

Solution

11 cm

1

$A_1 = s^2$

$= 11^2$

$= 121 \text{ cm}^2$

22 cm

2

$A_2 = s^2$

$= 22^2$

$= 484 \text{ cm}^2$
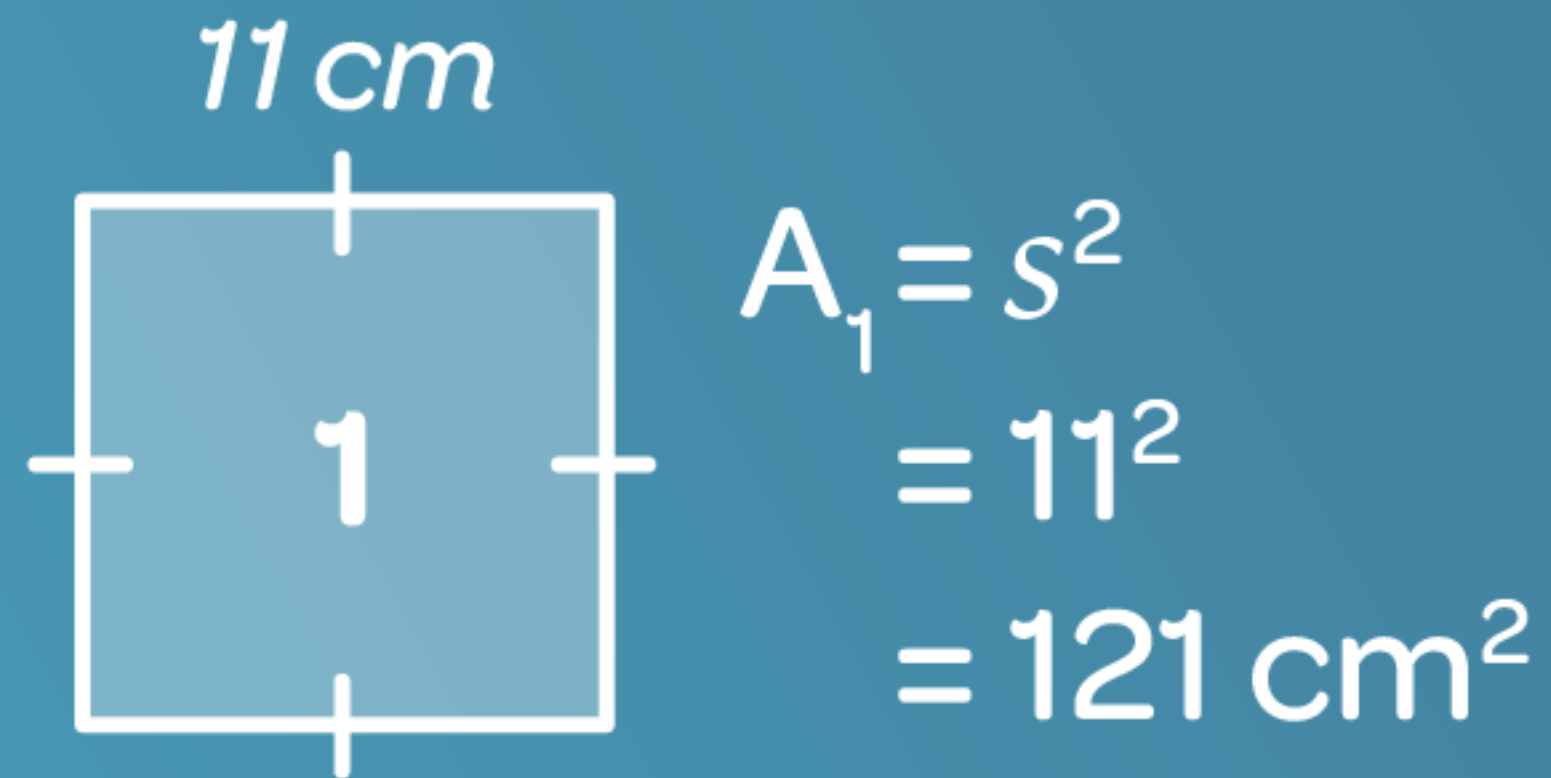
$A_{total} = A_1 + A_2$

$= 121 + 484$

$= 605 \text{ cm}^2$

The total area of both artworks is 605 cm².

Let's turn this problem solving task into a reasoning task

To turn a problem solving task into a reasoning task, we need to change the emphasis of the task.

To do this, we use cognitive verbs.

Students use reasoning skills
when they are asked to:

prove   deduce

analyse   justify

evaluate   generalise

explain   compare

infer   contrast

Reasoning Task 1

I hung two artworks on my bedroom wall. Both artworks were square. The side length of the first artwork was 11 cm. The area of the second was 484 cm^2.

Prove that the side length of the second artwork is double the side length of the first.

Prove: To provide certain evidence, eliminate all other possibilities

Prove should include phrases like 'is/is not' or 'will/will not'.

How can we prove that the side length of the second artwork is double the length of the first?

We can show that a square with an area of 484 cm² has a side length of 22, which is double 11.

Solution

The area of the second artwork is 484 cm². Find the side length:

$$A = s^2$$
$$484 = s^2$$
$$\sqrt{484} = \sqrt{s^2}$$
$$22 = s$$
$$s = 22$$

Determine whether the side length of 11 has doubled:

$$11 \times 2 = 22$$

Therefore, when the area is 484 cm², the side length is 22 cm. This is double the side length of the first artwork, 11 cm.

This proves that the side length of the second artwork is double the side length of the first.

Your Turn

Prove that only one of these numbers is both a square number and not even.

14   15   24   25   35   36

How can we prove that only one of the numbers is both a square number and not even?

We can show that one of these numbers meets both conditions and the other numbers do not.

Solution

List the numbers: 14, 15, 24, 25, 35, 36

Find all the square numbers:

$\sqrt{14} = 3.74...$ $\sqrt{25} = $ **5**

$\sqrt{15} = 3.87...$ $\sqrt{35} = 5.91...$

$\sqrt{24} = 4.89...$ $\sqrt{36} = $ **6**

The only square numbers are 25 and 36.

25 is not even; 36 is even.

25 is the only number that is both square and not even.

Reasoning Task 2

I hung two artworks on my bedroom wall. Both artworks were square. The side length of the first artwork was 11 cm and the side length of the second was double that.

Justify why the area of the second artwork is not double the first.

Justify: To make a convincing argument; follow a logical pathway; show linked steps; show all working

Justify should include 'because' statements.

How can we justify that the area of the second artwork has not doubled?

We can investigate the relationship between length and area.

Solution

Length is one dimensional (length). Area is two dimensional (length and width). With squares, the length and width are equal.
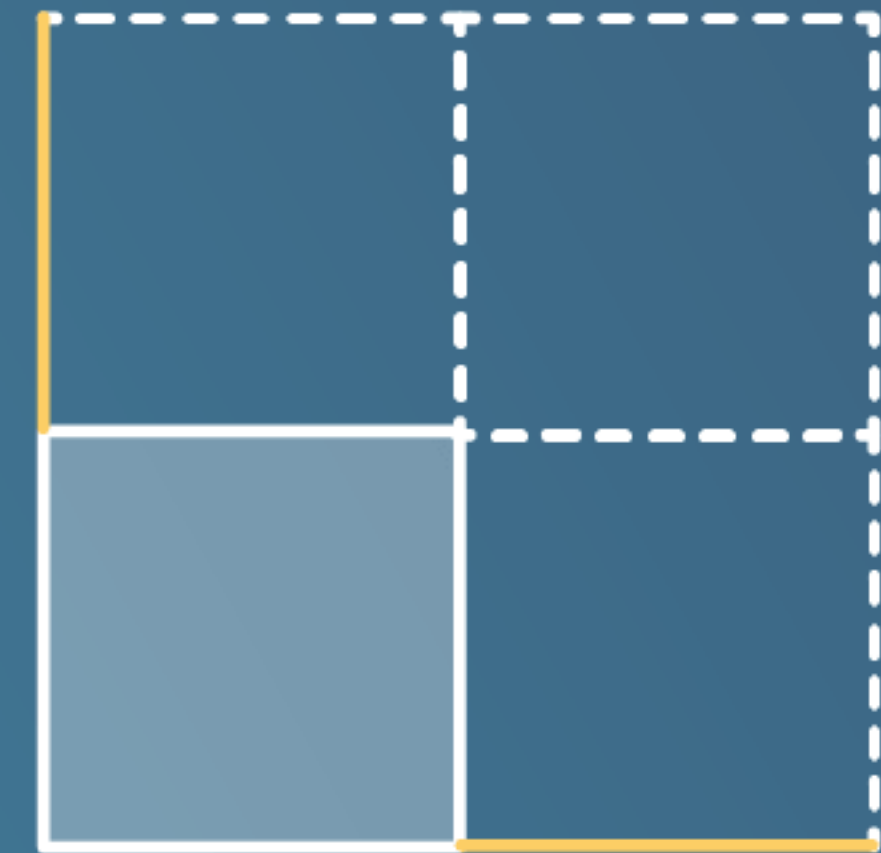
If we double only the length of a square, we extend it along one dimension. This doubles the area:

Solution

However, when we double the length and width of a square, we extend it across two dimensions. This multiplies the area by 4:

The area of the second artwork is not double the first because, when we extend by two dimensions (length and width), we quadruple the total area.

Reasoning Task 3

I hung three artworks on my bedroom wall. All artworks were square. The side length of the first artwork was 11 cm and the side length of the second was double that. The third was double again.

Make a **generalisation** about the areas of any additional artworks.

Generalise: A statement or conclusion based on data, or results that hold true in most cases

Generalise should include 'if/then' statements.

How can we generalise how the area will change as the dimensions of the artworks double?

We can look for a pattern between side length and area.

Solution

Calculate the area of squares when the side length is doubling:

Side length (cm)	11	22	44	88
Area (cm^2)	121	484	1936	7744

Determine the pattern:

$$\times 2 \qquad \times 2 \qquad \times 2$$

Side length (cm)	11	22	44	88
Area (cm^2)	121	484	1936	7744

$$\times 4 \qquad \times 4 \qquad \times 4$$

The pattern shows that as the side length is doubling, the area is increasing by a multiple of 4.

Therefore, we can generalise that if the side length of the artwork doubles, then the area quadruples.

Bit Maths provides targeted reasoning questions.

Students can work independently or elect to be guided through the Reasoning Wizard.

The Wizard scaffolds the reasoning process by unpacking the meaning of the cognitive verb in context, and supporting students to provide a well-reasoned answer.

Reasoning Activity

A real estate agent is trying to sell five different houses in a suburb.

The asking prices are:

Instead Boulevard	$480 000
Bird Lane	$1 400 000
Siganto Street	$650 000
Jowett Court	$710 000
Dawson Drive	$480 000

The agent wants to make the suburb appear more expensive than it is, in order to make it more attractive to his potential buyers. He wants to publish the highest average (mean, median and mode) asking price in the suburb, using only data relating to the five different houses he is selling.

Choose which measure of central tendency he should use and justify why. Remember, he needs the average price to sound high.

BEGIN

The task

A real estate agent is trying to sell five different houses in a suburb.

The asking prices are:

Instead Boulevard $480 000
Bird Lane $1 400 000
Siganto Street $650 000
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Dawson Drive $480 000

The agent wants to make the suburb appear more expensive than it is, in order to make it more attractive to his potential buyers. He wants to publish the highest average (mean, median and mode) asking price in the suburb, using only data relating to the five different houses he is selling.

Choose which measure of central tendency he should use and justify why. Remember, he needs the average price to sound high.

This Reasoning task requires you to:
- justify a decision for choosing a particular measure of central tendency.

This Reasoning question would be best answered by:
- showing calculations for all measures of central tendency
- completing a statement justifying the agent's choice.

Ensure you complete all required working in your workbooks, and be prepared to show your teacher should they ask.

The first step to good reasoning is ensuring we understand the question.

1 What is this question asking us to justify?

○ that house prices can vary greatly

○ that real estate agents set their own prices

○ the decision to use one particular measure of central tendency over another

○ the decision to choose one property over another

SUBMIT

The first step to good reasoning is ensuring we understand the question.

1 What is this question asking us to justify?

○ that house prices can vary greatly

○ that real estate agents set their own prices

◉ the decision to use one particular measure of central tendency over another ✅

○ the decision to choose one property over another

That's right! In this case, we must justify the agent's decision to choose the measure of tendency that provides the highest value for an average property price.

CONTINUE

2 What mathematical evidence should we provide to justify our choice?

○ calculate and show the mean average for the house sale prices only

○ calculate and show the mode average for the house sale prices only

○ calculate and show the median average for the house sale prices only

○ calculate and show the mean, median and mode values for the house sale prices

SUBMIT

2 What mathematical evidence should we provide to justify our choice?

○ calculate and show the mean average for the house sale prices only

○ calculate and show the mode average for the house sale prices only

○ calculate and show the median average for the house sale prices only

◉ calculate and show the mean, median and mode values for the house sale prices   ✔

That's correct. In order to justify our decision, we need to calculate all of the measures of central tendency, so we can compare them and find which value shows the highest average price.

CONTINUE

3 Let's find the mean of the house prices first.

To find the average using mean we [Choose... ▾] all of the values in the data set together and [Choose... ▾] this by the number of values in the set.

SUBMIT

3 Let's find the mean of the house prices first.

To find the average using mean we [add ▾] all of the values in the data ✅ set together and [divide ▾] this by the number of values in the set.

That's correct.

CONTINUE

4 Find the mean now. You can use a calculator, but make sure you show your working in your workbook.

The mean average price of the house is $ [　　　　　　] .

SUBMIT

4 Find the mean now. You can use a calculator, but make sure you show your working in your workbook.

The mean average price of the house is $ [744 000] .

That's correct. Let's calculate the median next.

CONTINUE

5 Let's find the median next.

To find the average using median we arrange all of the values in the data set in [Choose... ▾] and select the [Choose... ▾] value.

SUBMIT

5 Let's find the median next.

To find the average using median we arrange all of the values in the data ✓
set in [ascending order ▾] and select the [middle ▾] value.

That's correct.

CONTINUE

6 Find the median now.

Rearrange the asking prices into ascending order.

| $480 000 | $710 000 | $650 000 | $1 480 000 | $480 000 |

SUBMIT

6 Find the median now.

Rearrange the asking prices into ascending order.

| $480 000 | $480 000 | $650 000 | $710 000 | $1 480 000 | ✔ |

That's correct.

CONTINUE

7 The median average price of the houses is $ [   ] .

SUBMIT

7 The median average price of the houses is $ [650 000] . ✓

That's right! Let's calculate the mode next.

CONTINUE

8 To find the average using mode we look for the value or values that occur [Choose... ▾] than others.

SUBMIT

8 To find the average using mode we look for the value or values that

✔

occur [more frequently ▾] than others.

That's correct.

CONTINUE

9 Find the mode now.

The mode average price of the houses is $ [   ] .

SUBMIT

9 Find the mode now.

The mode average price of the houses is $ [480 000] .   ✓

That's correct.

CONTINUE

10 We've calculated all of the measures of central tendency. Let's review our data.

mean $ [   ]

median $ [   ]

mode $ [   ]

SUBMIT

10 We've calculated all of the measures of central tendency. Let's review our data.

mean $ [744 000]

median $ [650 000]

mode $ [480 000]

That's correct.

CONTINUE

11 Which measure of central tendency should the agent to use to show the highest average price?

○ mean

○ median

○ mode

SUBMIT

11 Which measure of central tendency should the agent to use to show the highest average price?

- ⦿ mean
- ○ median
- ○ mode

Perfect!
But why did the mean show the highest value of all the measures of central tendency?

CONTINUE

12 Sometimes values in a set of data can be much smaller or much larger than the other values in the set.

There is an unusual value in this set that affected the mean average. It is the house valued at [Choose... ▾]. It has a much [Choose... ▾] value than other houses in the data set.

SUBMIT

12 Sometimes values in a set of data can be much smaller or much larger than the other values in the set. ✓

There is an unusual value in this set that affected the mean average. It is the house valued at $1 400 000 ▾ . It has a much higher ▾ value than other houses in the data set.

Great! Let's complete a statement to justify our choice.

CONTINUE

Let's look at the original statement again:

A real estate agent is trying to sell five different houses in a suburb.

The asking prices are:

Instead Boulevard	$480 000
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Choose which measure of central tendency he should use and justify why.
Remember, he needs the average price to sound high.

13 Refer to your working to complete this statement:

My working shows that the agent should use [Choose... ▾] to find the average house price.

SUBMIT

Let's look at the original statement again:

A real estate agent is trying to sell five different houses in a suburb.

The asking prices are:

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Choose which measure of central tendency he should use and justify why.
Remember, he needs the average price to sound high.

13 Refer to your working to complete this statement:

My working shows that the agent should use [mean ▾] to find the average house price.   ✔️

CONTINUE

14 The mean house price for the properties is [Choose... ▾] .

The agent should use mean as this results in a [Choose... ▾] average price than if he uses median or mode for the average price.

SUBMIT

14 The mean house price for the properties is $744 000 ▾ .   ✓

The agent should use mean as this results in a higher ▾ average price

than if he uses median or mode for the average price.

CONTINUE

15 The mean results in a higher average price because of the unusual house price of Choose... ▾ .

SUBMIT

15 The mean results in a higher average price because of the unusual house price of $1 400 000 ▾ .

Let's recap: To develop reasoning skills, students need to:

1. understand what the problem is asking and the reasoning required to solve it

2. know how to write a well-reasoned answer.

As teachers, we need to model both.

To do so, focus on cognitive verbs when tackling reasoning.

Want more?

Register your interest at bitmaths.com.au.

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