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The Action of Mathematics: The Maths Proficiencies

MAV Conference 2019

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Genovieve Grouios

6th December 2019

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Warm Up - Balancing Numbers

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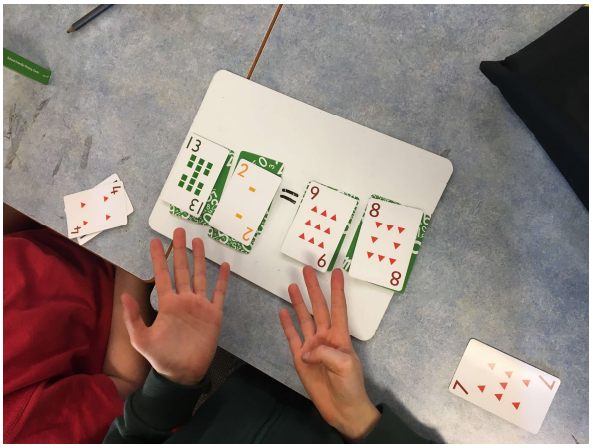

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Balancing Numbers - Instructions

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1) Divide a deck of cards into four piles, as shown
2) Turn over the top card in each pile
3) Two players play the game, with each player responsible for one side of the equal sign
4) Take turns removing one card from your designated side. Turn over the next card to try to make your side equivalent to the other side by adding the cards together on each side

- *An adaptation to this game is to allow different operations other than addition*

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Victorian Maths Curriculum

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Content – The *'nouns'* (what we teach)''

- Number & Algebra
- Measurement & Geometry
- Statistics & Probability

Proficiencies – The *'verbs'* (how we teach)

- Fluency
- Problem Solving
- Understanding
- Reasoning

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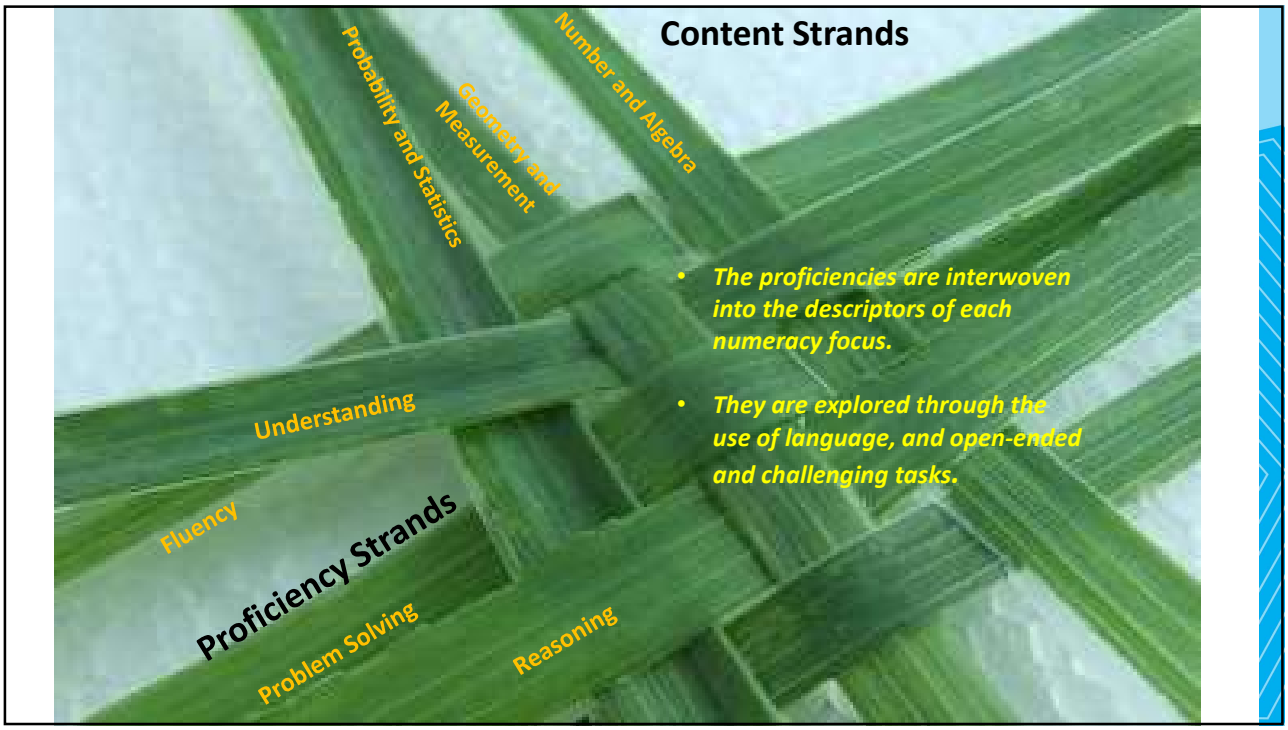

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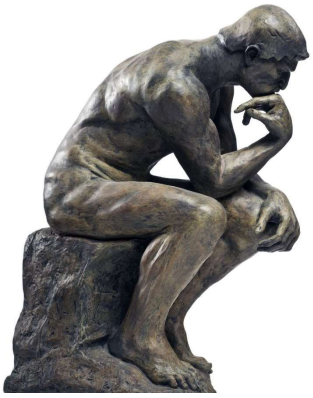

How Do we implement and encourage the proficiencies?

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1) Through rich, open-ended challenging Maths tasks

2) Through the use of properly planned questions

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Open-ended Challenging Maths Tasks

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Peter Sullivan

Challenging Mathematical Tasks supports the idea that students learn best when they work on problems that they do not yet know how to solve. Peter Sullivan's research shows that many students do not fear challenges in mathematics, but welcome them. Plus, rather than having teachers instruct them, these students prefer to work out solutions for themselves.

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Opened-ended Challenging Maths Tasks

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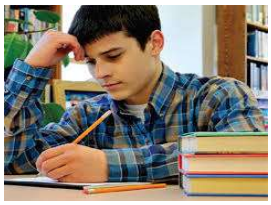
- Learning starts from an initial challenge with activities that allow for productive struggle, sustained thinking, decision-making and risk-taking by the students. They work by themselves first before working with another student(s).
- A zone of confusion may be created.
- Enabling and extending prompts are then introduced, if needed.



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Questioning is Vital to Student Learning

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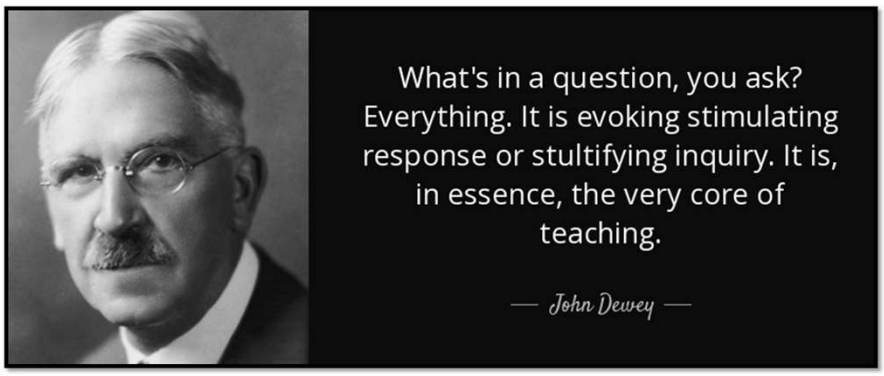

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Why Do We Use Questions?

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- **To scaffold students' understanding**
- **To clarify students' understanding of the strategies, knowledge and skills that they possess**
- **To probe students' thinking, and reasoning**

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Different Levels of Questioning

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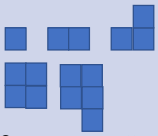
Essential Knowledge	Guiding Questions	Background Knowledge	Thinking Level
How can we classify numbers as odds and evens?	How can you represent this with materials?  Can you see a pattern with the even/odd numbers? Can you explain what happens?	When have you seen these numbers before e.g. 0, 2, 4, 6, 8?	Students apply this knowledge when working with conjectures such as the sum of 2 odd numbers is even. They will need to prove that this is always true.

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Effective Questioning Strategies in the Classroom

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- Planning is essential i.e. make sure that the questions are organised and connect to learning objectives. Spur-of-the-moment type questions are not as effective i.e. it's is harder to guide student thinking.
- Questions used in the classroom should allow students to freely explore ideas in a safe and positive environment and apply their developing knowledge and problem solving skills across domains.

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Effective Questioning Strategies in the Classroom

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- Questions should promote critical thinking, discussions and creativity.
- Some key things to consider when writing questions is to first identify the key mathematical ideas in the lesson.
- Next, consider the anticipated responses from students i.e. think about their prior skills, experiences and knowledge.
- Plan questions that are: ***factual, conceptual and speculative (open-ended)***

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What Does it Mean To Be Fluent in Mathematics?

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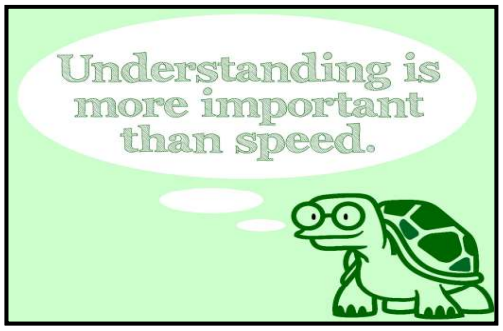

Many mathematicians work slowly, because they think deeply rather than quickly.
Jo Boaler

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Fluency

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- Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate *flexibility* in the computational methods they choose, *understand* and can explain these methods, and produce accurate answers *efficiently.*

- *It is not about being 'fast and accurate'. Although accuracy is important, speed is not.*

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Fluency (VCAA, 2019)

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- Making reasonable estimates
- Calculate answers efficiently
- Recognise robust ways of answering questions
- Choose appropriate methods and approximations
- Recall definitions ad use facts
- Can manipulate expressions and equations to find solutions

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Number Talks help develop Fluency

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- *'Number Talks are a brief daily practice where students mentally solve computation problems and talk about their strategies.'*
- *'It is about helping students learn to work flexibly with numbers and arithmetic properties.'*
- *'Helps them build a solid foundation and confident disposition for future mathematics learning.'*

Humphreys, C & Parker, R (2015)

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Solve this problem

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18 x 7

How many different ways can you solve this in your head?

- The teacher then asks the students to share and explain their strategy
- The teacher (or student) records the method on a whiteboard for everyone to see
- *Other strategies are shared and recorded as well*

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Fluency through Games

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Rather than rote or drill activities, fluency can be developed through games: E.g. Paul Swan card, dice and domino games

- **Snap – Number Partners to Ten**
- **Multo (Maths300)**
- **Domino Difference (ACU)**
- **7 Up! (ACU)**

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Fluency

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Questioning Prompts to use in the classroom:

1. If we changed the numbers what would happen?
2. What if….?
3. What could we do next?
4. Could you work this out a different way?
5. What instrument would you use to measure this table?

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Conceptual Understanding

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- *Students with conceptual understanding know more than isolated facts and methods.*
- *They make connections between related concepts and progressively apply the familiar to develop new ideas.*
- *They develop an understanding of the relationship between the 'why' and the 'how' of mathematics.*
- *Facts and methods are easier to remember and use, and can be reconstructed when forgotten because they are learned with understanding and are connected.*

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Conceptual Understanding (VCAA, 2019)

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- Connect related ideas
- Represent concepts in different ways
- Identify commonalities and differences between aspects of content
- Describe their thinking mathematically
- Interpret mathematical information

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Understanding

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Questioning Prompts to use in the classroom:

1. What is the problem asking you to do?
2. What do you know about this (e.g. measurement)?
3. Knowing this, what could you do now?
4. Which part are you finding difficult?
5. Can you use materials to help you?
6. How can you check it?

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Problem Solving

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- Mathematical problem solving takes place when a learner tackles a task for which they do not have a suitable readily-available solution method (NCTM, 2000).

- Promote a Growth Mindset to help reduce fears of Problem Solving.

It is better to work on one problem in many different ways, rather than working on many problems in the same way'.

George Polya

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Problem Solving (VCAA, 2019)

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- Use mathematics to represent unfamiliar or meaningful situations
- Design investigations and plan their approaches
- Apply their existing strategies to seek solutions
- Verify that their answers are reasonable

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Why is Problem Solving Important?

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- Problem solving is crucial to the use and application of mathematics in the world beyond school (Hodgen & Marks, 2013).
- Solving the specific problem must not become an end in itself, if the goal is to learn about how to solve future (as yet unknown) problems.
- The only true way of developing good problem solving skills is .... **TO SOLVE LOTS OF GOOD PROBLEMS.**

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Problem Solving

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Questioning Prompts to use in the classroom:

1. Can you show me what you are thinking?
2. What did you visualise when you were working on this problem?
3. What were you saying to yourself in your mind as you worked on this?
4. How does what you you've done help you in solving the problem?
5. How did you work it out? What strategy did you use?
6. What do you now know?

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Reasoning

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- Reasoning skills help students ***develop conceptual understanding*** and encourages them to ***be creative thinkers***. Three key reasoning actions are ***analysing***, ***generalising*** and ***justifying*** (reSolve 2019)
- It encourages students to make ***conjectures*** (asserting that something might be true at the stage when there isn't enough evidence to decide if it is true, then going through a process of checking this), ***formulate hypotheses***, ***articulating*** and ***formulating generalisations*** and ***explaining*** and ***convincing***. (Haylock 2014)
- They learn to ***make sense*** of mathematics, rather than believing that mathematics is a set of arbitrary rules and formulas. It helps students ***communicate*** their ideas and ***use mathematical language*** (Reys et al. 2017)

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Reasoning (VCAA, 2019)

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- Explain their thinking
- Deduce and justify strategies used and conclusions reached
- Adapt the known to the unknown
- Transfer learning from one context to another
- Prove that something is true or false
- Make inferences about data or the likelihood of events
- Compare and contrast related ideas and explain their choices

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Reasoning

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Questioning Prompts to use in the classroom:

1. Can you prove it to me?
2. Is there another way to work that out?
3. Would your method always work?
4. Can you work out a rule?
5. Does this rule always work? Prove it to me.
6. How could you explain this rule to someone else?
7. What do you notice about your results?

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Facilitating the Proficiencies

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Teddy Bear Rulers (Grades F-1)

Measurement (ACU)

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Question:

- I measured something in our room that is 5 teddies long. What could it be?
- Taking your 5 teddies around the room to measure the length of an item is a real hassle. What could we do to make this task easier?
- How could you use the teddies and the paper ribbon to make a teddy ruler?
- Go around the room now, looking for items that are 5 teddies in length
- Compare your ruler with a friend? How can you explain what is the same and what is different? Why do you think they are different.

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Facilitating the Proficiencies

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What proficiencies are being addressed?

What questions could you ask to facilitate the enactment of the proficiencies?

- Jot down some of your ideas.
- Discuss this with the people at your table.

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Understanding Measurement Ideas

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A important question to ask:

1. *Why are some teddy rulers a different size to yours?*

- The teddies were positioned differently, therefore not using an identical unit
- Some peoples' teddies were overlapping or had gaps
- Some people began their rulers at 1 instead of zero

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Teddy Bear Rulers – Key Concepts

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- **Iteration** – teddies need to be used over and over to measure something.
- **Identical units** – teddies must be the same size and used (oriented) in the same way.
- **Tiling** – there must be no gaps or overlaps when using the teddies to measure the length
- **Number and unit** – for every measurement there is a single number that represents that measurement.
- **Zero starting point** – when measuring you need to start at zero, with one representing the end of the first unit.

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Pattern Block Fractions (ACU)

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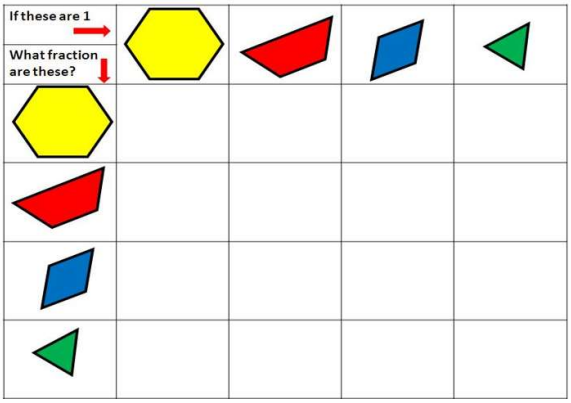
- Using your blocks and template find the value of each piece.
- Note that the value of a whole changes.



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Facilitating the Proficiencies

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Understanding Fraction Ideas

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A important question to ask:

1. What can you tell me about the whole?

- Create a design where the hexagon is ¾ of the area. How many different designs can you make? Create them and then draw and label them on paper Can you create another design where one-third is represented in different way?
- What about a third way?
- Create a design using 2 different pieces. What is the value of each piece? If we call your whole design '1', what is the value of each of the other pieces? Now convince the person beside you that you are correct.
- What do you notice about the relationship between the different pieces?
- What if the whole was represented by two hexagons combined, what would the other shapes now represent?
- Make your own whole and find the fractional parts of the other shapes.

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Key Concepts

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- **Meaning of symbols and the role of the numerator and denominator**. The denominator tells us the size or the name of the parts. The numerator is the numbers of parts of that name or size.
- **Number triad**: Words, symbol, concrete and visual
- **Equivalence:** This is am important concept which helps students operate with, order and compare fractions. Using concrete materials helps students make sense of the algorithm.
- **Mixed and improper fractions:** Using models helps students naturally understand what these numbers represent. You use a method of counting fractional parts and using correct fractional language.
- **Partitioning**: making equal shares by separating the whole into equal parts. They learn to describe these parts using fractional language. Using key language and fractional names helps students understand the magnitude of fractions.
- **Part-whole construct and region models.** Here the region changes and represents the whole unit. The parts are congruent. It is important to use various shapes so that students do not only associate fractions with pies or rectangles (Reys et al. 2019).

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Summary

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- The proficiencies are interwoven into the descriptors of each numeracy focus.
- They are explored through the use of language and open-ended and challenging tasks.
- Carefully planned questions can help facilitate the enactment of the proficiencies.

- Something to reflect on: In what ways are you supporting, planning, teaching and assessing the four proficiencies to develop your students' numeracy?
- Challenge yourself by focusing on one or more of the proficiencies as part of your own or your educational settings' improvement journey.

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Challenging Task Resources:

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ReSolve: Mathematics by Inquiry (Classroom Resources)

- https://www.resolve.edu.au/our-resources/classroom-resources

nrich Resources

- nrich.maths.org/roadshow

Maths300

- https://maths300.com/

Maths Assessment for Learning

- Rich Tasks and Work Samples, *ACU National* , 2006

- Sullivan, P. (2018). **Challenging *Mathematical Tasks. Unlocking the potential of all students.*** Oxford University Press, South Melbourne, Victoria

- Sullivan, P., Lilburn, P. (2017). ***Open – Ended Math Activities. Using good questions to enhance learning in Mathematics.*** Oxford University Press, South Melbourne, Victoria

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Other Useful Resources:

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Birth to Level 10 Numeracy Guide

- http://numeracyguidedet.global2.vic.edu.au/

Mathematics Curriculum Companion

- http://fuse.education.vic.gov.au/Resource/ByPin?Pin=N7NDQC&SearchScope=All

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