G11 Connecting Probability

Abstract

Y7 - Y10, Pedagogical Content Knowledge, Workshop
Heather Ernst and Anna Morton, Federation University Australia

Where does the topic of probability fit with the maths curriculum? It is often the topic squeezed into the end of a busy year but it can effectively be connected into many if not all mathematics topics across the secondary year levels.

This workshop will trial games, experiments and learning activities to engage and support the learning of probability throughout the whole year. Through examining and challenging a number of common misconceptions associated with probability, the benefits of integrating probability more broadly will be demonstrated. For example, probability can support the understanding of fractions and fluency with numbers but even more importantly it can assist with problem-solving and the development of reasoning skills.

The workshop is aimed at Year 7-10 teachers as they prepare students for the leap into complex VCE probability, but will also help demonstrate how probability can be incorporated into decision making in daily life.

Federation University

Connecting Probability

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The problems

Busy curriculum and probability left to the end of the year

Many misconceptions and misuse of language

Students not prepared for Mathematical Methods

So much to remember in mathematics

The problems Solutions

Busy curriculum and probability left to the end of the year

Connect probability to different parts of the curriculum in mathematics, and other curriculum areas.

Misconceptions and misuse of language

Actively address misconceptions and misuse of language

Students not prepared for Mathematical Methods

Informally introduce Mathematical Methods concepts

So much to remember in mathematics

Understand concepts, so they are remembered

Probability

What is your favourite probability activity? Pair/Share

Three ways of looking at probability:

Experimental, Theoretical, Subjective probability

Fairground Investigation

Instructions

Place counter at Start

Toss coin

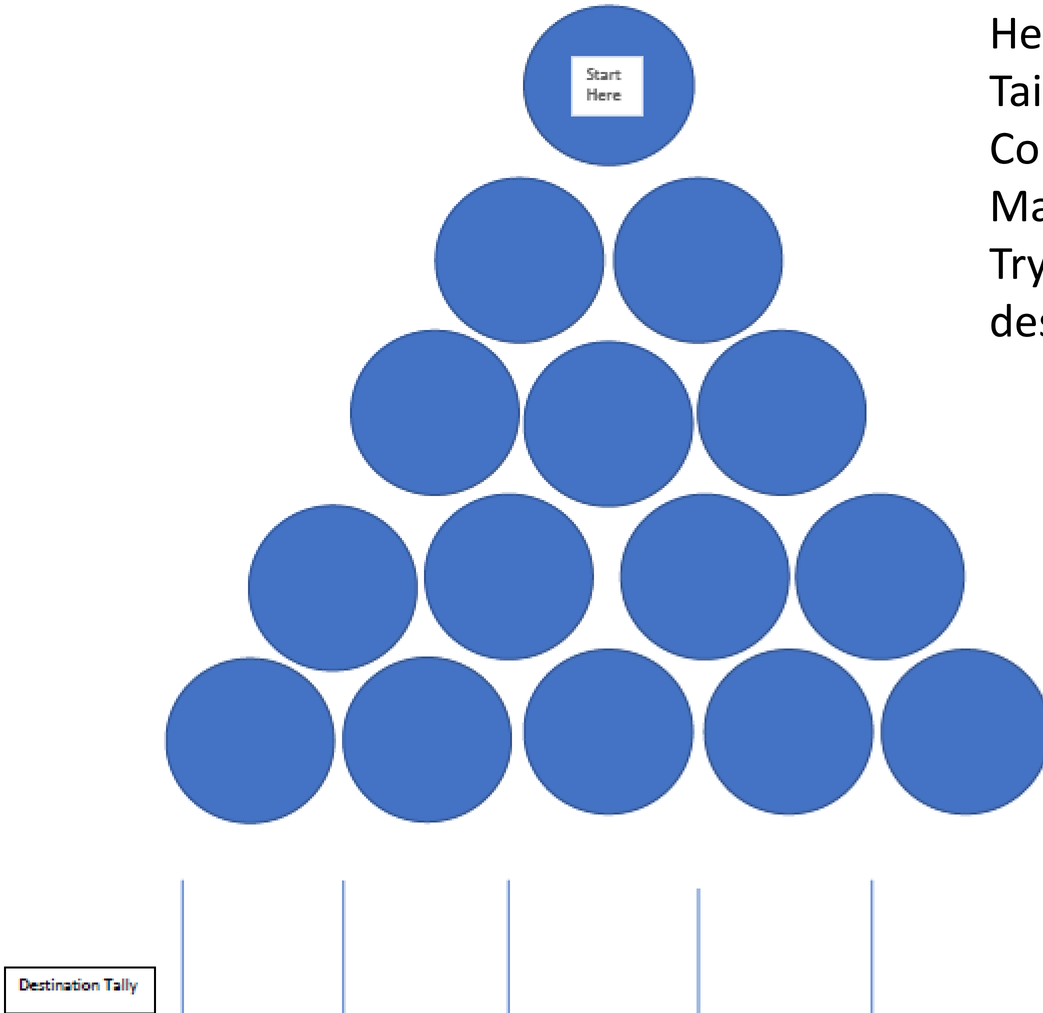
Heads – move left into next row

Tails – move right into next row

Continue until the last row of circles

Mark your destination (Tally I)

Try this out 50 times: marking your destination each time



Analysis:

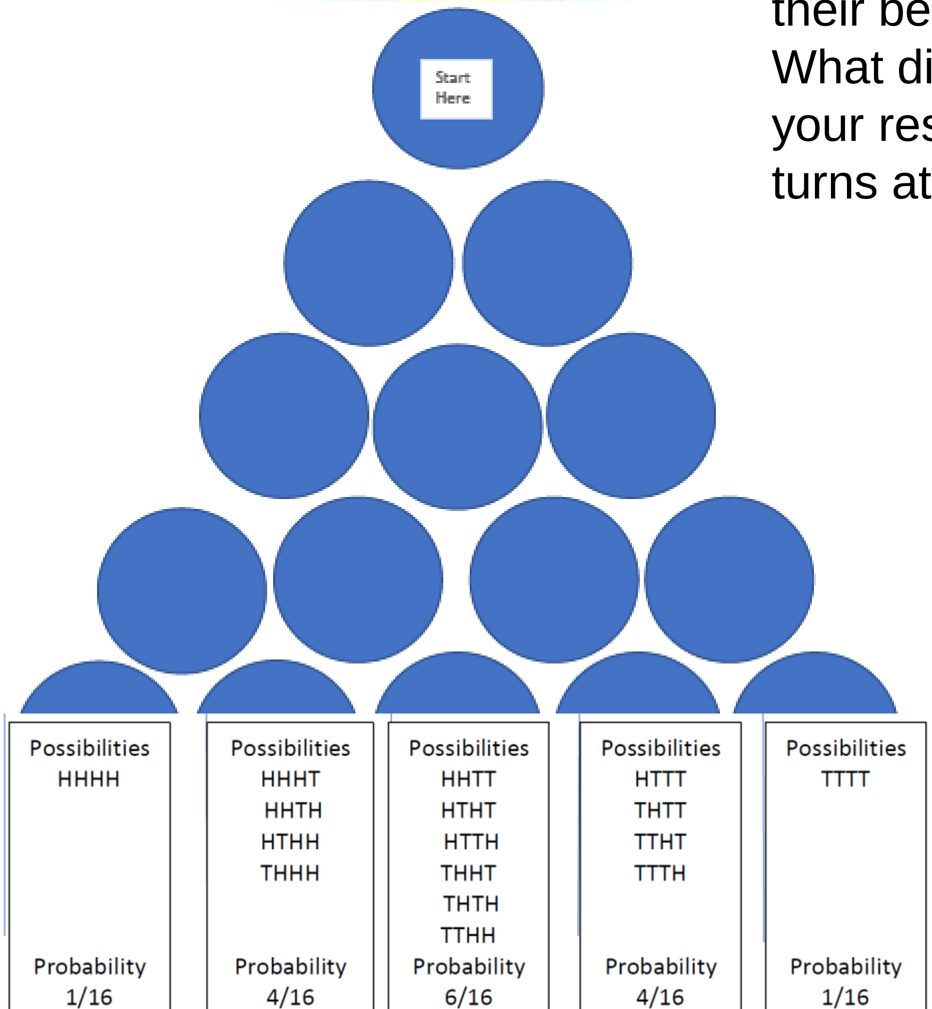
What have you discovered?

What is the probability of the counter finishing up in each of the slots?

How might you work this out?

Fairground Investigation

Further Questions:
Given your results explain where the fairground owners might place their best prizes?
What difference might it make to your results if you used 16, 32 or 48 turns at winning?

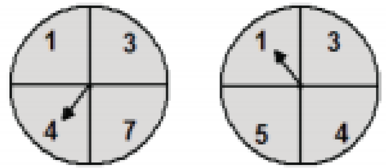


Destination Tally

NAPLAN probability – very little

YEAR 7 **NUMERACY (CALCULATOR ALLOWED)**

12 Marie spins these two arrows. She adds the numbers in the sections where the arrows stop and gets a **total** of 5.



Marie then spins the arrows again.

How many different ways can she get a **total of 8**?

1	2	3	4
⬭	⬭	⬭	⬭

Curriculum

Level 7

Construct sample spaces for single-step experiments with equally likely outcomes (VCMSP266)

Assign probabilities to the outcomes of events and determine probabilities for events (VCMSP267)

Level 8

Identify complementary events and use the sum of probabilities to solve problems (VCMSP294)

Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and' (VCMSP295)

Represent events in two-way tables and Venn diagrams and solve related problems (VCMSP296)

Curriculum

Level 9

List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (VCMSP321)

Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or' (VCMSP322)

Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians (VCMSP323)

Level 10

Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (VCMSP347)

Use the language of ‘if ....then, ‘given’, ‘of’, ‘knowing that’ to investigate conditional statements and identify common mistakes in interpreting such language (VCMSP348)

Mathematical Methods Unit 1

Area of Study 4

Probability and statistics

In this area of study students cover the concepts of event, frequency, probability and representation of finite sample spaces and events using various forms such as lists, grids, venn diagrams, karnaugh maps, tables and tree diagrams. This includes consideration of impossible, certain, complementary, mutually exclusive, conditional and independent events involving one, two or three events (as applicable), including rules for computation of probabilities for compound events.

This area of study includes:

- random experiments, sample spaces, outcomes, elementary and compound events
- simulation using simple random generators such as coins, dice, spinners and pseudo-random generators using technology, and the display and interpretation of results, including informal consideration of proportions in samples
- probability of elementary and compound events and their representation as lists, grids, venn diagrams, karnaugh maps, tables and tree diagrams
- the addition rule for probabilities, $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$, and the relation that for mutually exclusive events $\Pr(A \cap B) = 0$, hence $\Pr(A \cup B) = \Pr(A) + \Pr(B)$
- conditional probability in terms of reduced sample space, the relations $\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$ and $\Pr(A \cap B) = \Pr(A \mid B) \times \Pr(B)$
- the law of total probability for two events $\Pr(A) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B')\Pr(B')$
- the relations that for pairwise independent events A and B, $\Pr(A \mid B) = \Pr(A)$, $\Pr(B \mid A) = \Pr(B)$ and $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$.

Mathematical Methods Unit 2

Probability and statistics

In this area of study students cover introductory counting principles and techniques and their application to probability and the law of total probability in the case of two events.

This area of study includes:

- addition and multiplication principles for counting
- combinations: concept of a selection and computation of ${}^nC_r$ application of counting techniques to probability.

Probability and statistics

In this area of study students cover discrete and continuous random variables, their representation using tables, probability functions (specified by rule and defining parameters as appropriate); the calculation and interpretation of central measures and measures of spread; and statistical inference for sample proportions. The focus is on understanding the notion of a random variable, related parameters, properties and application and interpretation in context for a given probability distribution.

Mathematical Methods Units 3 and 4

This area of study includes:

- random variables, including the concept of a random variable as a real functi examples of discrete and continuous random variables
- discrete random variables:
  - specification of probability distributions for discrete random variables using graphs, tables and probability mass functions
  - calculation and interpretation and use of mean (μ), variance (σ^2) and standard deviation of a discrete random variable and their use
  - bernoulli trials and the binomial distribution, $\mathrm{Bi}(n, p)$, as an example of a probability distribution for a discrete random variable
  - effect of variation in the value/s of defining parameters on the graph of a given probability mass function for a discrete random variable
  - calculation of probabilities for specific values of a random variable and intervals defined in terms of a random variable, including conditional probability
- continuous random variables:
  - construction of probability density functions from non-negative functions of a real variable
  - specification of probability distributions for continuous random variables using probability density functions
  - calculation and interpretation of mean (μ), median, variance (σ^2) and standard deviation of a continuous random variable and their use
  - standard normal distribution, $\mathrm{N}(0, 1)$, and transformed normal distributions, $\mathrm{N}(\mu, \sigma^2)$, as examples of a probability distribution for a continuous random variable
  - effect of variation in the value/s of defining parameters on the graph of a given probability density function for a continuous random variable
  - calculation of probabilities for intervals defined in terms of a random variable, including conditional probability (the cumulative distribution function may be used but is not required)
- Statistical inference, including definition and distribution of sample proportions, simulations and confidence intervals:
  - distinction between a population *parameter* and a sample *statistic* and the use of the sample statistic to estimate the population parameter
  - concept of the sample proportion $\hat{P} = \frac{X}{n}$ as a random variable whose value varies between samples, where X is a binomial random variable which is associated with the number of items that have a particular characteristic and n is the sample size
  - approximate normality of the distribution of $\hat{P}$ for large samples and, for such a situation, the mean p, (the population proportion) and standard deviation, $\sqrt{\frac{p(1-p)}{n}}$
  - simulation of random sampling, for a variety of values of p and a range of sample sizes, to illustrate the distribution of $\hat{P}$
  - determination of, from a large sample, an approximate confidence interval $\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$ for a population proportion where z is the appropriate quantile for the standard normal distribution, in particular the 95% confidence interval as an example of such an interval where $z \approx 1.96$ (the term standard error may be used but is not required).

Connect to mathematics and other context, and VCE

Measurement,

With the Yellow Post-it-notes

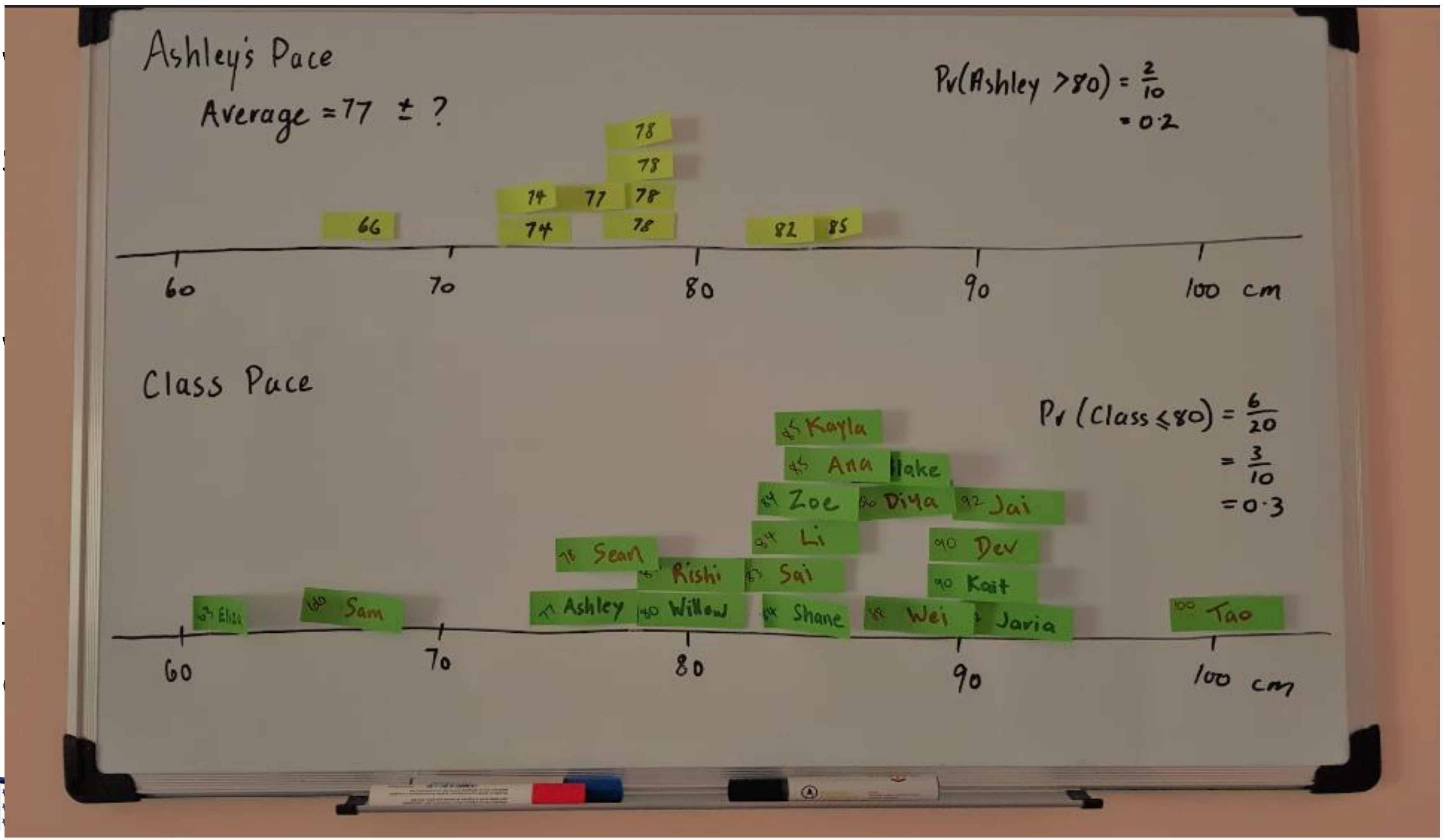
Several people to measure <u>Anna's</u> hand span
Put the measurements on a Post-it-note
Find the average, make a histogram

With the Green Post-it-notes

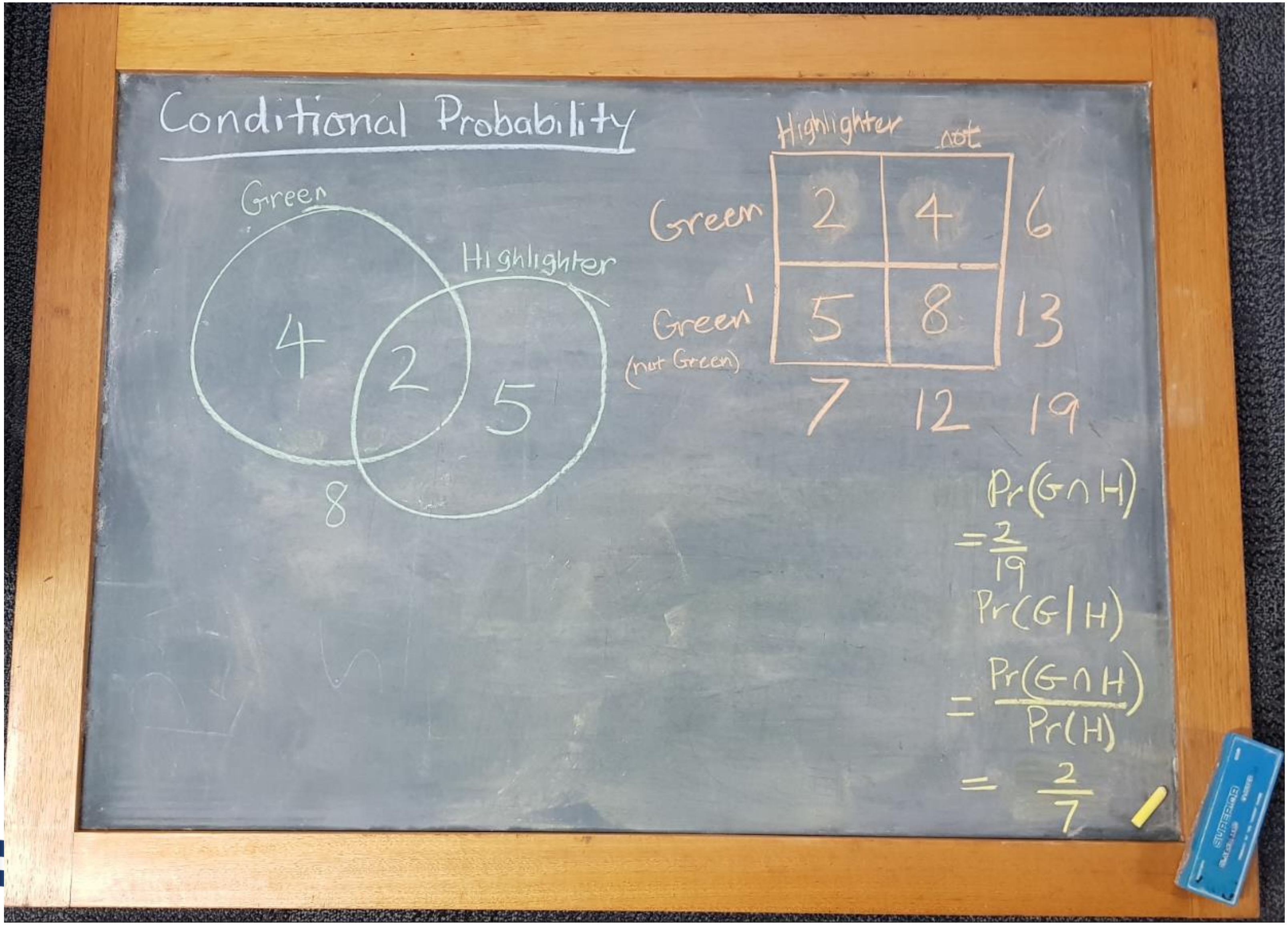
Measures your own hand span
Put the measurements on a Post-it-note
Find the average, make a histogram

Connect to mathematics and other context, and VCE

Measurement, errors, confidence intervals



Introduction to mathematical methods- language



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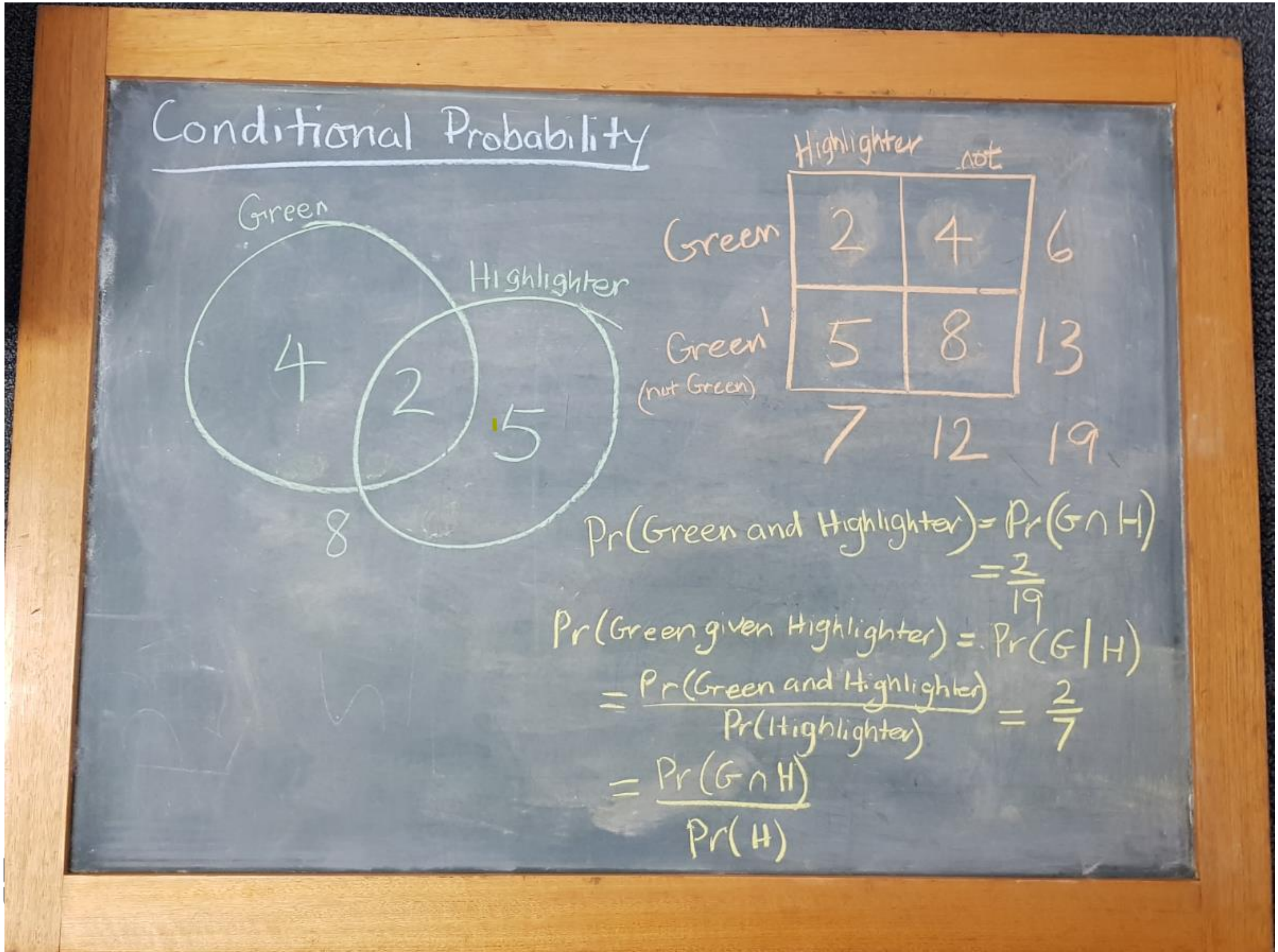
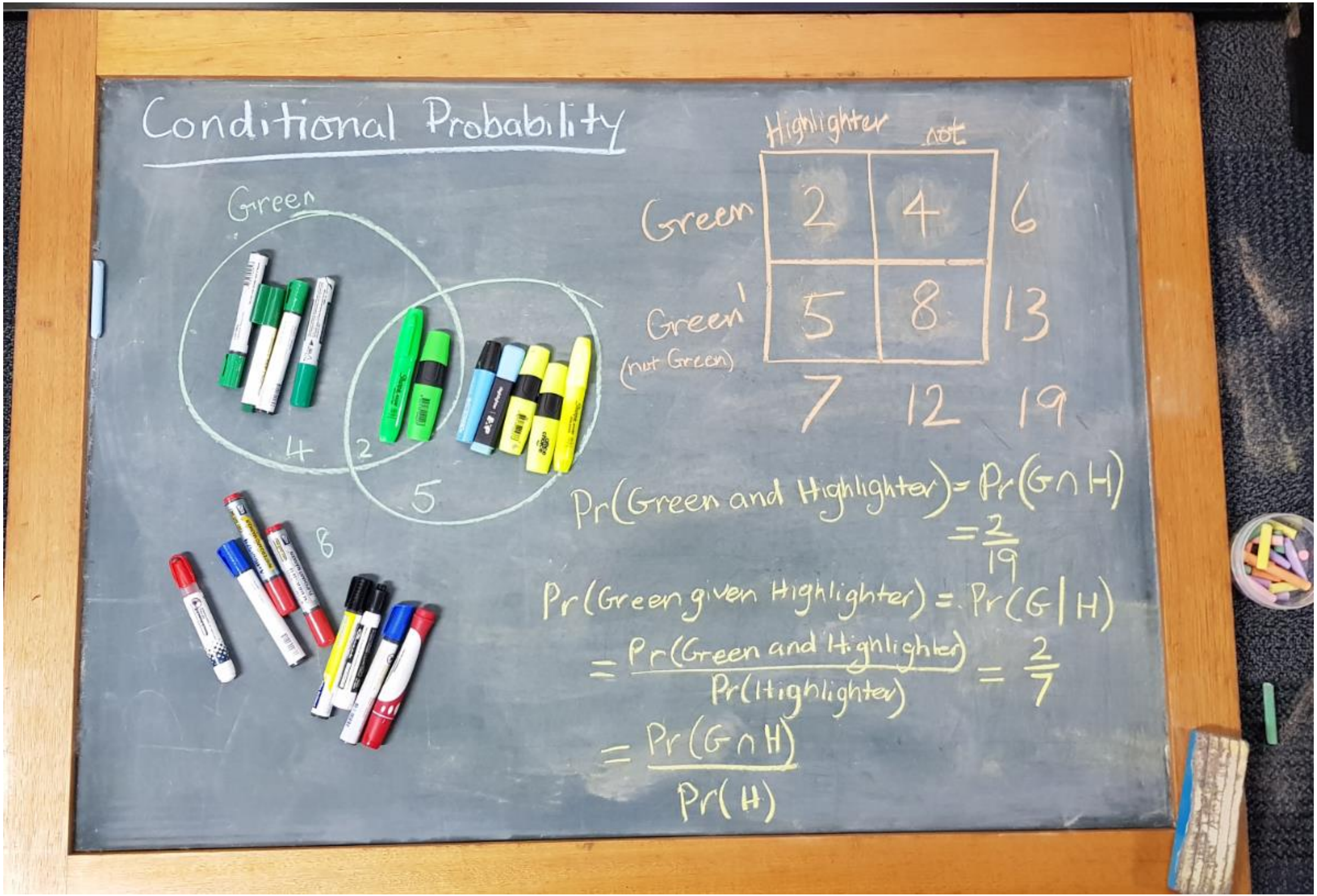
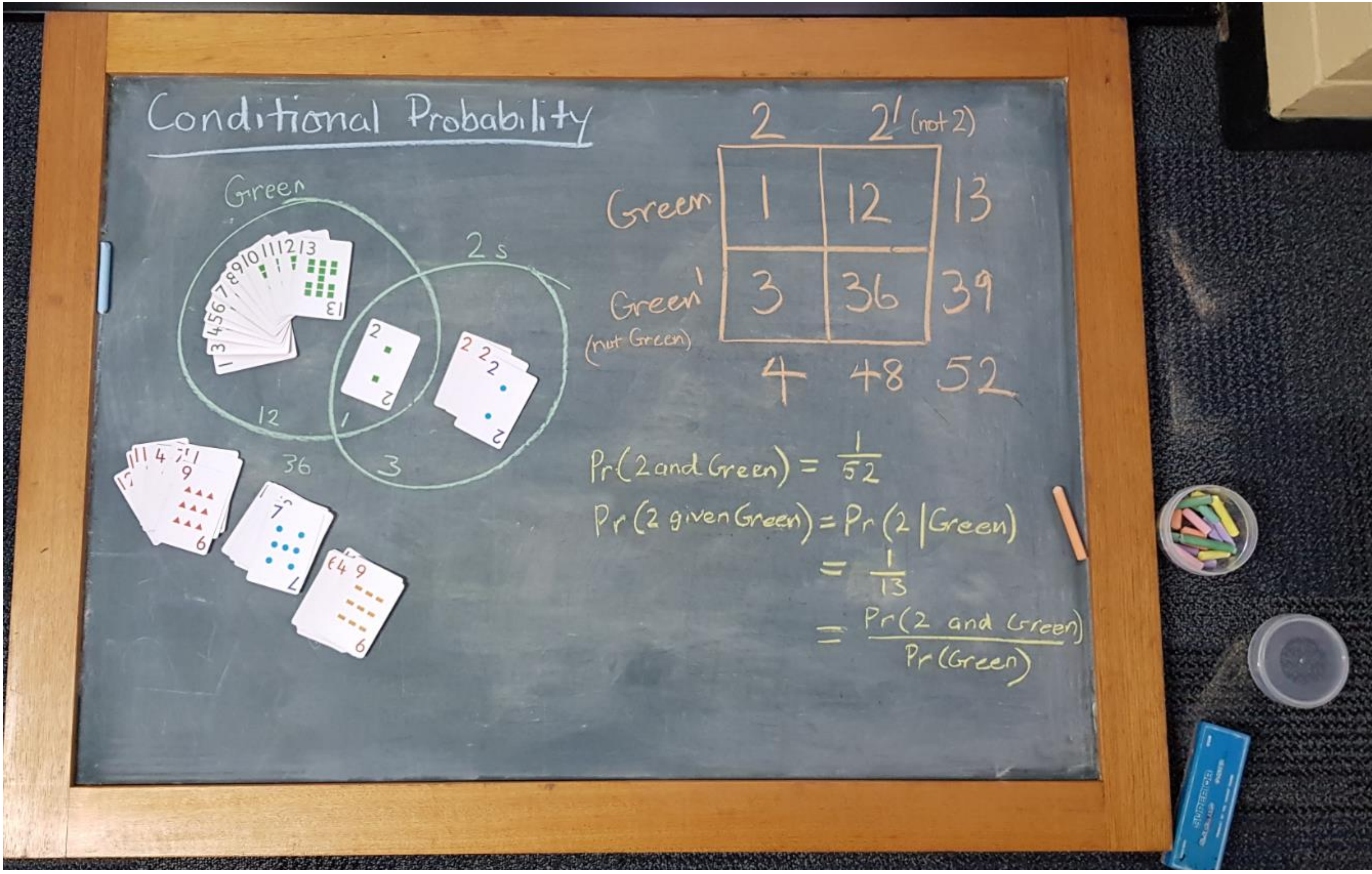


Introduction to mathematical methods- concrete



Introduction to mathematical methods- concrete

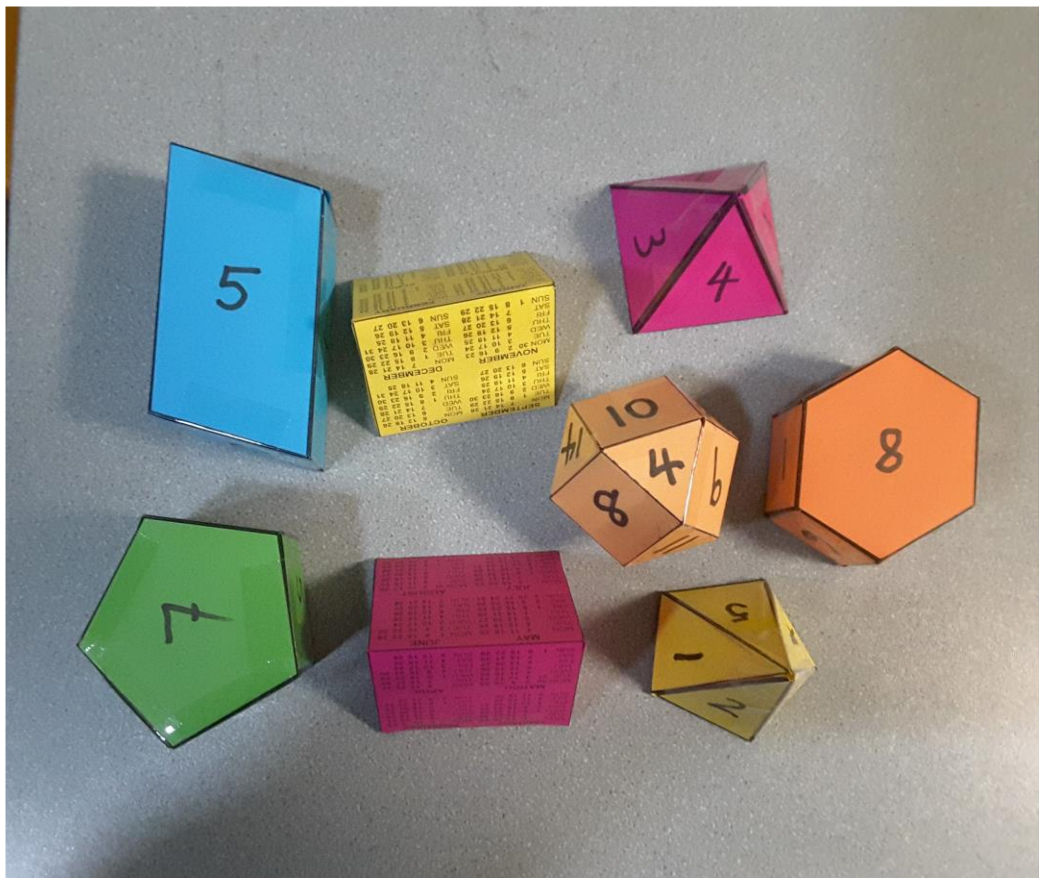


Connect to mathematics and other contexts

Geometry

Addresses the “Equally likely” misconception

Make the shapes
Name the shapes
What the chance of landing on each side?



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Connect to mathematics and other contexts

Geometry

YEAR 9 **NUMERACY (CALCULATOR ALLOWED)**

14 This block has 6 faces which are numbered from 1 to 6.

Vicky throws the block 1000 times to test it and records the outcomes.

Number on top face	1	2	3	4	5	6
Frequency	150	360	146	144	68	132

What is the probability of rolling a 2 based on Vicky's results?

$\frac{1}{6}$ ○ $\frac{1}{60}$ ○ $\frac{9}{25}$ ○ $\frac{3}{500}$ ○

Lu-lu

https://www.uccs.edu/Documents/pipes/mccoyprob-games.pdf

PROBABILITY GAMES from Diverse Cultures

LEAH P. McCOY, STEFANIE BUCKNER, AND JESSICA MUNLEY

TO MAKE MATHEMATICS RELEVANT AND meaningful for all students, it is important that we embrace a wide variety of real-world applications. Diverse cultures provide rich and interesting contexts in which students can experience and explore mathematics. One of the five Process Standards is Connections (NCTM 2000). The probability activities discussed here help students make connections across mathematics concepts through games from diverse cultures.

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Each lesson followed a common format. First, students learned about the game, including the history and background, as well as instructions for playing. Second, the teacher demonstrated the game to the class. Then students were placed in small groups of two to four, given appropriate game materials, and instructed to play. They were then introduced to a probability concept related to the game, either as an integral part of the strategy or as an experiment where data were collected as the game was played. Students collected and analyzed the data, and reported their results on the group worksheets as both short answers and longer explanations.

The lessons were designed and tested in prealgebra classes in a rural public middle school. Materials were either teacher-made or inexpensively purchased. We used the games as independent lessons, but a "game fair," where small groups of students rotate through the games in different learning centers, could also be designed.

Lu-lu

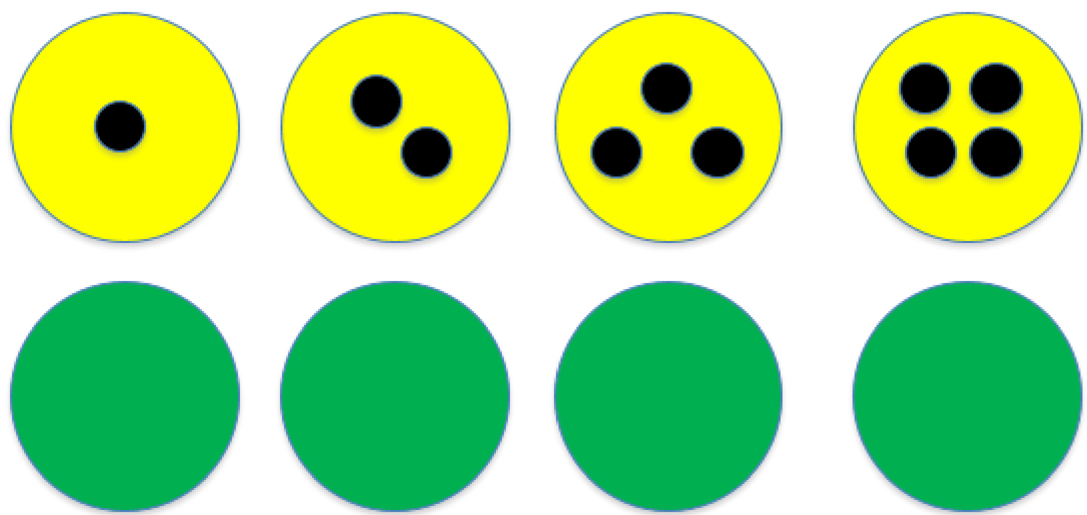
Hawaiian traditional game

Experimental/ Theoretical/ Subjective probability

Four counters, dots on one side and plain on the other.

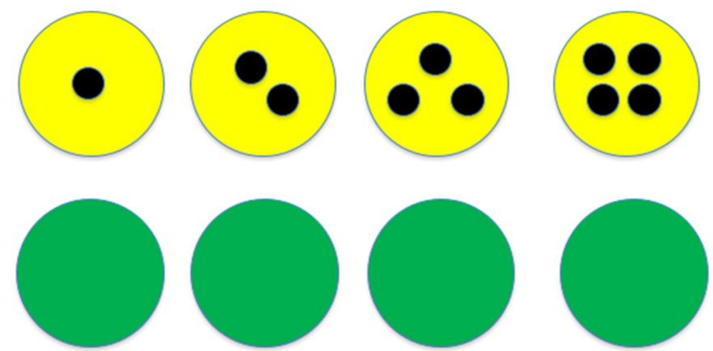
Shake and toss, add up the dots

Toss seven times, the winner is the closest to 50 dots



Lu-lu

What questions could you ask about this game?



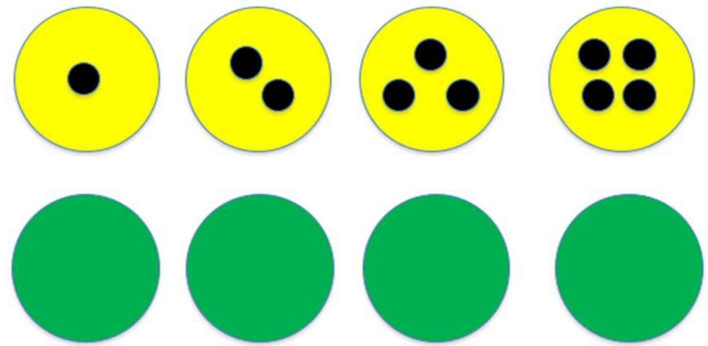
https://www.uccs.edu/Documents/pipes/mccoyprob-games.pdf

Lu-lu

What questions could you ask about this game?

How many ways are there to get a three?

What scores are possible?



What is the expected value?

How could you expand this game?

Expand - A pair of throws

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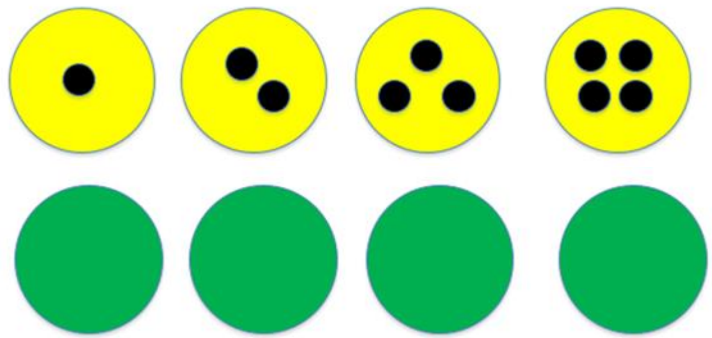
https://www.uccs.edu/Documents/pipes/mccoyprob-games.pdf

Lu-lu



Expected value?

				Sum
				0
1				1
1	2			3
1	2	3		6
1	2	3	4	10
1		3		4
1	2		4	7
1		3	4	8
1			4	5
	2			2
	2	3		5
	2		4	6
	2	3	4	9
		3		3
		3	4	7
			4	4
			Average	5

Sample space	Freq
0	1
1	1
2	1
3	2
4	2
5	2
6	2
7	2
8	1
9	1
10	1
Total	16

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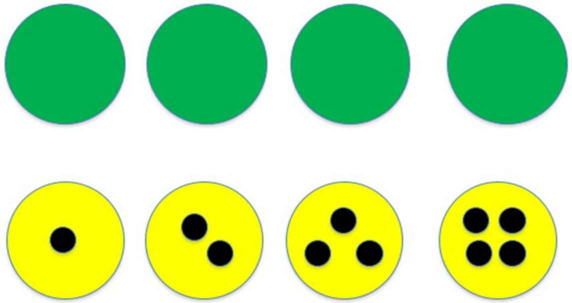
Lu-lu



Level 7 Assign probabilities to the outcomes of events and determine probabilities for events (VCMSP267)

Level 7 Construct sample spaces for single-step experiments with equally likely outcomes (VCMSP266)

Level 10 Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (VCMSP347)

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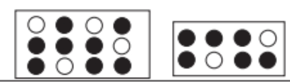
Misconceptions

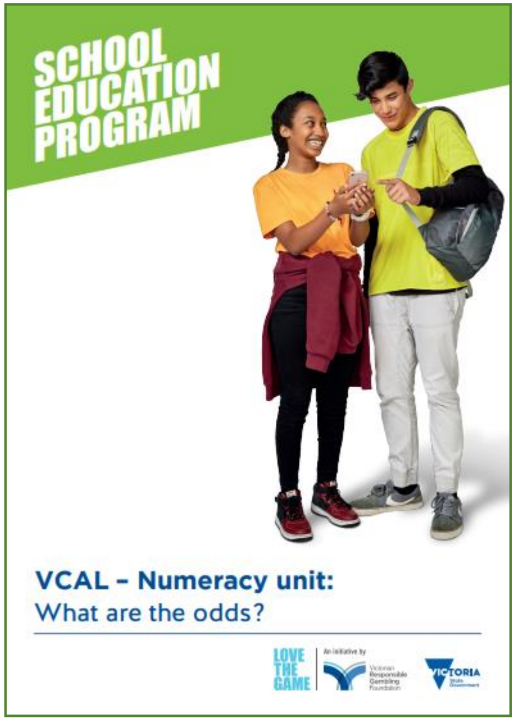
How would you explain why this misconception is not true?

http://www.cimt.org.uk/projects/mepres/allgcse/as5act1.pdf

1. I've spun an *unbiased* coin 3 times and got 3 heads. It is more likely to be tails than heads if I spin it again.	**2.** Aytown Rovers play Betown United. Aytown can win, lose or draw, so the probability that Aytown will win is $\frac{1}{3}$.
3. There are 3 red beads and 5 blue beads in a bag. I pick a bead at random. The probability that it is red is $\frac{3}{5}$.	**4.** I roll two dice and add the results. The probability of getting a total of 6 is $\frac{1}{12}$ because there are 12 different possibilities and 6 is one of them.
5. It is harder to throw a six than a three with a die.	**6.** Tomorrow it will either rain or not rain, so the probability that it will rain is 0.5.
7. Mr Brown has to have a major operation. 90% of the people who have this operation make a complete recovery. There is a 90% chance that Mr Brown will make a complete recovery if he has this operation.	**8.** If six fair dice are thrown at the same time, I am less likely to get 1, 1, 1, 1, 1, 1 than 1, 2, 3, 4, 5, 6.

9. There are more black balls in box A than in box B. If you chooses 1 ball from each box you are more likely to choose a black ball from A than from B. A  B	**10.** I spin two coins. The probability of getting heads and tails is $\frac{1}{3}$ because I can get Heads and Heads, Heads and Tails or Heads and Tails.
11. John buys 2 raffle tickets. If he chooses two tickets from different places in the book he is more likely to win than if he chooses two consecutive tickets.	**12.**  Each spinner has two sections – one black and one white. The probability of getting black is 50% for each spinner.
13. 13 is an unlucky number so you are less likely to win a raffle with ticket number 13 than with a different number.	**14.** My Grandad smoked 20 cigarettes a day for 60 years and lived to be 90, so smoking can't be bad for you.
15. It is not worth buying a national lottery card with numbers 1, 2, 3, 4, 5, 6, on it as this is less likely to occur than other combinations.	**16.** I have thrown an unbiased dice 12 times and not yet got a six. The probability of getting a 6 on my next throw is more than $\frac{1}{6}$.

Probability and Gambling



[VCAL Numeracy skills (foundation, intermediate and senior)]

What are the odds?

In this unit students will learn about the randomness of gambling games/activities along with the limited chances of winning given the size of gambling losses in Australia and the difficulty in predicting outcomes.

Students will gain an understanding:

- that 'chance has no memory'
- that many gambling games involve random processes
- how gambling agencies/venues make profits

VCAL Numeracy Unit: What are the odds?

Spreadsheets for demonstrating gambling outcomes and data:

- A day at the races
- Card sharp
- Melbourne Cup
- Pokies
- Setting limits
- Sports betting agency

PowerPoint presentations (pdf format) of lesson overviews:

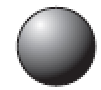
- Lesson 1: Chance has no memory
- Lesson 2: Who are the real winners?
- Lesson 3: Pokies
- Lesson 4: Sports betting
- Lesson 5: Horse racing

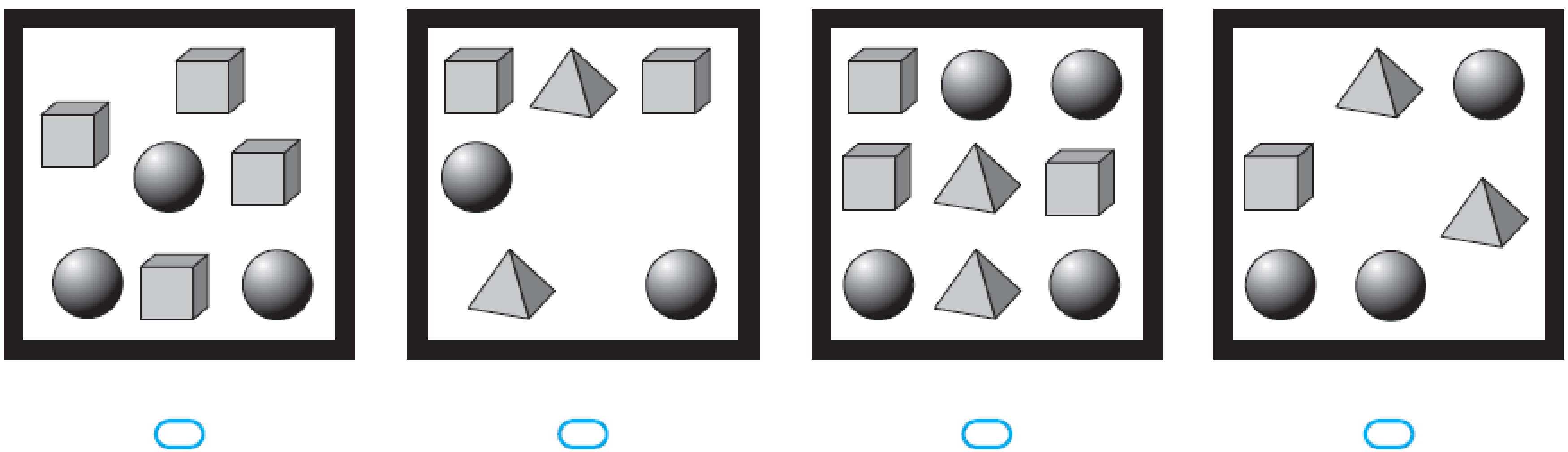
This unit was developed and piloted in partnership with the Mathematical Association of Victoria.

https://responsiblegambling.vic.gov.au/reducing-harm/schools/resources-teachers/

NAPLAN

Con takes an object from each box without looking.

Which box gives Con the **best chance** of taking a ?



Bingo

Times-table Bingo,

Children make their own Bingo boards. Would it be better to include: 24 or 15 or 31?

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

B	I	N	G	O
11	25	42	49	63
9	23	31	58	68
4	29	FREE	54	99
3	27	45	56	72
1	19	43	50	61

Misconceptions and language

Equally likely

Fractions

If I secretly use a spinner and tell you the results, can you tell be which spinner I have used?

Concentrate on correct, change and fraction/percentage language.

Spinners

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