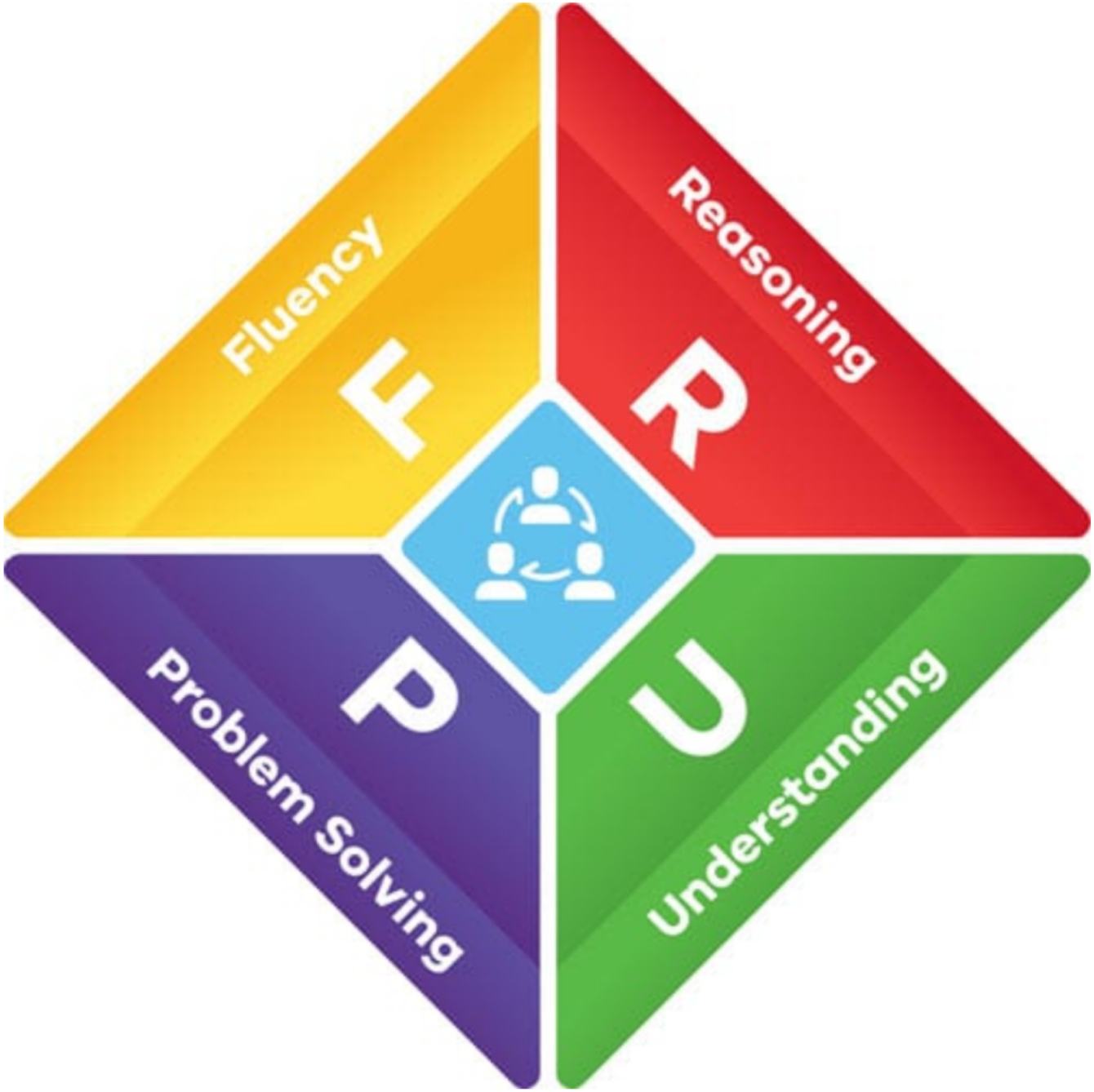
Big ideas: Connecting across the mathematical curriculum

Mike Askew
University of the Witwatersrand
MAV Conference
6 December 2019


Fluency
F
Reasoning
R
Problem Solving
P
Understanding
U

What is a Big Idea?

What is a Big Idea?

- Big Ideas are both mathematically and conceptually ‘big’.
- Mathematically - connect seemingly disparate ideas
- Conceptually - connect learning

What is a Big Idea?

- Has currency across all the years of primary schooling.
- Students get to revisit Big Ideas across the years.
- All students can be engaged in thinking about a Big Idea at different developmental levels.

Some Big Ideas

- Position on the number line
- Equivalence
- Pattern
- Place value
- Meanings and symbols
- Estimation
- Arithmetical reasoning
- Classification

Position on the number line

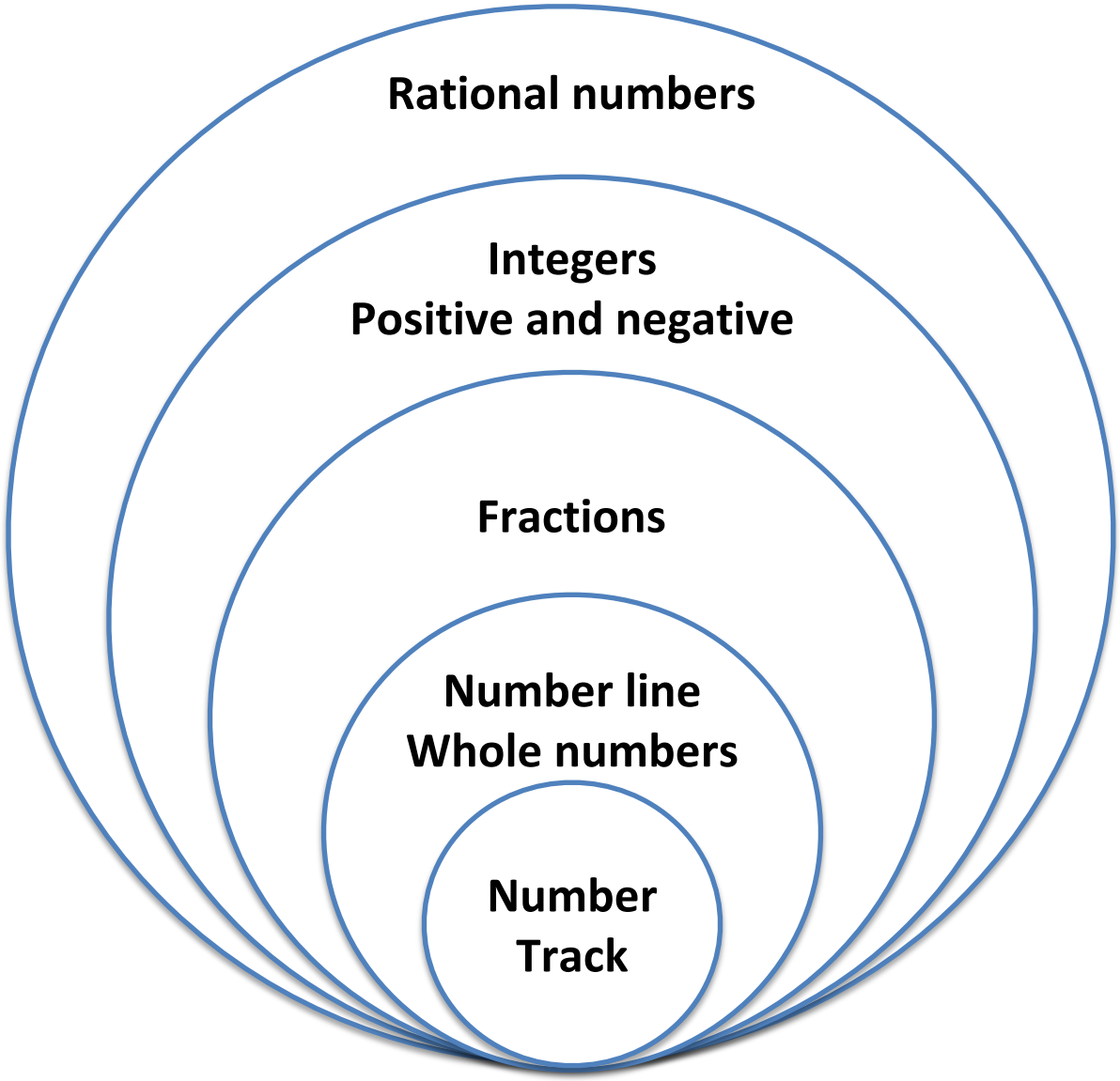
The (real) numbers have a unique position on the number line.

Position on the number line

- Putting numbers on a line links discrete and continuous quantities.
- Positioning numbers on the number line helps develop understanding of the number system.
- The number line helps learners connect different representations of numbers.
- Placing numbers at equal spaces on a number line is a key skill and marker of understanding.

Big Ideas

- Links two distinct types of quantity: quantities that are discrete (pebbles, learners, cows) and quantities that are continuous(sea water, height, milk).
- We can count discrete quantities but we have to measure continuous quantities.
- Mathematicians realising that discrete counting numbers could be placed on a continuous line brings together the separate domains of counting and measuring.


Rational numbers
Integers
Positive and negative
Fractions
Number line
Whole numbers
Number
Track

ROOTS OF/ROUTES TO FRACTIONS

Fractions arise from:

- Measuring
- Fair sharing

How much chocolate?

A bar of chocolate is cut up into three equal pieces and Joe eats two of the pieces.

How much chocolate does Joe get to eat?

Joe and two friends share two bars of chocolate equally.

How much chocolate does Joe get to eat?

Which is larger?

$\frac{29}{30}$ or $\frac{30}{31}$

Which is closer to 1?

$\frac{29}{30}$ or $\frac{30}{31}$

Hungry children

Four hungry children come home from school.

They can only find 2 slices of bread.

The toast the bread and share it equally.

Draw a picture to show how much toast each gets.

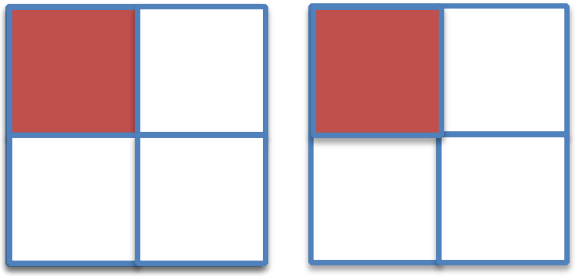
Hungry children

Four hungry children come home from school.

They can only find 2 slices of bread.

The toast the bread and share it equally.

Draw a picture to show how much toast each gets.



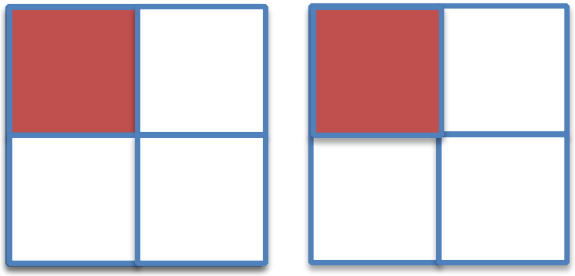
Hungry children

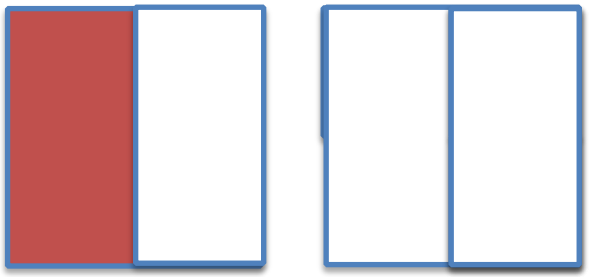
Four hungry children come home from school.

They can only find 2 slices of bread.

The toast the bread and share it equally.

Draw a picture to show how much toast each gets.





Big ideas

There are 2 jars: one contains 25 M&Ms, the other 75 M&Ms. The contents of both jars is poured into a third jar. How many M&Ms in that jar?

There are 2 jars: one contains 1 litre of water at 25°C, the other 1 litre at 75°C. The contents of both jars is poured into a third jar. What is the temperature of the water in that

Additive & Multiplicative Relations

Quantities can be related and operated on additively or multiplicatively and there are important differences between these.

Four rules of arithmetic:

Addition
Subtraction

Multiplication
Division

Four rules of arithmetic:

~~Addition~~
~~Subtraction~~

ADDITIVE RELATIONS

~~Multiplication~~
~~Division~~

MULTIPLICATIVE RELATIONS

How many numbers?

I have bag of 7 pears. I add 6 more pears to the bag. How many pears are in the bag?

$$7 + 6 = 13$$

I'm putting pears into bags of 7. How many pears do I need to fill 6 bags?

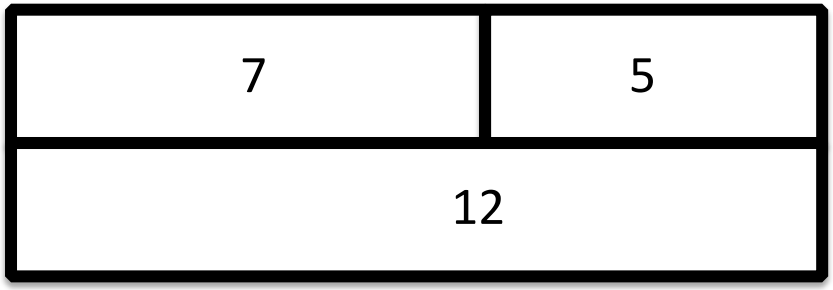
$$7 \times 6 = 42$$

Additive relations

Key big ideas/representations

- Part-part-whole relations – bar diagram
- One variable – number lines

ALL ADDITIVE PROBLEMS ARE PART-PART-WHOLE RELATIONS

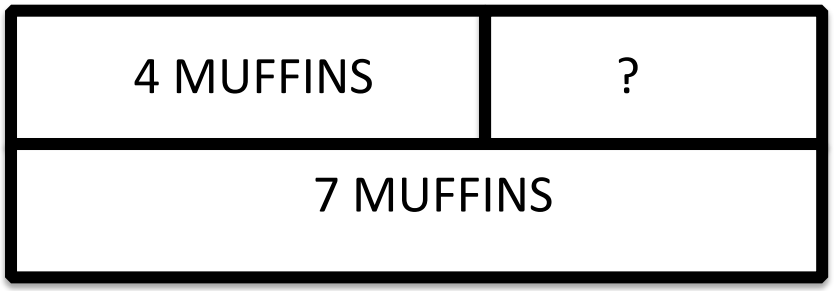


	END UNKNOWN
CHANGE	Mike has 4 muffins, he bought 3 more. How many does he have now?
COLLECTION	Mike has 4 blueberry muffins, and 3 chocolate. How many muffins does he have?
COMPARE	Mike has 4 muffins, Hamsa has 3 more than Mike. How many does Hamsa have?

Adapted from Carpenter
, Fennema, Franke, Levi,
& Empson, 1999

ALL ADDITIVE PROBLEMS ARE PART-PART-WHOLE

Mike has 4 muffins
Hamsa has 7 muffins.
How many more than Mike does Hamsa have?



4 + [ ] = 7
7 – [ ] = 4
[ ] + 4 = 7
7 – 4 = [ ]

Model of …

Model for …

Tool for thinking

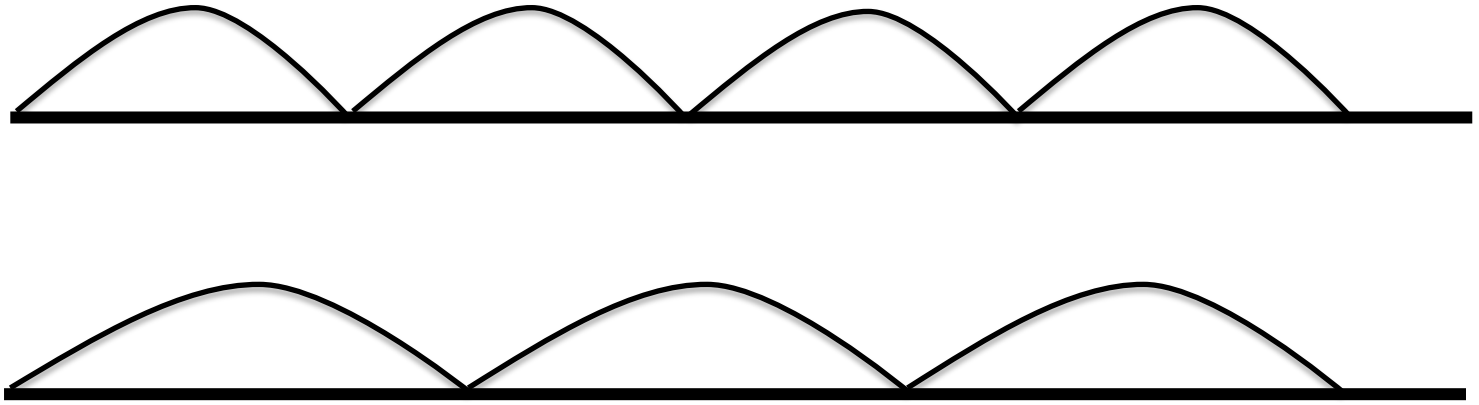
Multiplicative relations

Key big ideas/representations

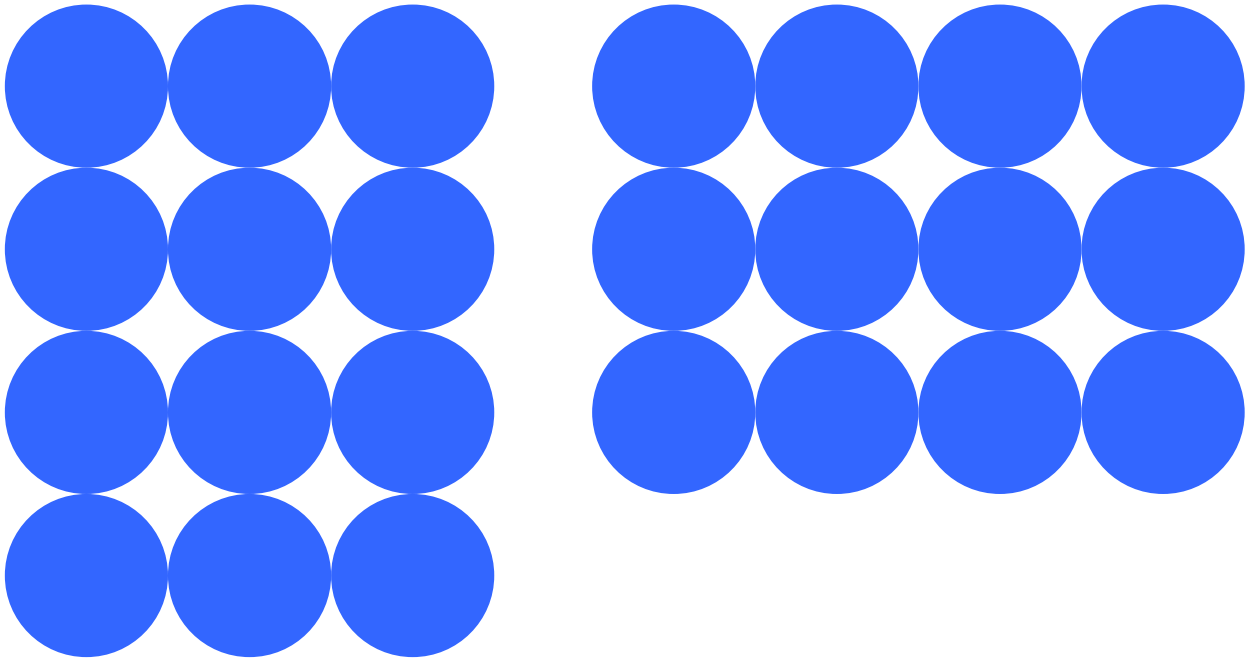
- Simple ratio and scalings

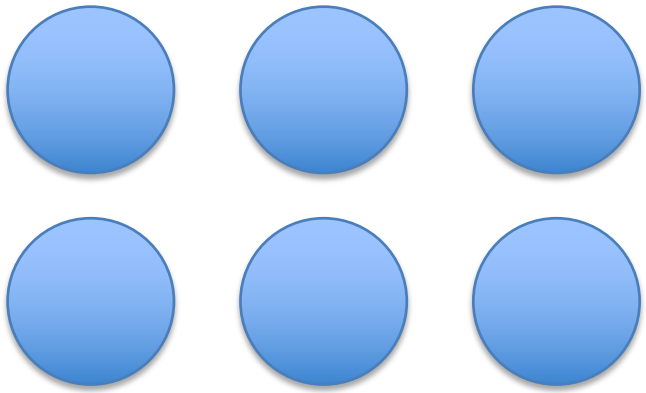
- Two variables – Arrays, double number lines, ratio tables

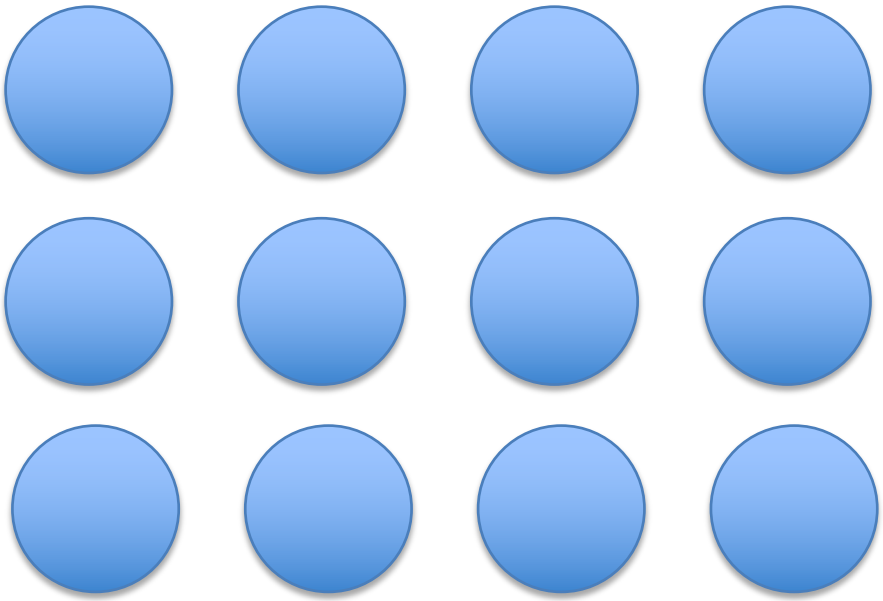
3 x 4 = 4 x 3

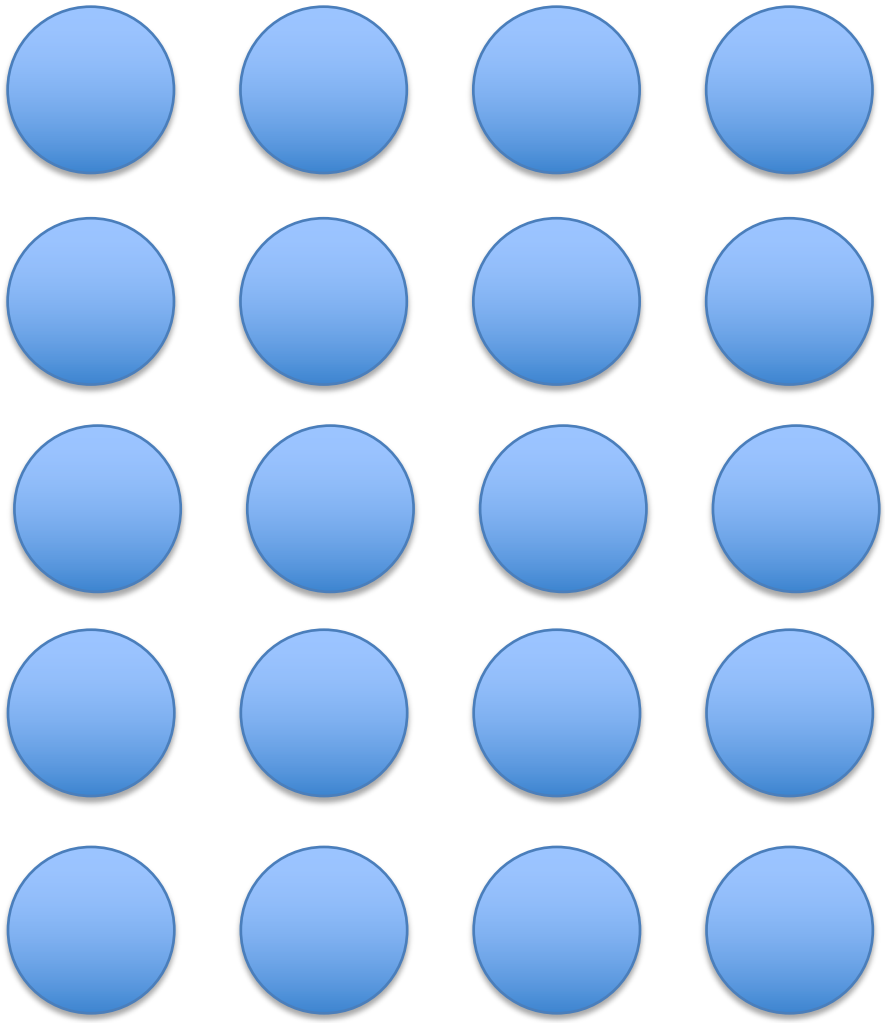


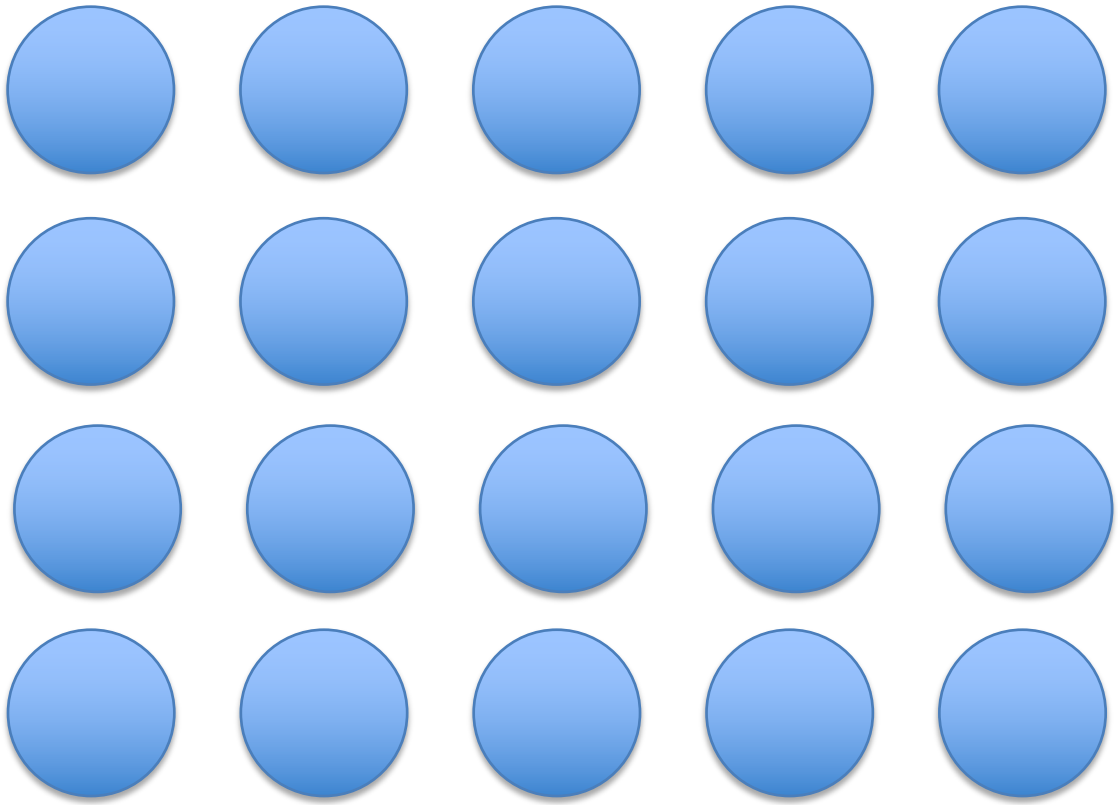
$$3 \times 4 = 4 \times 3$$

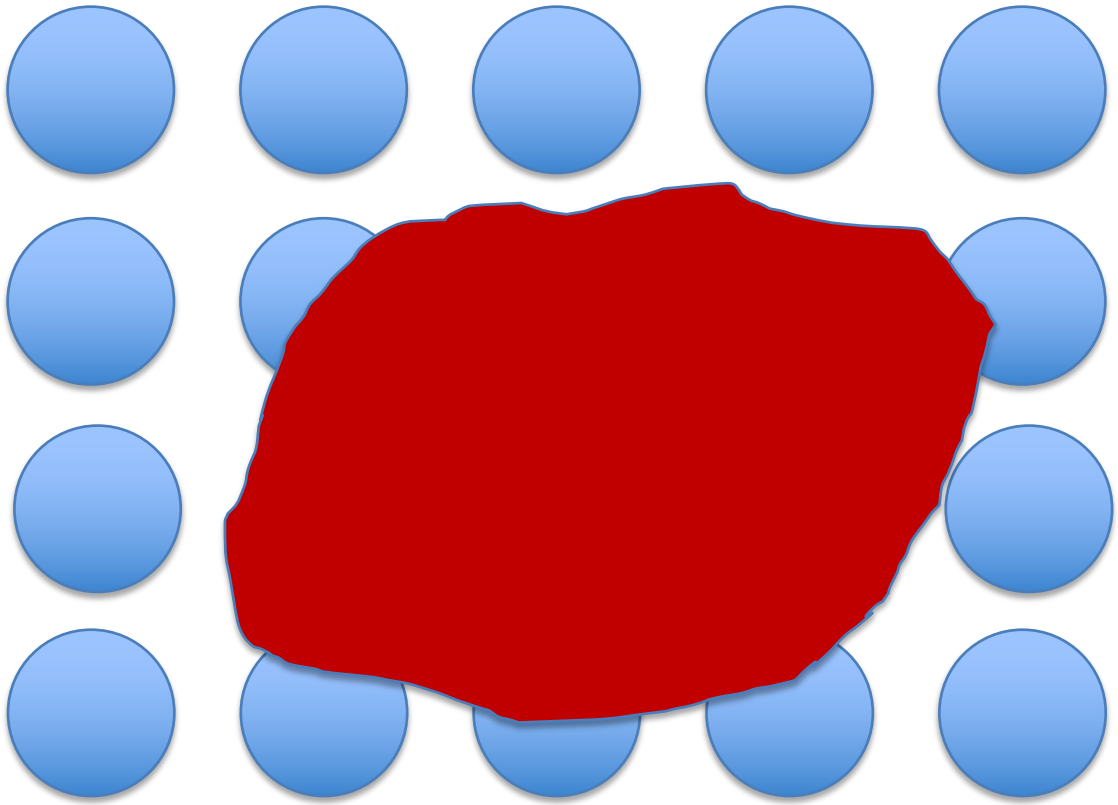


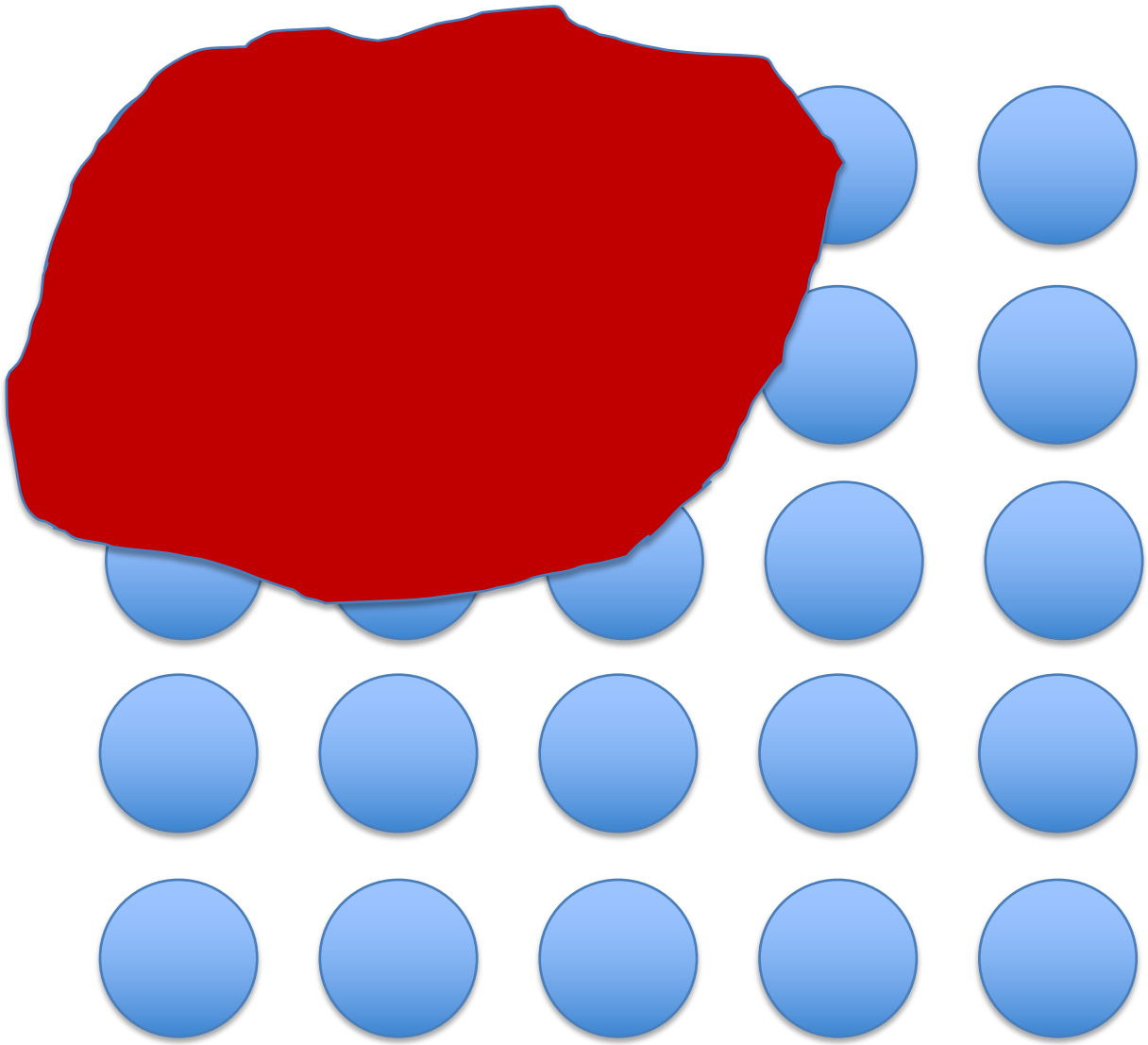








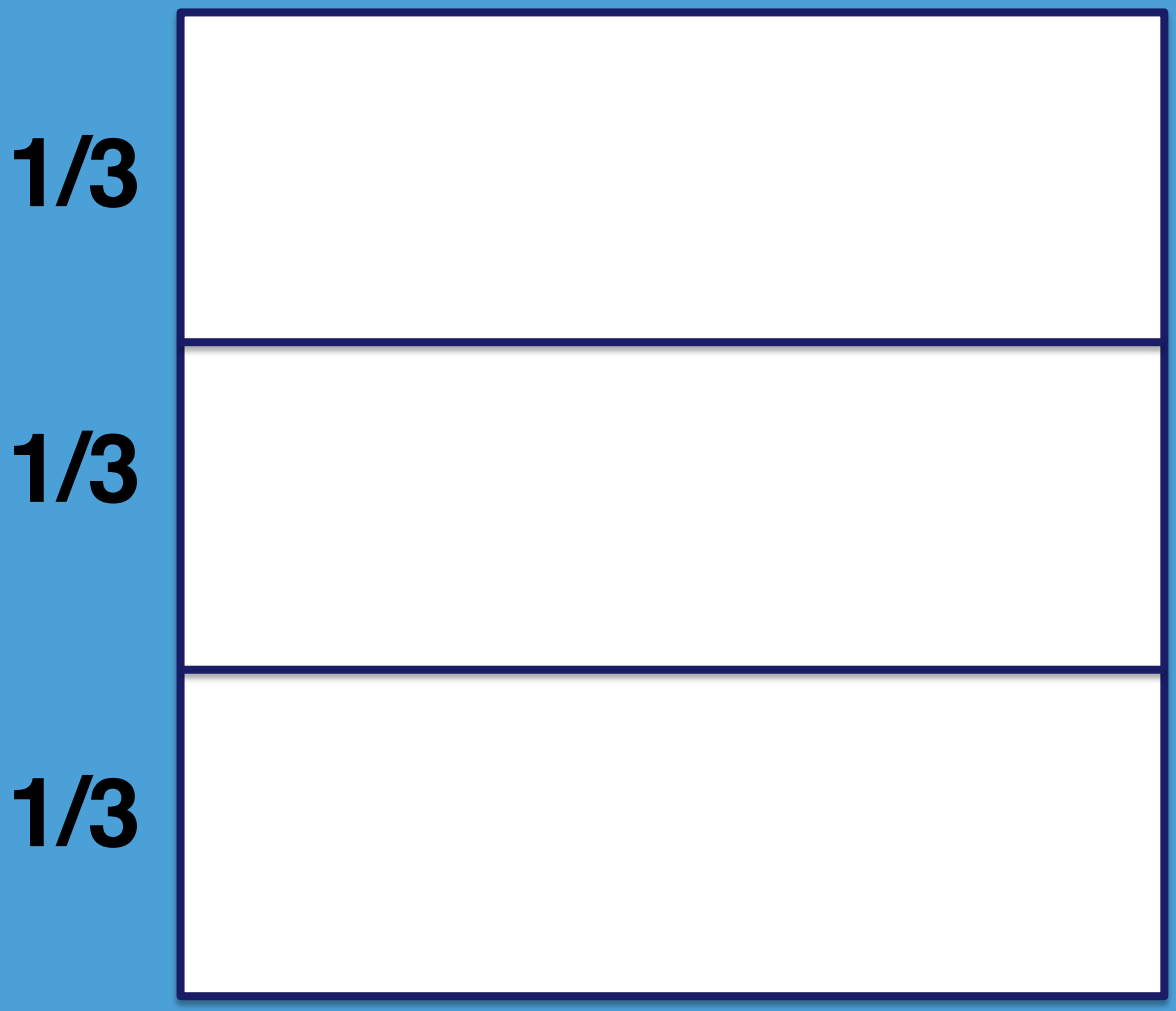


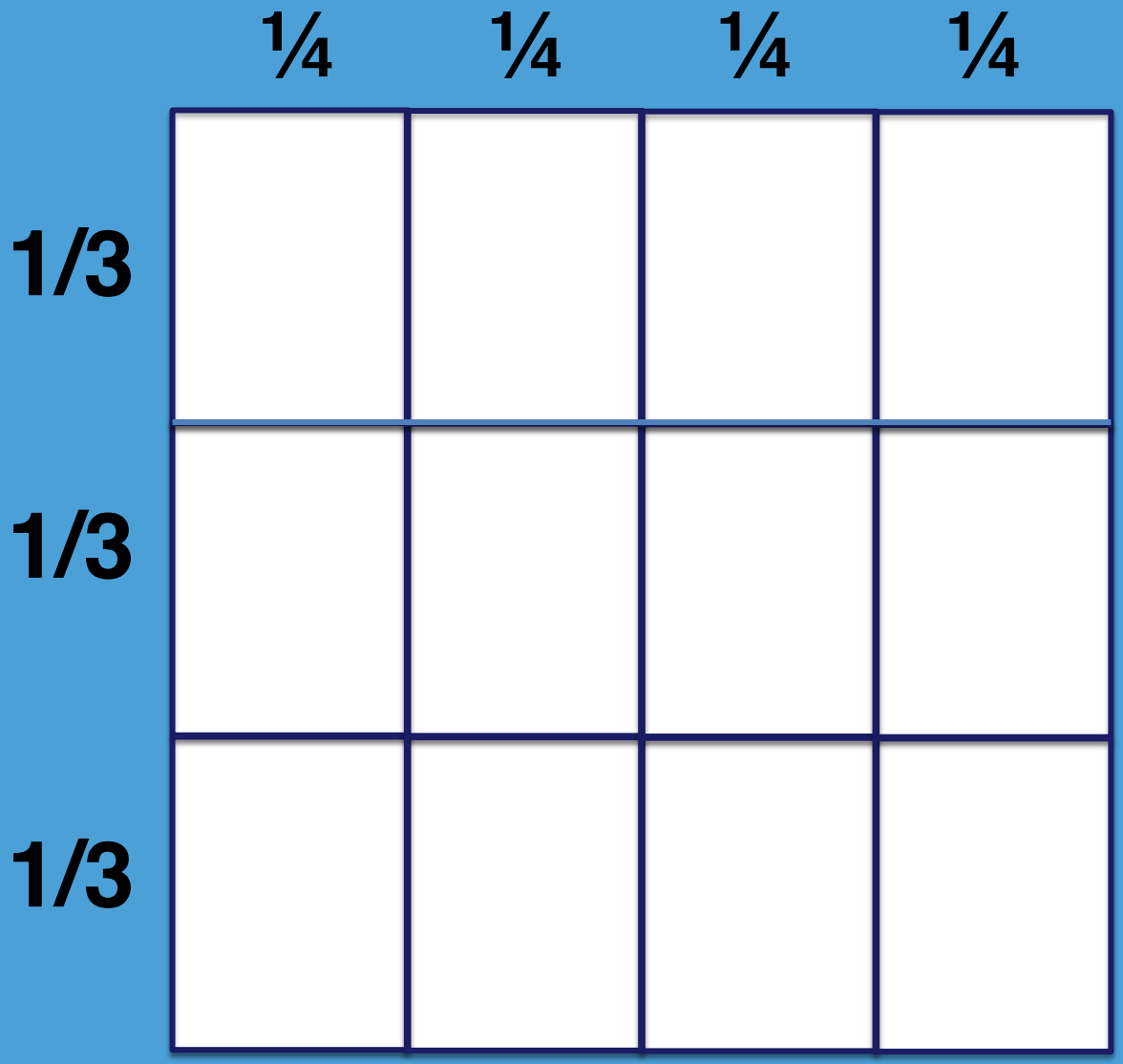


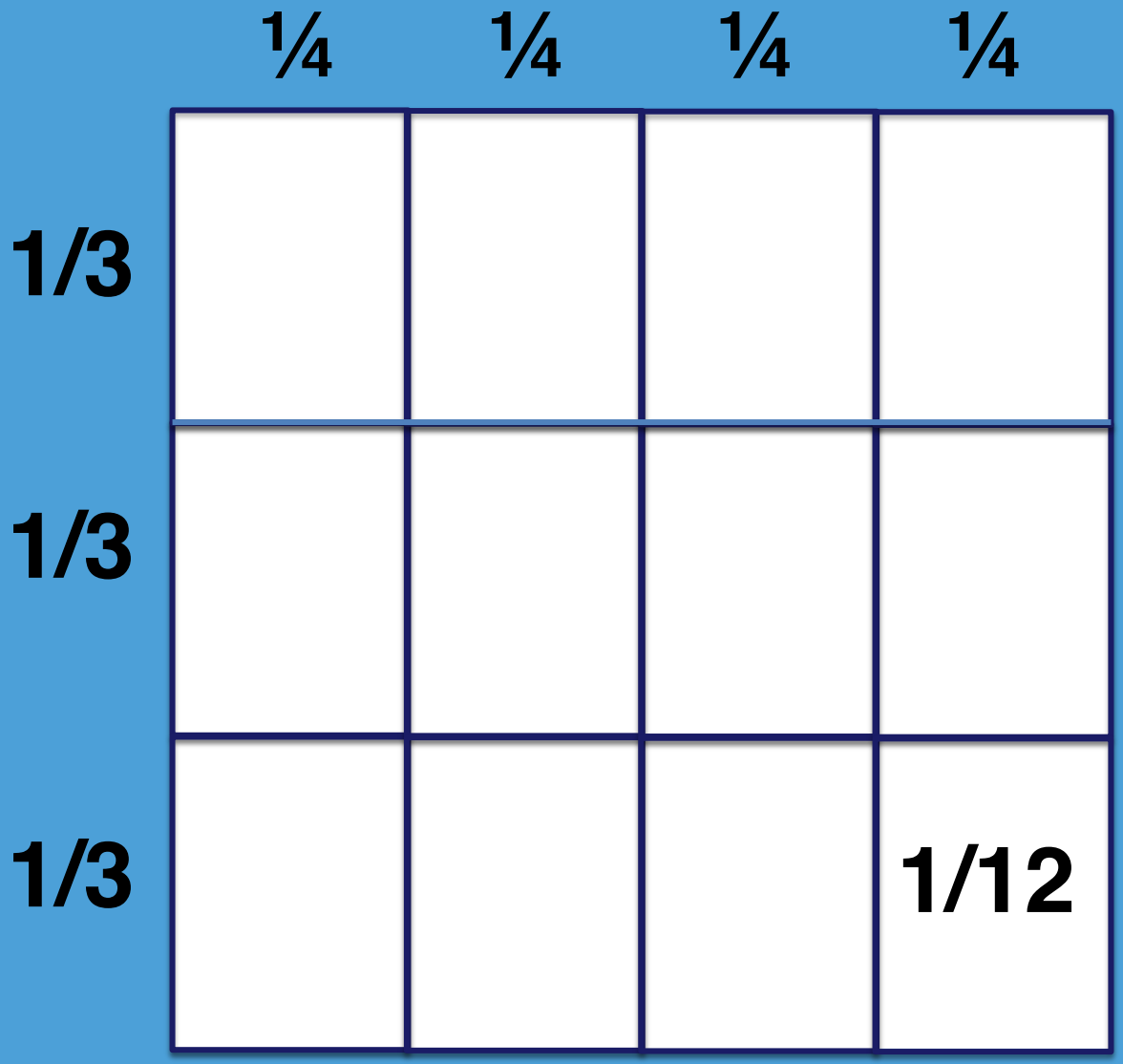
I want to varnish a painting that is 2/3 m by 3/4 m.

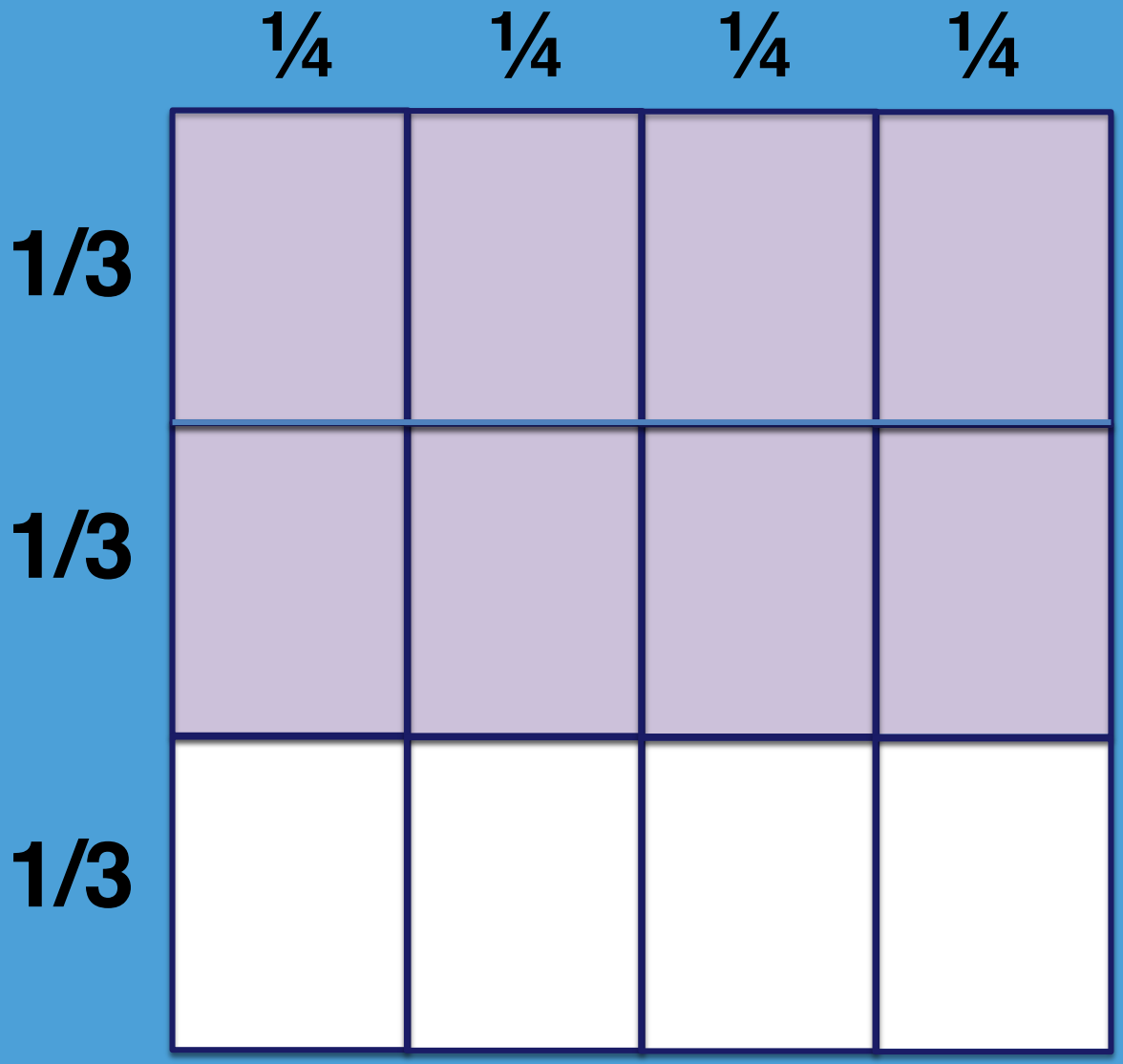
Tins of varnish come in three sizes, to cover ¼ m^2, ½ m^2 or 1 m^2.

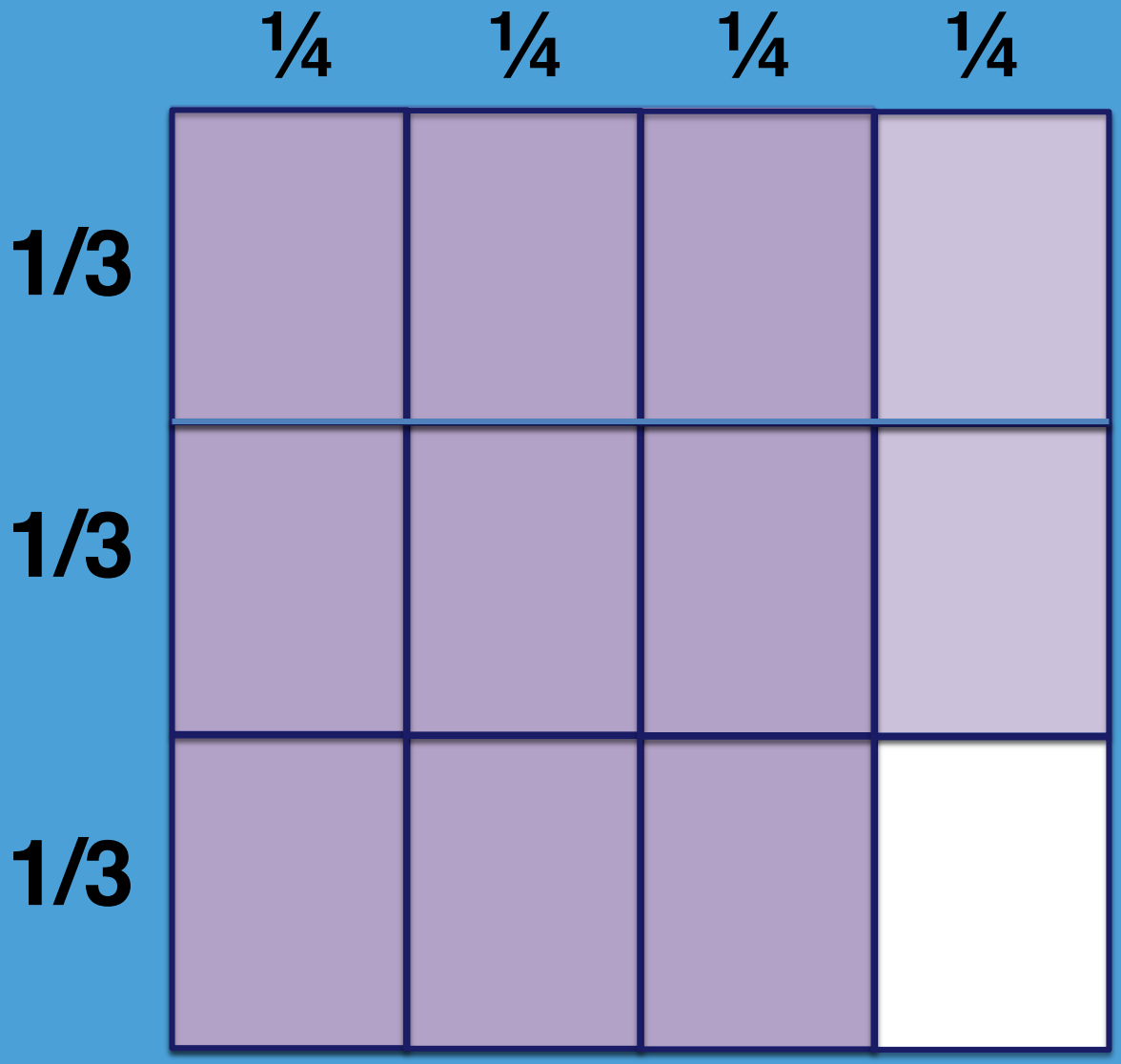
What is the smallest tin I could buy and not waste any varnish?

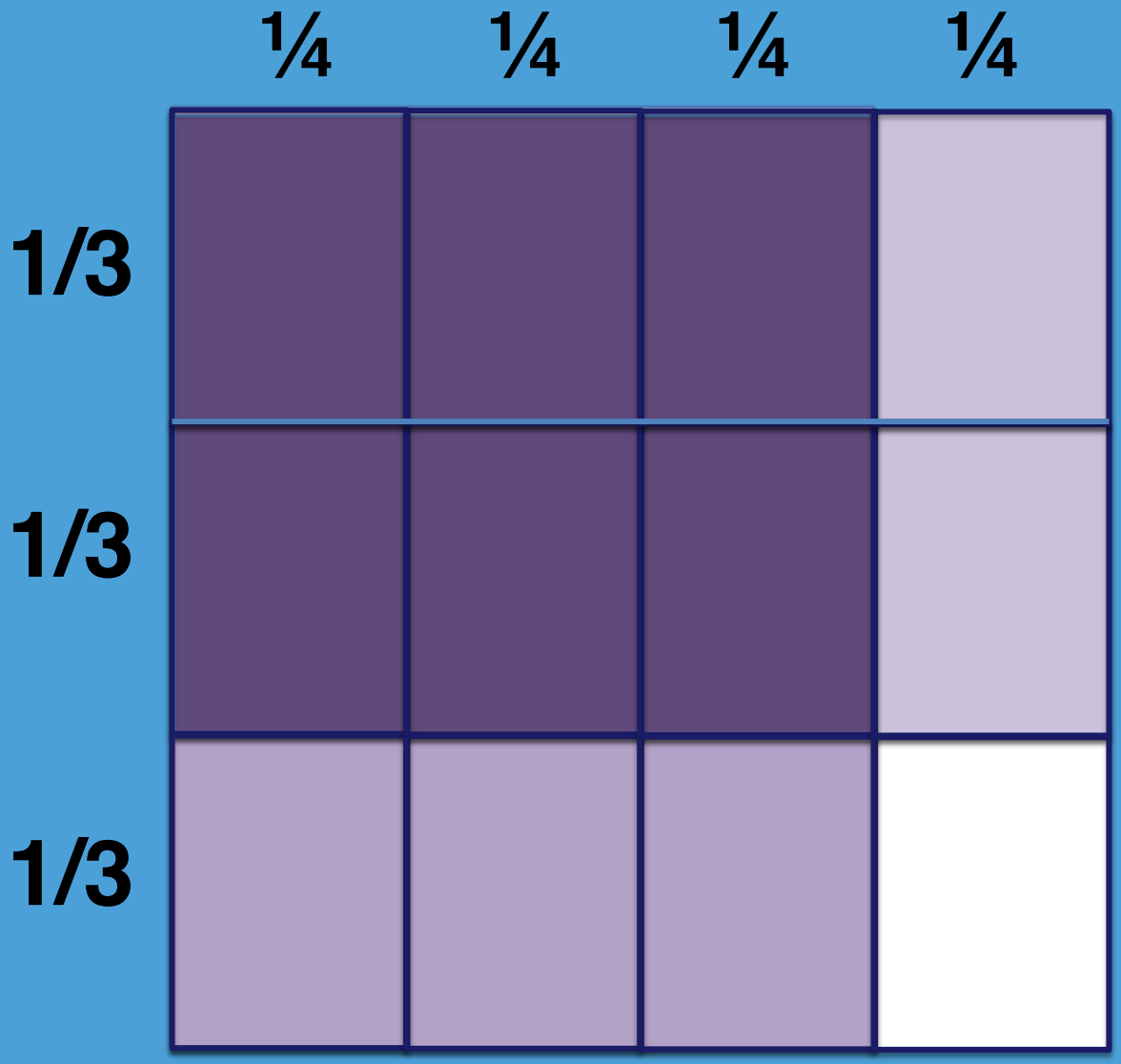

1/3
1/3
1/3

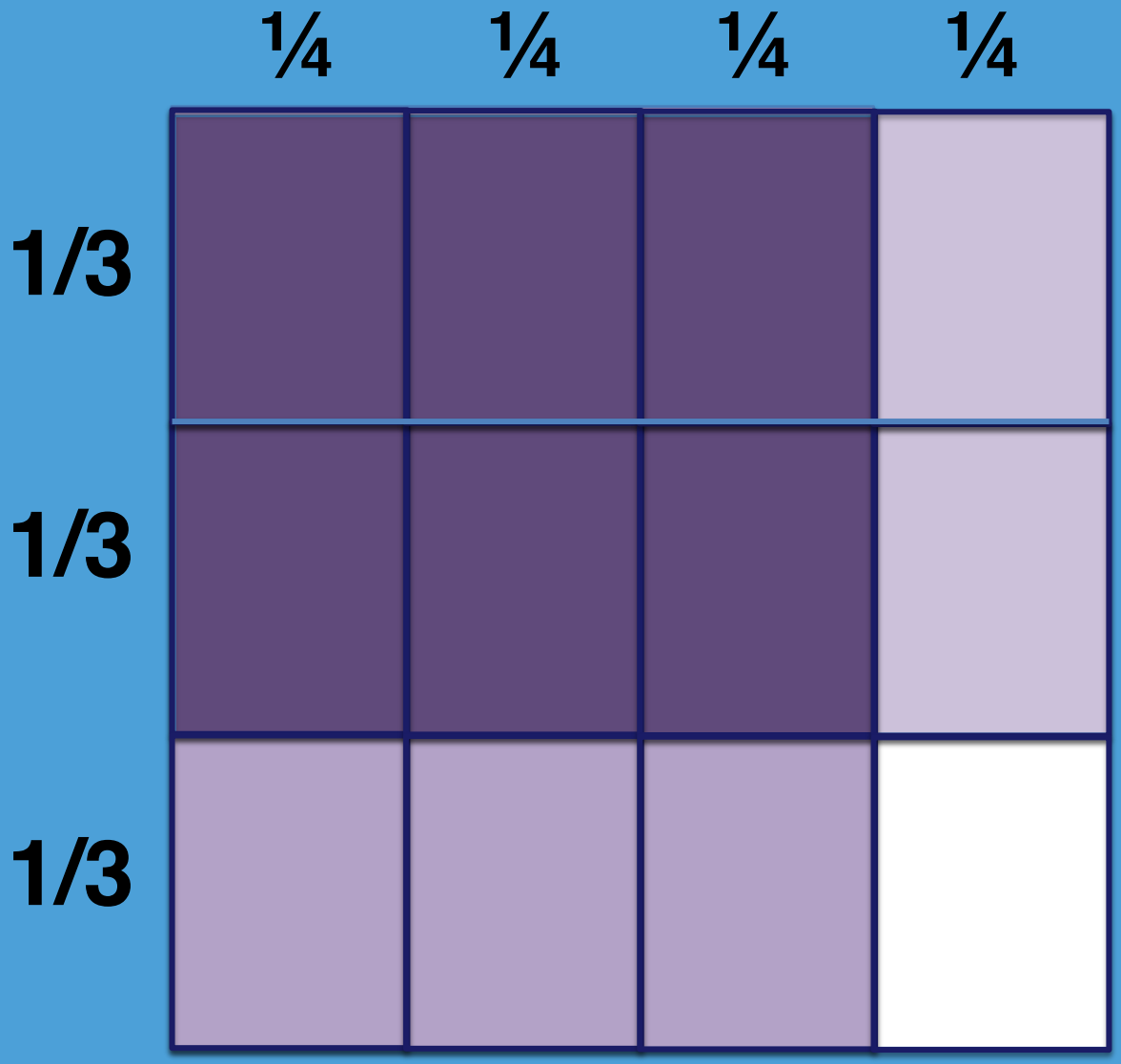

¼
¼
¼
¼
1/3
1/3
1/3


¼
¼
¼
¼
1/3
1/3
1/3
1/12


¼
¼
¼
¼
1/3
1/3
1/3


¼
¼
¼
¼
1/3
1/3
1/3

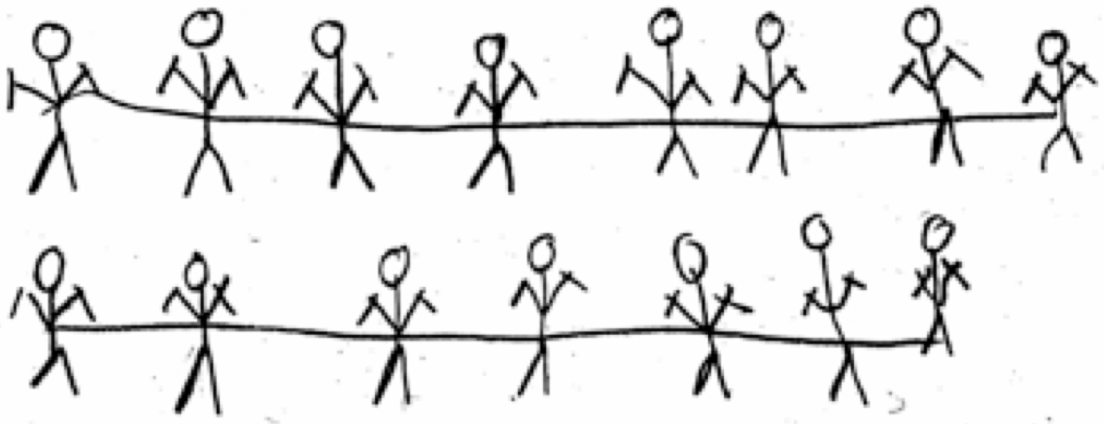

¼
¼
¼
¼
1/3
1/3
1/3

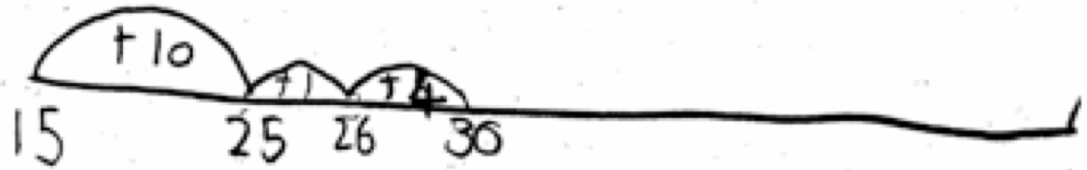


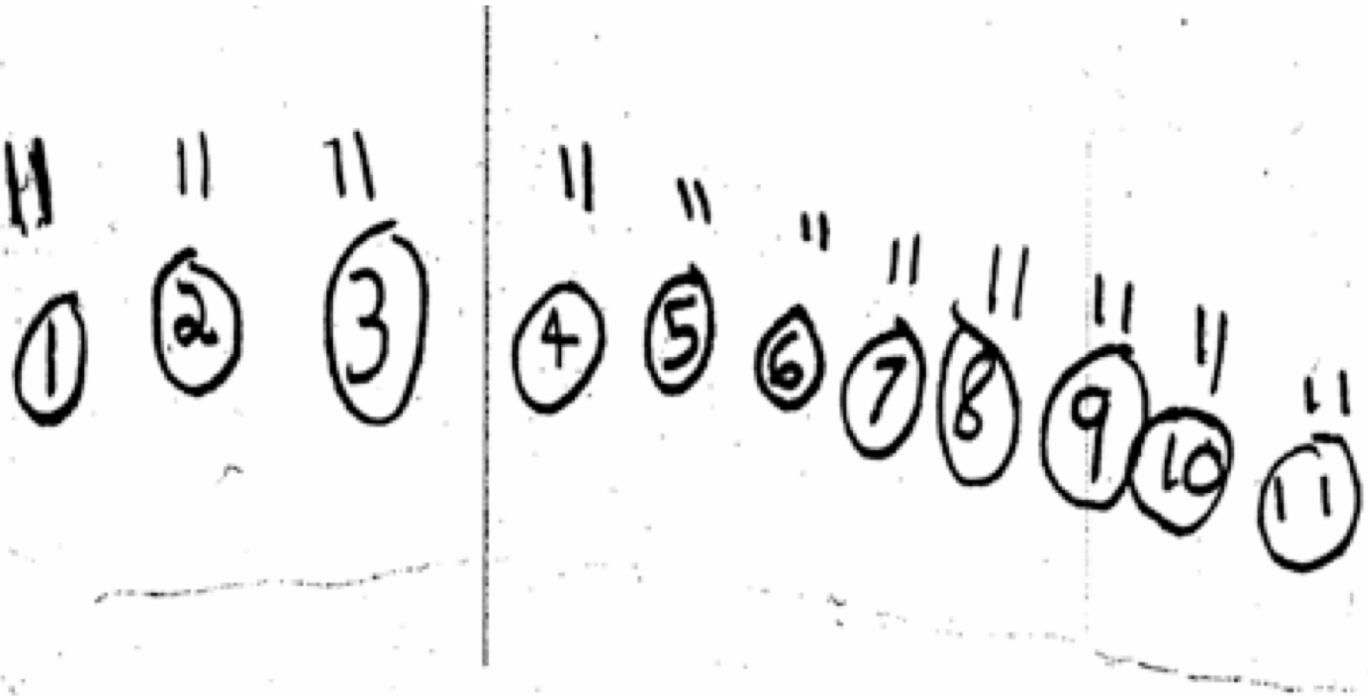
$$2/3 \times 3/4 = 6/12 = 1/2$$

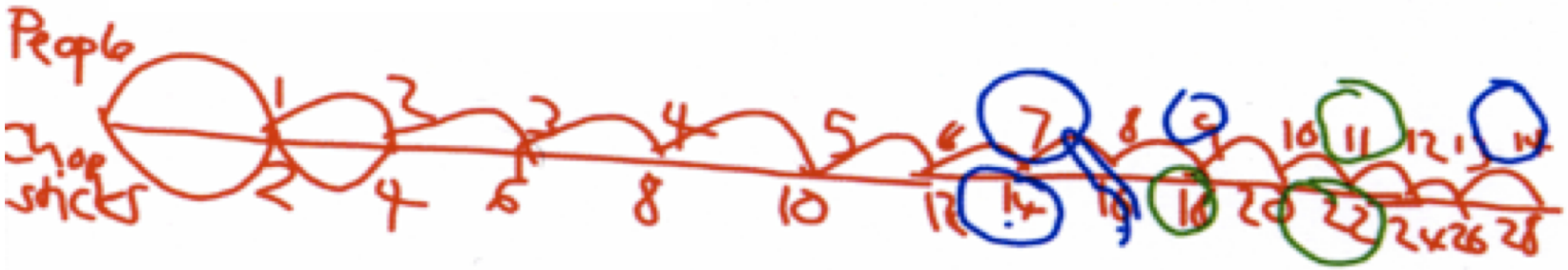
Making + Connections

2/3 x 3/4




+10
+1
+4
15
25
26
36


1
2
3
4
5
6
7
8
9
10
11

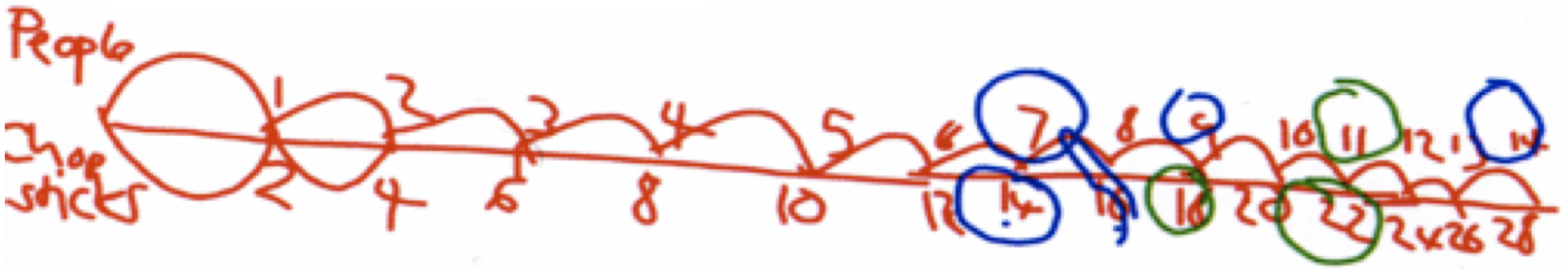

People
Shoe
sticks

Double 50
$= 100$

$50 \times 50 = 2500$

$100 = 50 \times 2$ ✓

$100 = 2 \times 50$ ✓



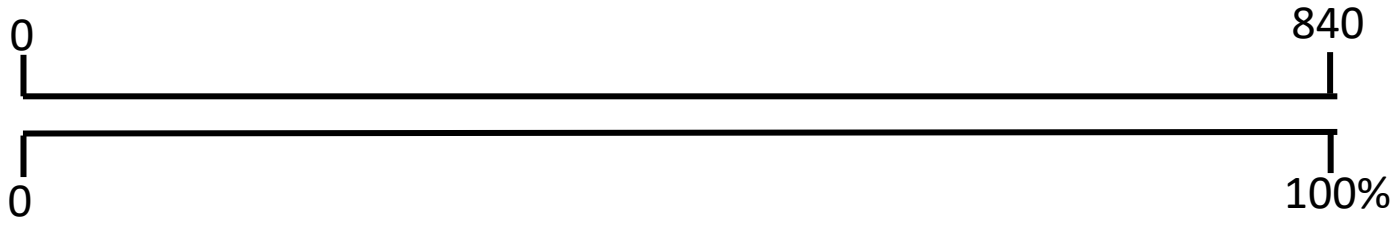
Jumping frogs

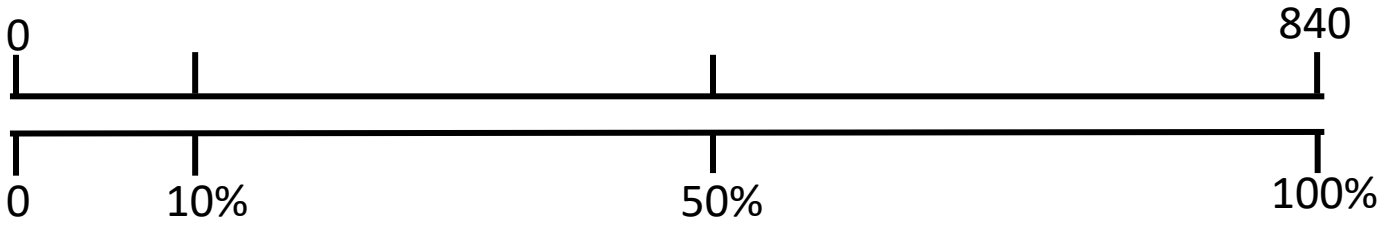
For every 1 jump a mother frog makes

Her baby frog has to make two jumps to keep up.

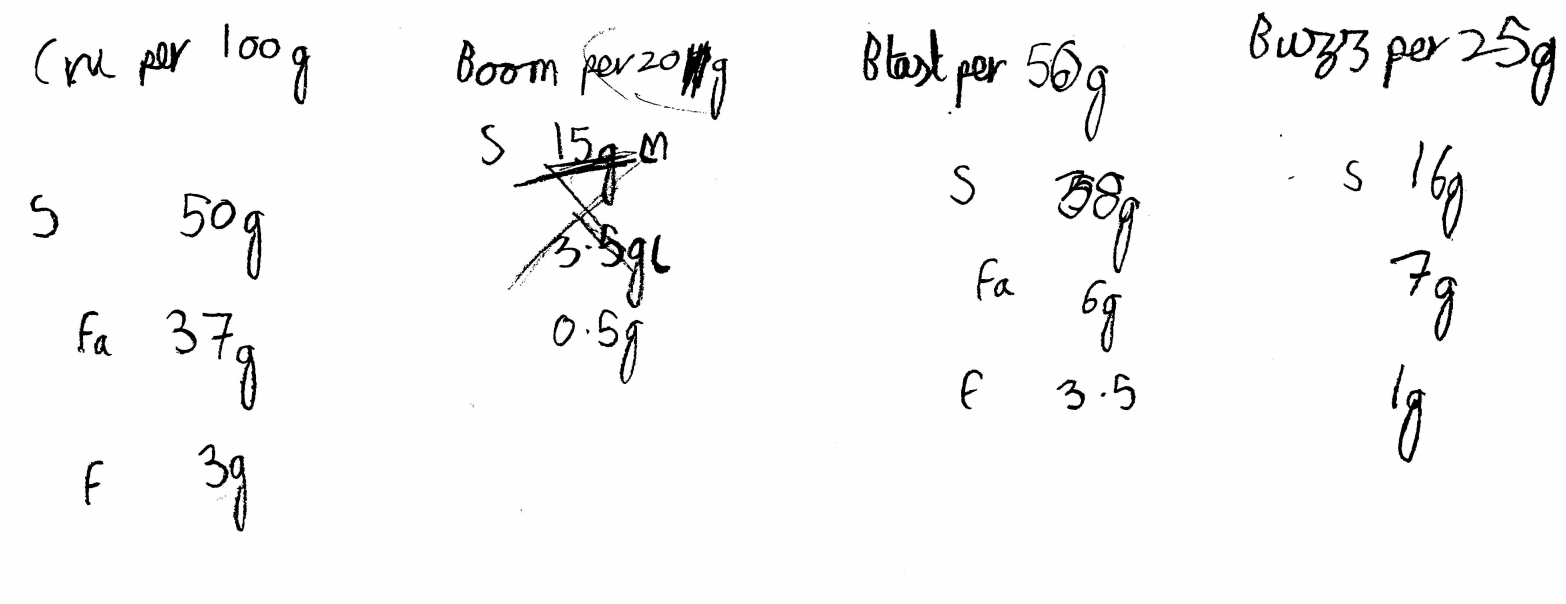
The mother frog makes 6 jumps

How many jumps does the baby frog make?

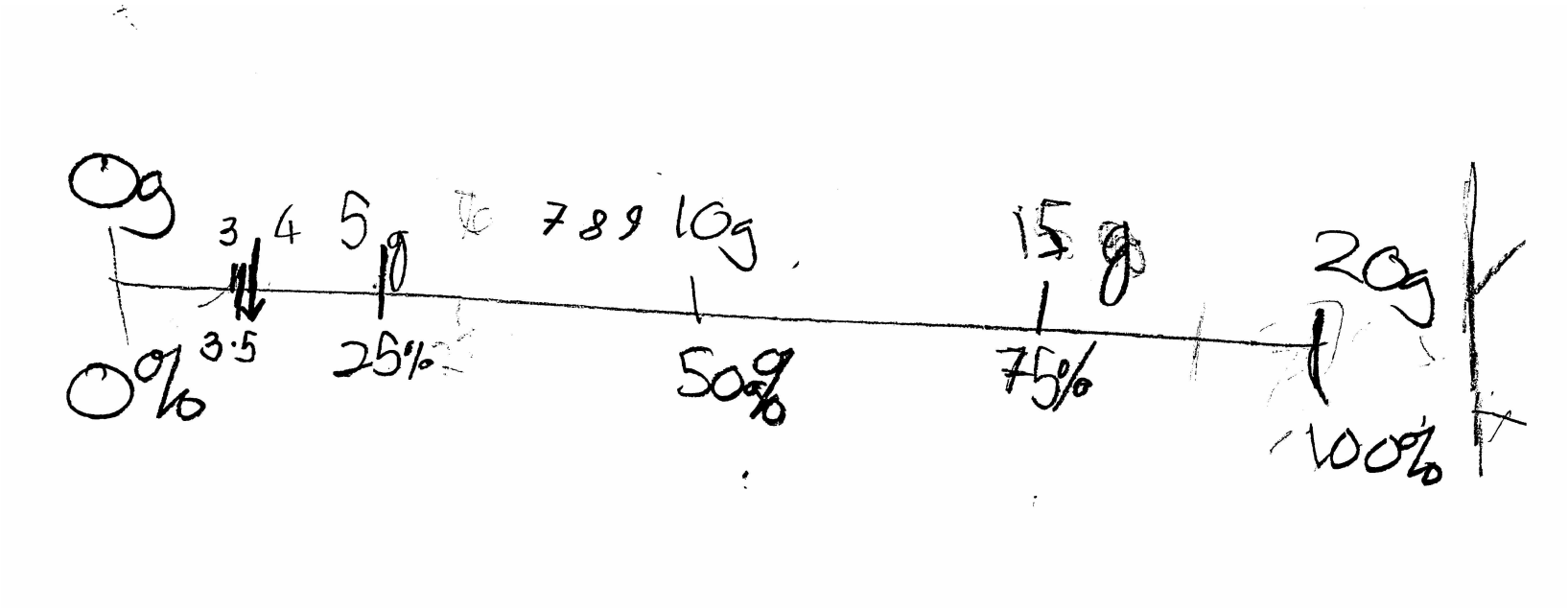

0
840
0
100%

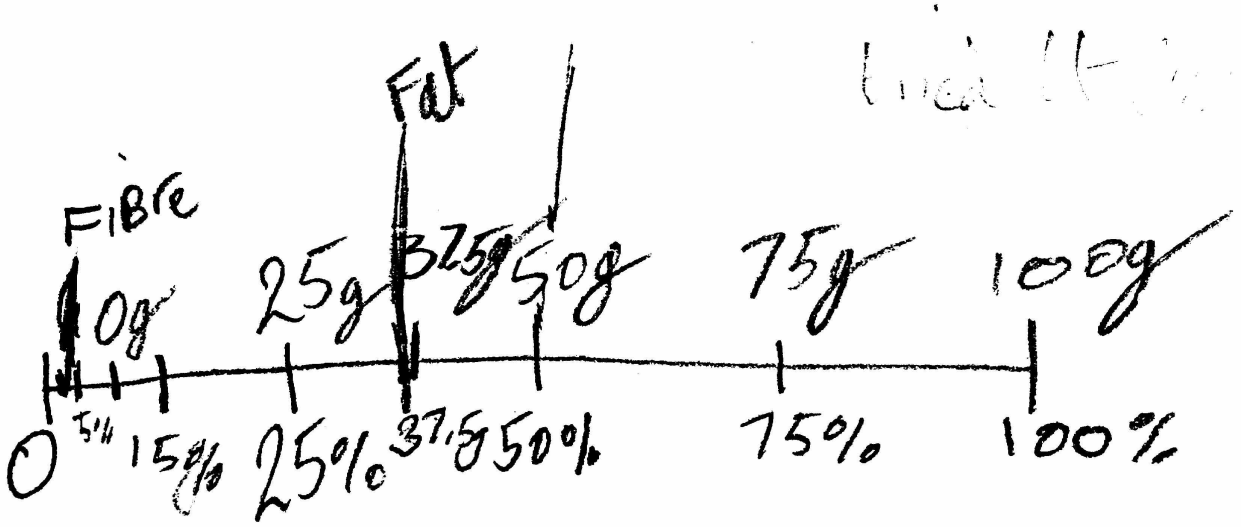

0
840
0
10%
50%
100%

Health buy?

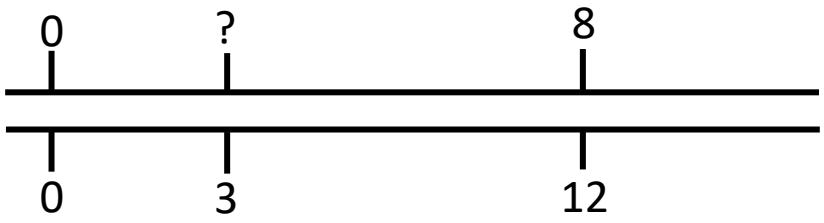

Cru per 100g
S 50g
Fa 37g
F 3g
Boom per 20g
S 15g
3.5gL
0.5g
Blast per 56g
S 28g
Fa 6g
F 3.5
Buzz3 per 25g
S 16g
7g
1g

Double number line





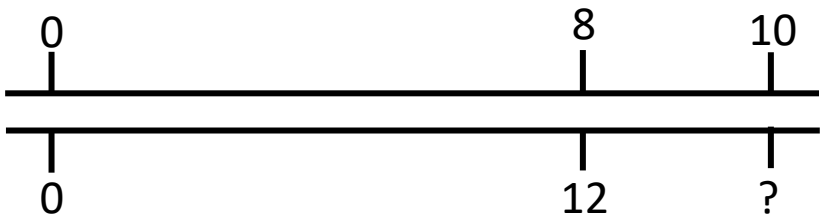
S = 50%



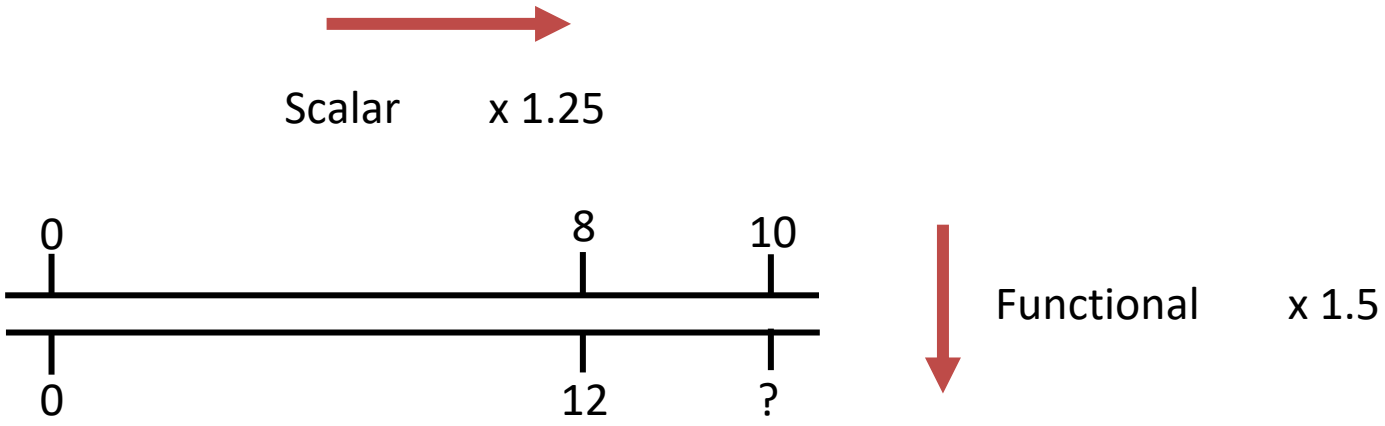
A father and daughter are walking side by side on a straight path.
Which line represents the daughters steps?

US$8 is worth about NZ$12.

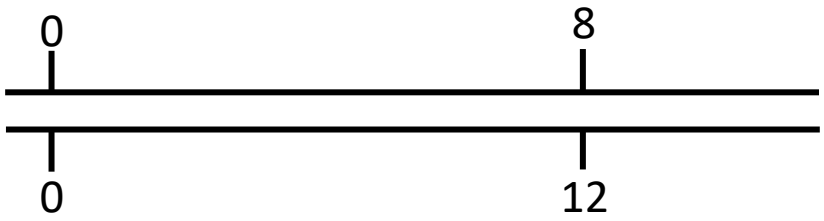
Küchemann, D., Hodgen, J., & Brown, M. (2011)
https://www.ncetm.org.uk/files/108357774/ncetm_ks3_cc_3_1.pdf.

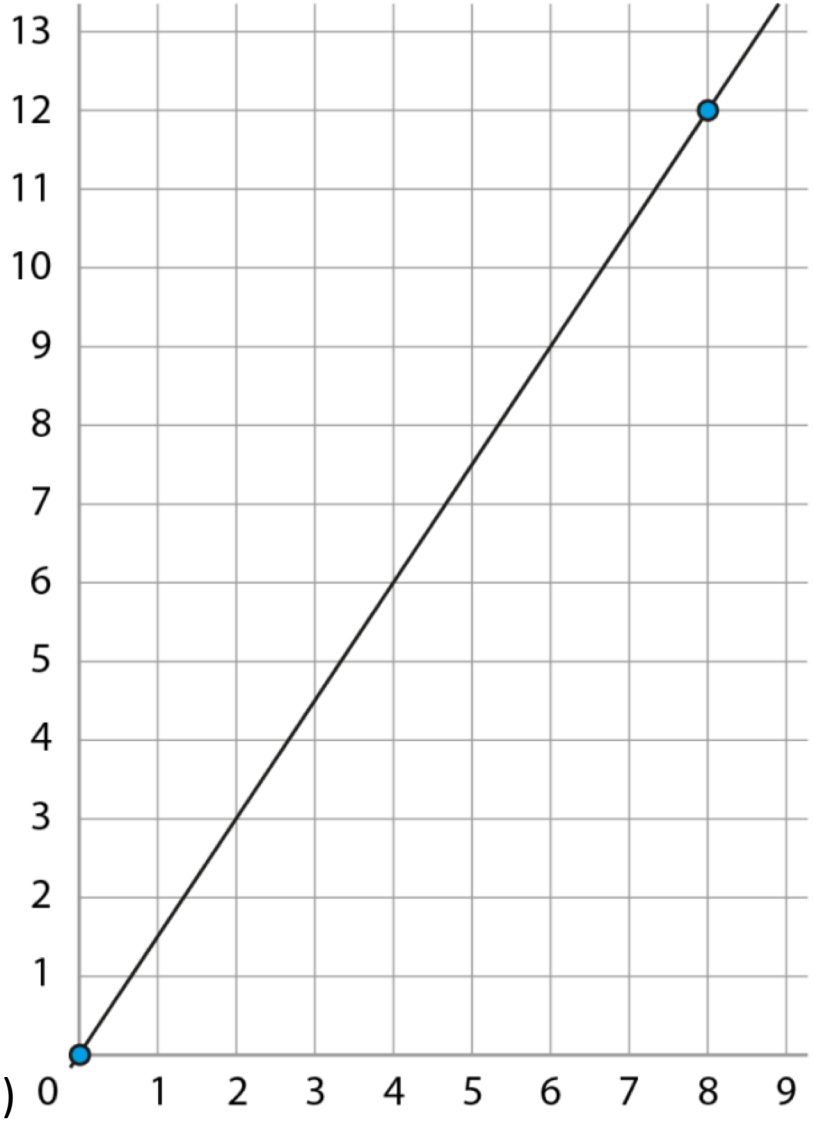


Küchemann, D., Hodgen, J., & Brown, M. (2011)
https://www.ncetm.org.uk/files/108357774/ncetm_ks3_cc_3_1.pdf.



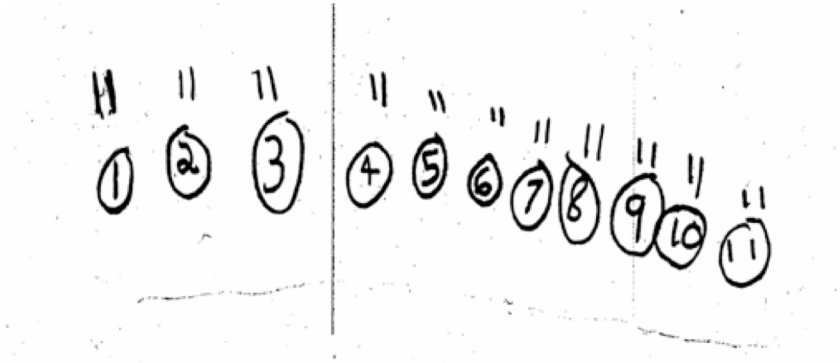
Küchemann, D., Hodgen, J., & Brown, M. (2011)
https://www.ncetm.org.uk/files/108357774/ncetm_ks3_cc_3_1.pdf.

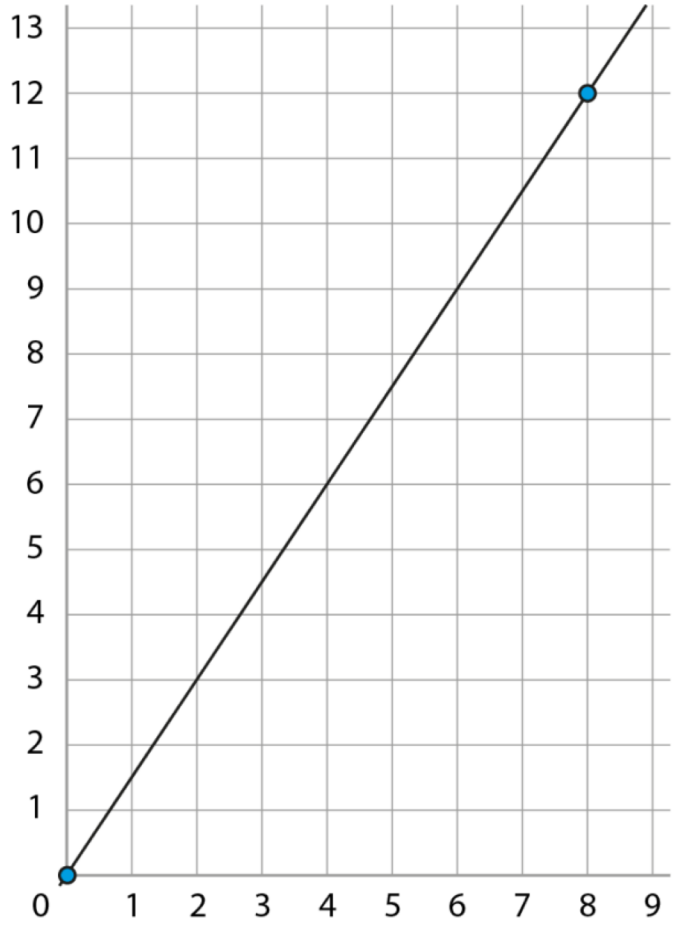




Küchemann, D., Hodgen, J., & Brown, M. (2011) https://www.ncetm.org.uk/files/108357774/ncetm_ks3_cc_3_1.pdf.

Making + Connections





Ratio table

Bags	Pears
1	7
6	?

Multiplicative relations

Bags	Pears
1	7
6	?

- 6 bags each hold 7 pears. How many pears are their altogether?
- Multiplication

Multiplicative reasoning

Bags	Pears
1	?
6	42

- 42 pears are are shared equally into 6 bags. How many pears does each bag contain?
- Partitioning – number of groups known, but not how many in each.
- Division as sharing

Multiplicative reasoning

Bags	Pears
1	7
?	42

- Pears are being put into bags of 7. If there are 42 pears how many bags can be filled?
- Quotitioning – size of group known (quota), but not how many can be served.
- Division as repeated subtraction.

Explicit ratio – joins multiplication and division

Bags	Pears
1	7
6	42

Isomorphism of measures

M1	M2
1	a
b	c

No distinction as to what types of numbers

Quantities in each measure may be integers, fractions or decimals

Division (1)

Division by multiplier

Seconds	Metres
1	?
3.3	13.9

A boat moves 13.9 metres in 3.3 seconds. What is its average speed in metres per second?

Division (2)

Division by multiplicand

Inches	Cm
1	2.54
?	7.84

An inch is about 2.54 cms. About how long in inches n 7.48 cms?

Working with Big Ideas is a way of dealing with classroom diversity and promoting inclusive classrooms.

Thank you

info@mikeaskew.net

Mikeaskew.net

References

NCETM Secondary Mastery PD materials

https://www.ncetm.org.uk/files/108357774/ncetm_ks3_cc_3_1.pdf.

Carpenter, T., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth NH: Heinemann.

Küchemann, D., Hodgen, J., & Brown, M. (2011). Using the double number line to model multiplication. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings of the Seven th Congress of the European Society for Research in Mathe matics Education* (pp. 326-335). Rzeszów, Poland: Universi ty of Rzeszów.