GEOMETRY + TECHNOLOGY = PROOFS

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Why Proofs?

- Proving and refuting mathematical claims constitute a significant element in the development of deductive thinking.
- Natural connection exists between generalization and deductive thinking.
- Formulating generalizations are difficult for students who lack deductive thinking.

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About Logical Thinking

- Students can best demonstrate their ability to think logically through the process of proof. It is in the process of proof that students are required to fully justify every step of their reasoning.
- In the high school curriculum proofs are mostly considered in the course of geometry. In the American secondary school curriculum there is practically no place for proofs in the traditional courses of algebra and precalculus.
- Since geometry only takes a small fraction of the high school mathematics curriculum, it is understandable why development of logical thinking and reasoning in secondary school mathematics becomes problematic.

Difficulties in Teaching Proofs

- Lack of motivation
- Lack of an algorithm
- Difficulty in choosing a method of proof
- Where to start?
- Multi-step planning
- Finding conjecture
- Problem interpretation
- Additional constructions

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Most Common Dynamic Geometry Software (DGS)

- TI-Nspire CAS by Texas Instruments
- Cabri by Cabrilog
- Geometer's Sketchpad by Key Curriculum Press (no longer supported)
- GeoGebra by International GeoGebra Institute (open source)

Enables students to

- discover results for themselves
- formulate conjectures and intermediate results
- examine special cases
- generate new ideas

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Features of DGS

- Dynamic geometry capabilities that are
  - numeric based
  - construction based
- CAS system is not built-in or CAS is not integrated with dynamic geometry

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Symbolic Geometry Software
Geometry Expressions™

- an interactive geometry system
- CAS is integrated with geometry
  - constraint-based rather than construction-based
  - symbolic rather than numeric
  - takes a geometric configuration and outputs algebraic expressions for quantities measured from the model

Geometry Expression™ is a trademark of Saltire Software (http://geometryexpressions.com)

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How Geometry Expressions Works?

- It provides the value of and/or an expression for goal parameters.
- It provides the value of and/or an expression for introduced parameters.
- It provides graphical/geometric objects based on algebraic expressions
- It provides algebraic expressions for graphical/geometric objects.

- Example: Pythagorean theorem

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Role of Geometry Expressions

- Enable students to
  - discover results for themselves
  - formulate conjectures and intermediate results
  - examine special cases
  - generate new ideas
- Assist students in
  - verifying validity of conjectures
  - proving conjectures
- In some cases, results provided by the software could be considered a proof

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Role of Technology in Development of Proofs

DGS

- Provide **a geometric approach** to strengthening reasoning skills by allowing students to explore geometric objects visually and dynamically and to generate and confirm conjectures on the basis of their observations.

Geometry expressions

- Provide opportunities for developing **an algebraic approach** to proofs due to its capability to produce symbolic algebraic outputs for geometric objects that can guide students as they develop strategies for proofs.

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Proofs Techniques (Methods)

- Geometric method - based on visual representation
- Algebraic method - based on investigation of various algebraic and trigonometric expressions
- Coordinate method - based on introduction of a system of coordinates and the use of formulas from coordinate geometry
- Vector method - based on using operations with vectors
- Transformations method - based on both isometric (rigid) and non-isometric (non-rigid) transformations on a plane

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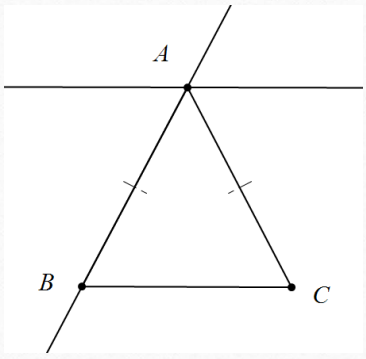
Geometric method

- Problem Statement: Given an isosceles triangle ABC, $AB = AC$. The angle bisector is constructed for the exterior angle at vertex A. What is the relationship between the angle bisector and the side BC?
- This problem is solved geometrically without additional constructions.



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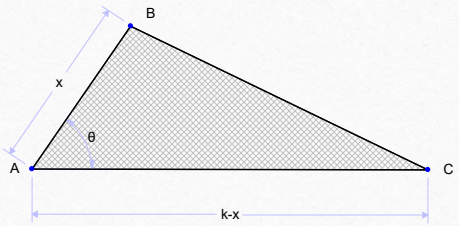
Algebraic method

- Problem Statement: In a triangle *ABC*, with fixed angle *BAC*, *AB* + *AC* is constant. Among all such triangles, find one with the largest area.
- This problem is solved algebraically without additional constructions.



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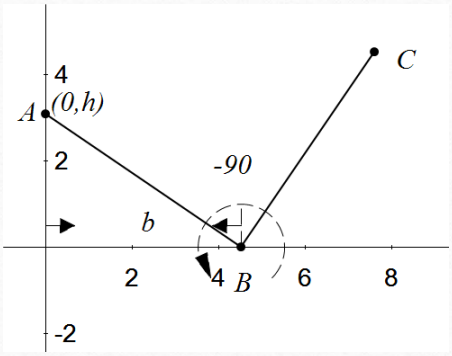
Coordinate method

- Problem Statement: Given a line and two points. Point *A* is not on the given line and point *B* is on the given line. Point *C* is chosen so that *AB* and *BC* are equal and perpendicular. What is the locus of the point *C*, if the point *B* moves along the given line?
- The problem is solved using the coordinate method.



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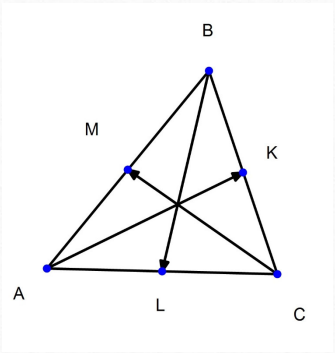
Vector method

- Problem Statement: Is it possible to construct a triangle from the three medians of a given arbitrary triangle?
- This problem is solved by vector method. In vector terms, the necessary and sufficient condition for constructing a triangle is equivalent to the condition that the sum of three non-collinear vectors be equal to the zero-vector.



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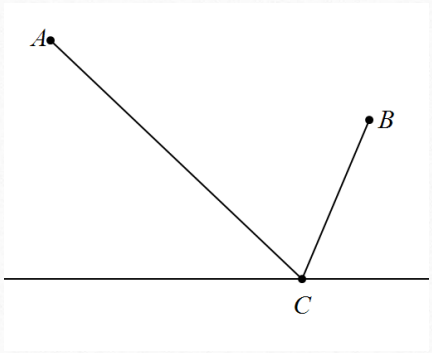
Transformations method

- Problem Statement: Points *A* and *B* are on one side of a given line. Find a point *C* on the line, such that *AC* + *CB* is the smallest.
- This is a classic optimization problem with many applications. The problem is easy to solve using reflection.



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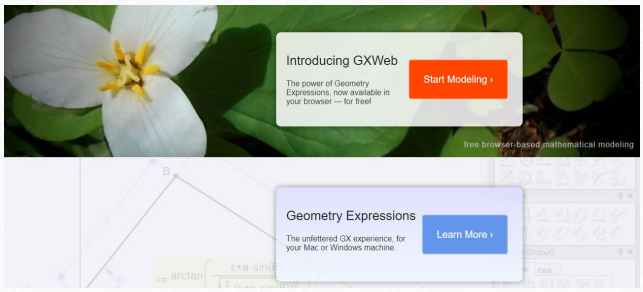
Sources

- Lyublinskaya, I., Ryzhik, V., Funsch, D. (2009) *Developing geometry proofs with Geometry Expressions.* Tigard, OR: Saltire Software, Inc. 297p. (**see amazon.com or geometryexpressions.com**)
- For more information check out http://www.geometryexpressions.com/



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