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KEYNOTE PRESENTATION

**Greg Oates**
**UTAS**

*Intuition and cognitive conflict: From 'Uh-Oh' to 'Ah-Ah'*

# “Where the Bloody Hell Are You?”


Cataract Gorge

Launceston

Little Dutch

BIG Dutch

Auckland

# OVERVIEW

My Story - Some anecdotes

Intuition, Misconceptions & Errors: Making Conjectures & Safe Classrooms

Learning from History

Real-life (?)  Examples

Possible Sources of Misleading Intuitions

Summary

# My Story: "How did I get Here?"

- Secondary School: (NZ)
  - A student in a Year 9 accelerated class who burst into tears: "I never get these right"
  - A parent complaint to the Principal after I failed to solve a year 12 *Trig Identities* problem in the moment
- University of Auckland Bridging Mathematics course in mid-1990's: Early lectures aimed to prompt students to think mathematically and dispel mathematical myths, e.g.
  - ODD Number + ODD Number = what?
  - <u>Convince</u> **yourself**
  - <u>Convince</u> **your neighbour**
  - How do you <u>know</u> **it's always right**?
  - Can you *Show* or *Prove* it?

# My Story Continued

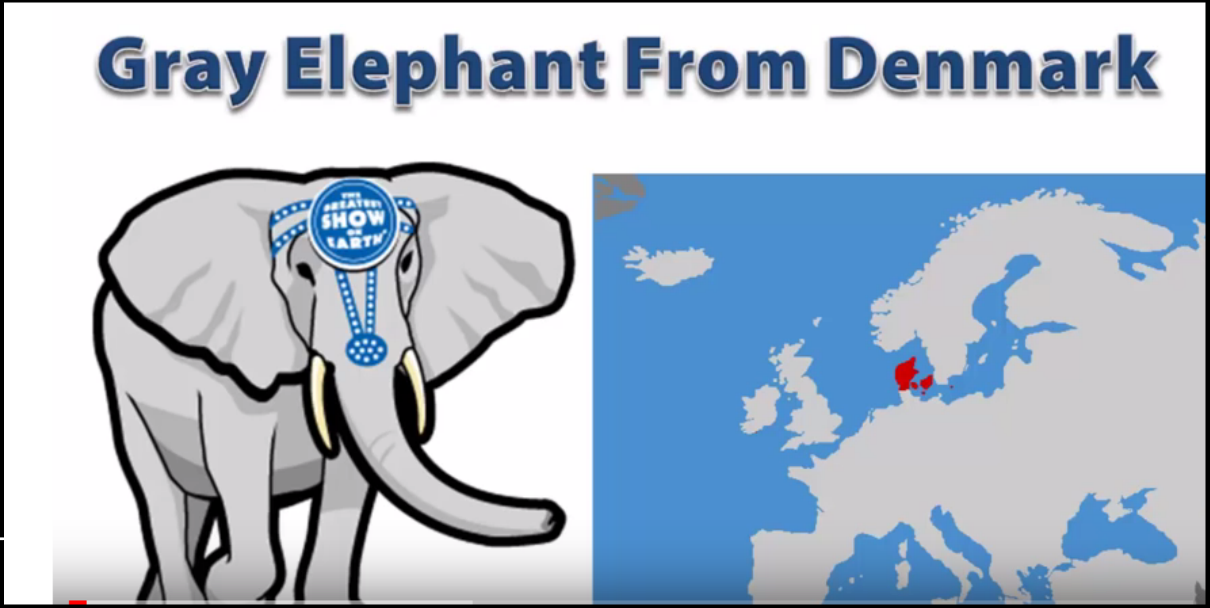
- (UOA continued:)
  - *"Railway Problem"*
    ('Intuitively misconceived solutions to problems, Avital & Barbeau, 1991). Student reactions were sometimes extreme. Started me wondering about intuition….. *"Just show us the formula"*
  - Misconceptions:
    e.g. Cannot cross an asymptote or reach a limit?
- UTAS BEd (Primary) student orientation-2017:

  "Think of a Number Mind Reader" ice-breaker –
  many students too scared to say what they think, or make a conjecture, some not even tried as soon as I mentioned "Maths".

# Icebreaker: A Math-Magic Trick

- Think of a number from one to fifty (keep it smallish if you are worried about adding/subtracting/multiplying etc, but you can always use your phone ☺);
- Multiply your number by 9
- Now add the digits of your answer together if more than one, e.g. if your answer was 63, then 6 + 3 = 9;
- Now subtract "5" from your answer
- Now whatever number you have, choose the letter of the alphabet, starting with "A" =1, through to "Z" = 26, e.g. if your answer was 13, the  your letter would be "M".
- Think of a country that starts with your letter? Eg for "M", might be Mongolia
- Think of an animal that starts with the second letter of your country, e.g. Orangutan
- What colour is your animal?
- Tell your neighbour

Who is thinking of a ……. Why?



# Intuition, Misconceptions & Errors Making Conjectures

- **Myths about Mathematics:**
  Many students quickly learn (believe) that Maths is about getting the correct answer, quickly, and without making mistakes (being "*good at maths*");

- **Intuition is often either not considered** (eg "what do you think"), **or under-valued** (what is the difference in value between an intuition and a guess?)

- **We often don't differentiate between "errors" and "misconceptions", and even within "errors"** e.g. a numerical error (calculation); procedural error; modelling error; "error in thinking about the problem"

# "Safe Classrooms"

- Many teachers and mathematics education leaders already adopt approaches that involve students tackling challenging problems which may lead to ***Cognitive Dissonance*** or ***Cognitive Conflict**** and promote the value of ***Productive Struggle****,* e.g. Charles Lovitt & Maths300; Malcolm Swan, Peter Sullivan, reSolve – primary focus on developing student reasoning, inquiry, curiosity…

- My main premise here has a slightly different focus:

**Safe Classrooms –**
**where it's ok to take a risk, to be "wrong"**

We should start early with students deliberately & explicitly **experiencing problems that provoke cognitive conflict, lead to errors and challenge misconceptions,** as part of legitimate mathematical enquiry.

# Mathematics Metaphors

- *Mathematics is a "frypan, I use it often": Utilising metaphors in research* (ICME 11, Miller-Reilly, 2010)

  Charles: What does using or doing maths feel like?

  "*skiing on blue ice, with no edges, blindfolded*"

  Mathematics is most like:

  "*a hyena, a scavenger-predator, rearing its head when I least want it, always succeeding in removing my self-confidence and sense of self. I hate it*".

- A metaphor for "*safe classrooms*" is more like an off-road driving enthusiast: The challenge is to get through a circuit, but there is an expectation you will get stuck- the challenge is to get "unstuck" (Wiggins, Delta keynote November 2019)

# Mathematics Metaphors:

# Stuck → Unstuck

# Example 1:
# Algebra Patterns for many year levels

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 4 | 7 | 10 |

- How many in the sequence do we need to see before we can be confident we know the pattern?
- Similarly for functions: How many points do we need to be confident we know the type of function, e.g. *linear? Quadratic? Cubic? …*

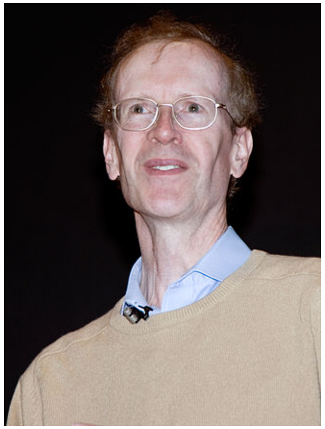
# Learning from History

Descartes, in his *Rules for the Mind, Rule IX*, describes our valuing of deduction over intuition as "*a mental disorder which prizes the darkness higher than the light.*"

*"Intuitive seems the opposite of rigorous, or logical, or formal...Logic seems to be clearly understood. Intuition appears elusive… Intuition and logic seem to be in opposition only because of the very limited powers of the human mind"*

(Otte, 1990)

Many problems in history have taken a long time to solve, e.g. *Fermat's Last Theorem*; The *Four Colour Theorem*



Andrew Wiles Proof: 1993-1994

# Conjectures form the backbone of mathematical thinking.

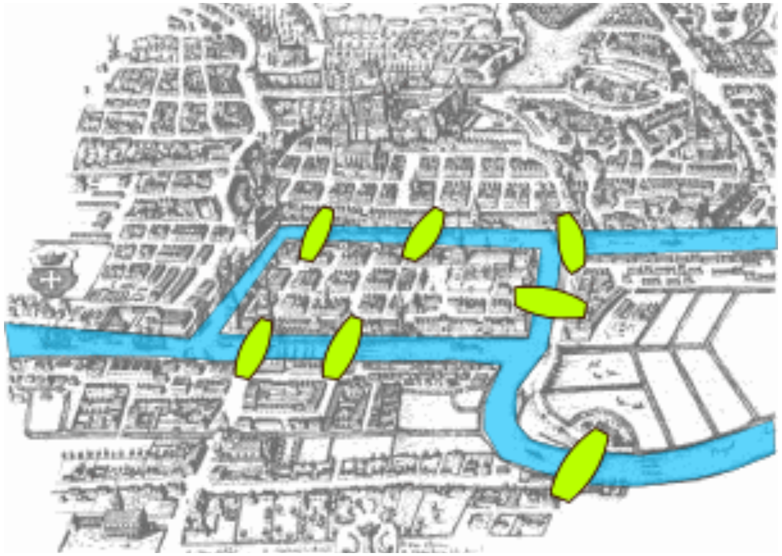
(Mason, Burton & Stacey, 1982 p.72)

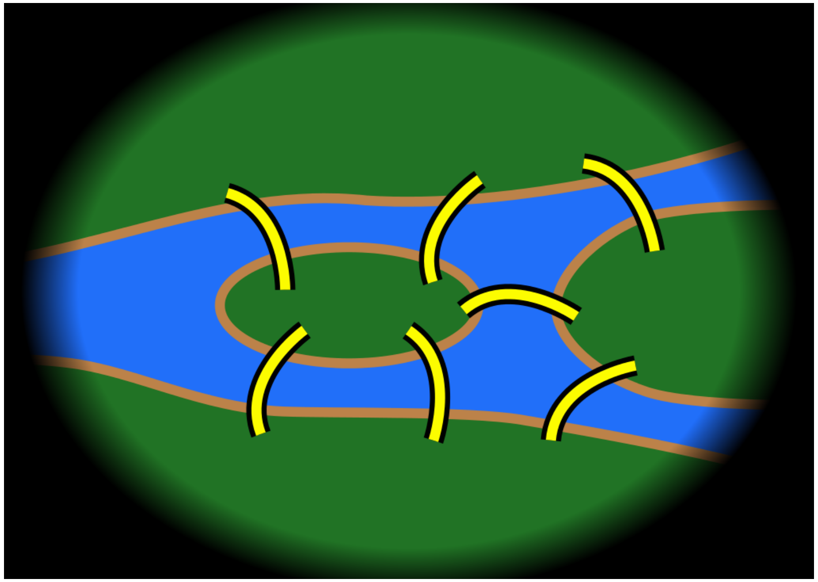
- Some property is thought to be true.
- A conjecture about it often begins as a vague feeling lurking in the darkness at the back of the mind.
- Gradually it is dragged forward by attempting to state it as clearly as possible, and exposed to the strong light of investigation.
- If it is found to be false it is either modified or abandoned.
- If it can be convincingly justified then it takes its place in the series of conjectures and justifications that will eventually make up the resolution.

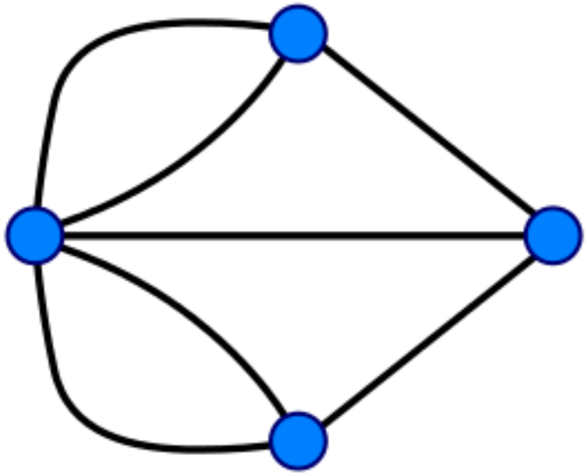
# Example 2:
# Seven Bridges of Königsberg

The Seven Bridges of Königsberg is a historically notable problem in mathematics.

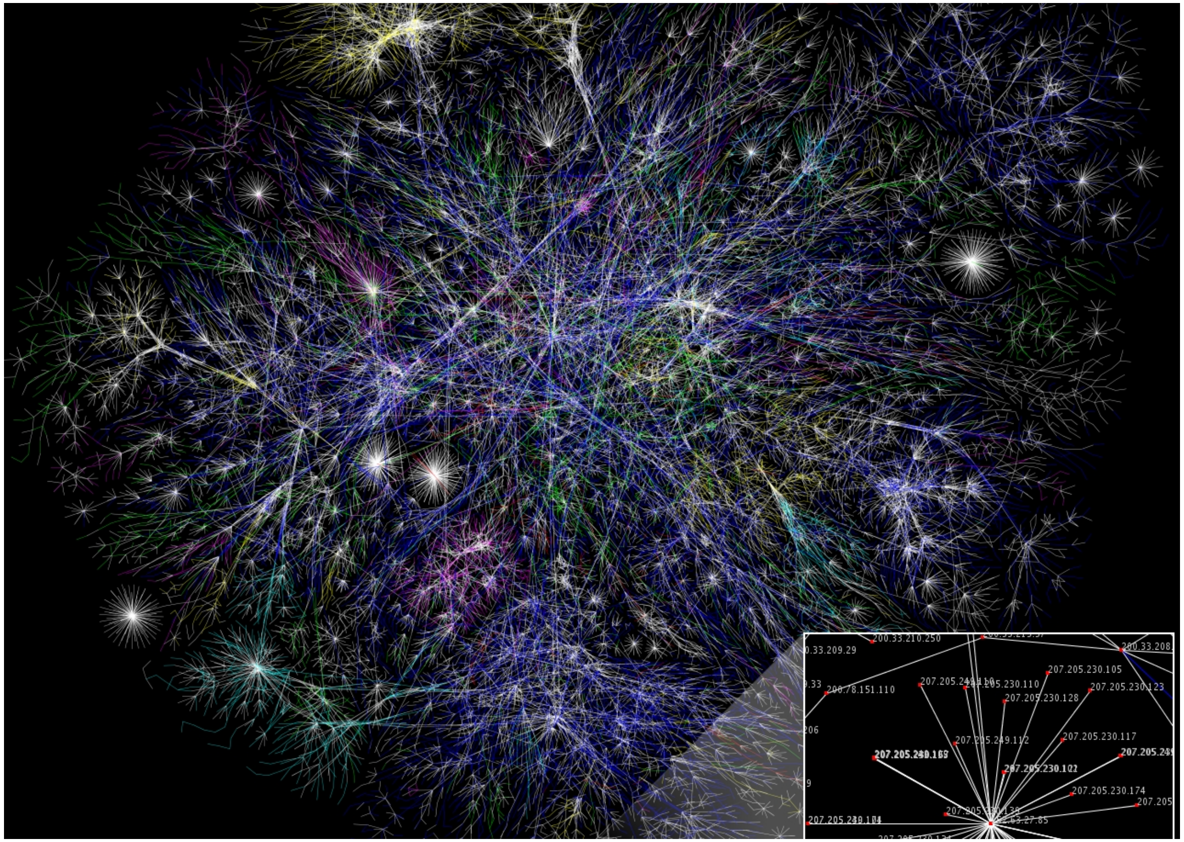
The problem was to devise a walk through the city that would cross each of those bridges once and only once.







# Network Theory & Search Engines



Partial map of the Internet based on the January 15, 2005 data (opte.org)

Each line is drawn between 2 nodes, representing IP addresses. The length of the lines indicate the delay between those 2 nodes.

# Example 3 (Algebra): Moser's Circle Problem

Try this out for yourself:

– Draw a circle, put one dot on the circumference.
How many regions are there?

– Put a second dot anywhere else on its circumference & join the two dots with a straight lines.
How many regions are there now? (try and draw as straight as possible)

| | | |
|---|---|---|
| 1 | 2 | Number of Dots |
| 1 | 2 | Number of Regions |

# Moser's Circle Problem

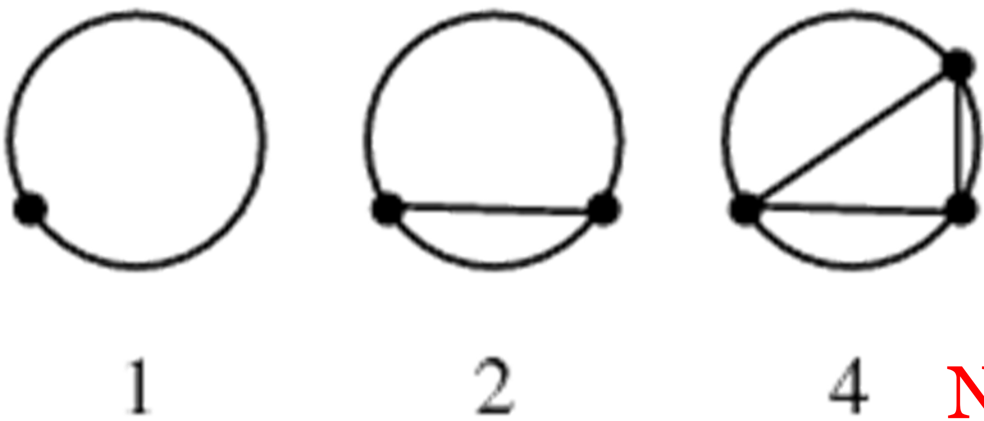
– Draw a third dot and join all dots up to every other dot (straight lines). How many regions now?

– **Make a prediction** for four dots, check your answer…

– **Make a conjecture** for any number of dots…

– **Try it out** for 5 dots, 6 dots (can you do 7?)…

– What do you notice?



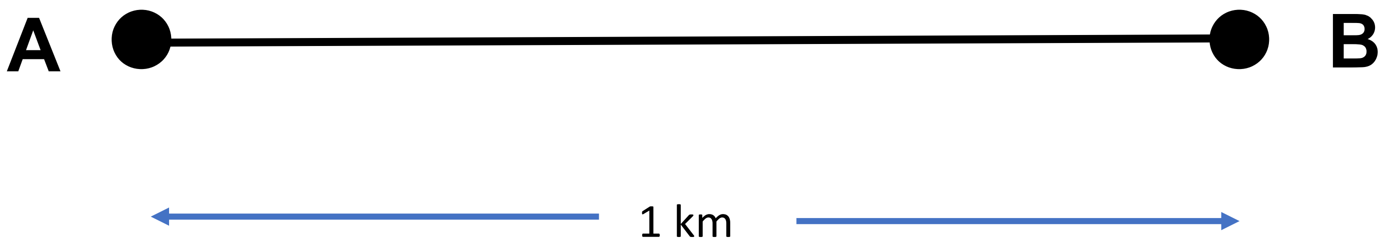

Number of Regions

| Dots ($D$) | Regions ($R$) | Formula? |
|---|---|---|
| 1 | 1 | ? |
| 2 | 2 | $R = D$? |
| 3 | 4 | $R = 2^{D-1}$? |
| 4 | 8 | $R = 2^{D-1}$ |
| 5 | 16 | $R = 2^{D-1}$ |
| 6 | 31 (32) | **$2^5 = 32$?** |
| 7 | 57 (64) | **$2^6 = 64$?** |

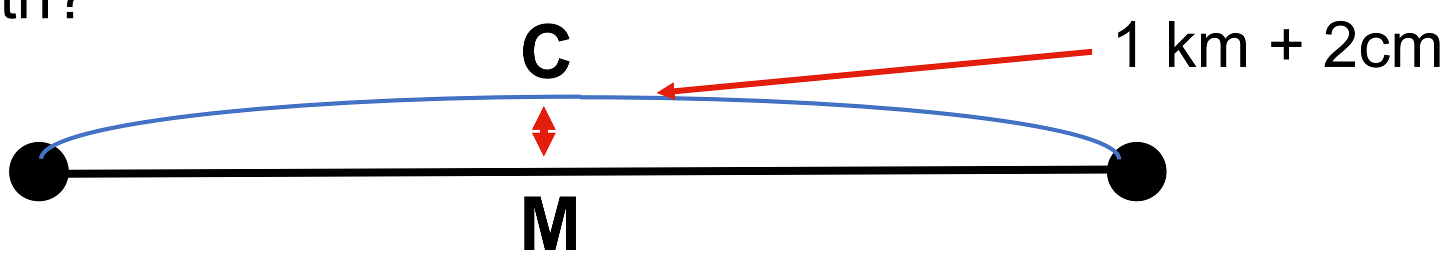
g(n)= (1/24) [n^4-6n^3+23n^2-18n+24]

# Example 4: The Railway Problem

A railway track 1 km long is fixed at points **A** & **B** as shown below:

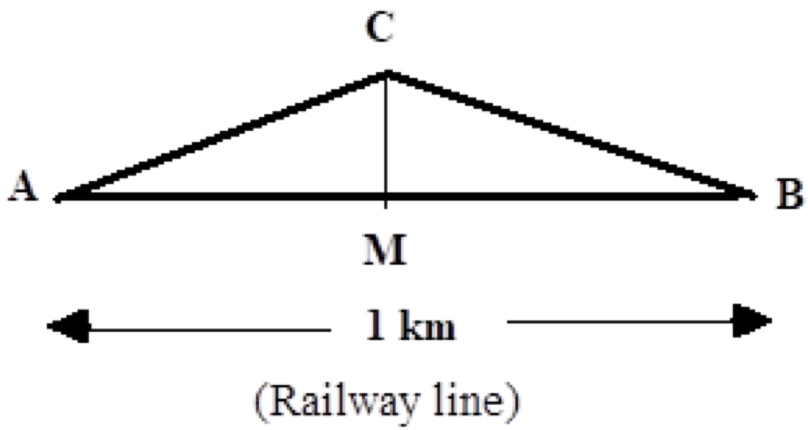



The railway line is heated by the sun, and expands by 2 cm, causing it to buckle upwards. If the track buckled uniformly about its midpoint **M**, would it be high enough at point **C** for (i) a mouse, (ii) a cat, (iii) a small child, or (iv) a horse; to pass underneath?



# Example 4: The Railway Problem

- PROOF: How might we prove it?
- ASSUMPTIONS: What are our assumptions?
- MODEL: How might we model it?




Model using Pythagoras: AM = 500m; AC = 500.01m

$$500^2 + MC^2 = 500.01^2 \Rightarrow MC = \sqrt{(500.01^2 - 500^2)}$$
$$= 3.16\text{m}$$

# Possible Sources of Misleading Intuitions

- My experience:
  - Lack of confidence
  - Maths Anxiety – repeated ‘failure’ (Charles), e.g. think they are wrong although the Konisberg Bridge has no solution?

- Avital & Barbeau (1991: p. 2)

1. *Lack of analysis*
2. *Unbalanced perception*
3. *Improper analogy*
4. *Improper generalisation*
5. *The solver may impute a false symmetry to the situation or misinterpret what symmetry is present.*

# Further Examples

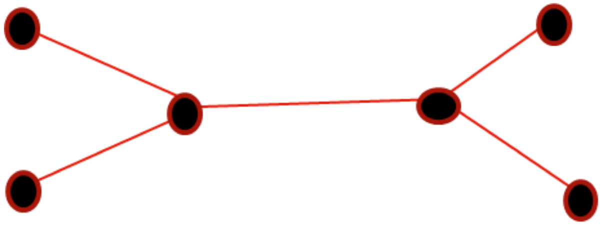

- Goodwill Hunting: Graph Theory.

  "Draw all the homeomorphically irreducible trees of size 10"

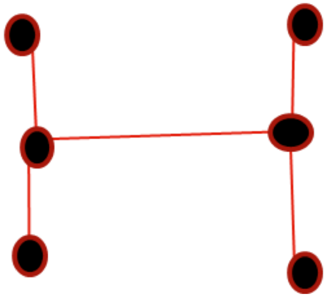
- Helen Chick *Mind-Reading & Coin Toss Sequence*

- Derren Brown: Ten heads in a Row"

- Monte Hall Problem



Homeomorphic Trees



# Summary

- **Start Early:**
  it's never too late, we just need a softer approach
- **Start Big**
- **Link to History**
- **Provoke and Celebrate Students' Misconceptions:**
  make these normal, so students feel safe safe to take risks!
- **Be Brave – Model it Yourself:**
  make conjectures which may or not be correct, but seem feasible. Make it plain the teacher is not necessarily "the expert who knows ALL the answers", but instead, as "an authentic mathematician who is looking for the answers".

…

# Final Comments

- We need to continually monitor students' take-aways from such approaches – *the teacher as facilitator.*
- Not in conflict with mastery-mastery comes from confidence in your solutions.

*…direct experiences which lead to the making of conjectures are important, but the recognition that errors and misconceptions do occur should make both teachers and students wary. Reflection, review, and revision must be part of the process, so that mathematical learning is seen as a dialectic between intuition and formalism with each informing the other.*

(Avital & Barbeau, 1991, p. 7)

# Thank you
# Nga Mihi

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