Mathematical Methods Unit 3 Application task – modelling product functions and designing buildings to withstand Earthquakes. The application task is to be of 4–6 hours duration over a period of 1–2 weeks.

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**Areas of study**

The following content from the areas of study is addressed through this task.

|  |  |
| --- | --- |
| **Area of study** | **Content dot point** |
| Functions and graphs | 1, 5 |
| Algebra | 5 |
| Calculus | 4, 5 |
|  | - |

**Outcomes**

The following outcomes, key knowledge and key skills are addressed through this task.

|  |  |  |
| --- | --- | --- |
| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| **1** | 1, 2, 6, 10, 12 | 7, 10, 11, 12, 13 |
| **2** | 1, 2, 4 | 1, 2, 4, 5 |
| **3** | 1, 2, 3, 4, 6 | 2, 3, 4, 5, 6, 8, 9, 10, 11 |

Component 1 PART A (One lesson) Calculator required (23 marks)

*Introduction of the context through specific cases or examples*

In 2018 worldwide there were 16 major Earthquakes with a magnitude between 7.0 -7.9 which can cause extensive damage and loss of life. <https://en.wikipedia.org/wiki/List_of_earthquakes_in_2018>

*The magnitude M of an Earthquake can be defined by the Richter scale where the independent variable is energy.*

M = 0.67 where E is the energy in Kilowatt hours

1. Calculate the energy E, (to two D.P.) of an earthquake with magnitudes of 5, 6 and 7 and complete the table below  **(2 marks)**

|  |  |  |
| --- | --- | --- |
| M | E |  |
| 5.00 | 5.19 | - |
| 6.00 | 1.61 | 31 |
| 7.00 | 5.02 | 31 |

1. Hence from (a), state the geometric increase in energy E for the corresponding linear increase of 1 in Magnitude M**. (1 mark)**

31

1. How much greater in energy was earthquake on September 28th 2018 with a magnitude of 7.5 in Indonesia compared to an Earthquake in Haiti on October 7 2018 with a magnitude of 5.9 ? That is find **(2 marks)**

For 7.5 E = 2797200834

For 5.9 E = 11446269

 = 2797200834/11446269 = 244 times greater

1. Given your answers above, explain why logarithms are used for modelling earthquakes. **(1 mark)**

Logarithms rescale large numbers into convenient form for comparisons

1. For 𝛼x), complete the table below for the three different values of . State the period and dilation from the y axis. Comment on the effect of increasing on the period. **(3 marks)**



|  |  |  |
| --- | --- | --- |
| sin( | Period | Dilation from y axis |
| 2 |   | 1/2 |
| 3 | 2/3 | 1/3 |
| 4 |   | 1/4 |

1. The period is the reciprocal of the frequency, The resonant frequency of buildings is about 1 Hertz ( 1 cycle /second) . An earthquake having this frequency is more likely to destroy a building. State the corresponding value of in for an earthquake of this frequency. **(1 mark)**

sin(2

1. Determine the constants ,n, h and k given the vibration of a concrete building

is modelled by A(t) = where the earthquake occurs at 9a.m. in the morning and it takes second between the the first maximum vibration of 6mm, and the first minimum of 2 mm. Assume time begins from 9 a.m. when the first maxima arrives **(4 marks)**

average value of vibration = (6+2)/2 = 4 = k

 =2 ,

period = 2 = 2 n =

As maximum is at t=0, and period = 2 seconds ¼ cycle left = 0.5 seconds

1. What does the model predict the size or amplitude of the vibration to be after 32 seconds?

Given Earthquakes don’t last long in time, explain whether the model is realistic. **( 2 marks)**

A(t) =

A(32) = 6 mm

1. To improve the model the term is includedto give A(t) =sin(t).

Investigate the height of the first maximum of the graph at t = 1 second for

. (**4 marks)**

|  |
| --- |
|  Table 1 |
|  | for t = 14 D.P. | Height of 1st maximum4 D.P. | Position of 1st maximum((t)4 d.p. | Position of second maximum(t)4 d.p. |
| 1 | 0.3679  | 1.2750 | 0.4019 | 2.4019 |
| 0.7 | 0.4966 | 1.4445 | 0.4302 | 2.4302 |
| 0.3 | 0.7408 | 1.7293 | 0.4697 | 2.4697 |

(j)What is the maximum value of sin(t)? ( 1mark)

2

1. What is the affect on the maximum height of A(t) =sin(t) as approaches zero. Briefly explain why. ( 2marks)

As , 1 and maximum value of A(t)

1. From the tabled values above, does changing change the period? **(1 mark)**

**Changes the height but does not change the period.**

1. Briefly explain how including the term is an improvement for modelling Earthquakes. **(2 marks)**

COMPONENT ONE PART B Calculator Required ( ONE LESSON). ( 16 marks)

*Introduction of the context through specific cases or examples*

(a)Using calculus confirm the position of the first two maximum points below, for

A(t) =sin(t). **Show working.**

Show how the value of affects the position of the maximum stationary points using evidence for your answer. Briefly discuss why this occurs . (5 marks)

|  |
| --- |
| Table 1 from Part A of SAC |
|  | for t = 14 D.P. | Height of 1st maximum4 D.P. | Position of 1st maximum((t)4 d.p. | Position of second maximum(t)4 d.p. |
| 1 | 0.3679  | 1.2750 | 0.4019 | 2.4019 |
| 0.7 | 0.4966 | 1.4445 | 0.4302 | 2.4302 |
| 0.3 | 0.7408 | 1.7293 | 0.4697 | 2.4697 |

A(t) =sin(t).

 =sin(t) +cos(t)

 2 ( in(t) + cos(t))

2 2 ( in(t) + cos(t))

 sin(t) + cos(t)=0

t)=

For t = 0.4019 for n=2 t = 2.4019

For t = 0.4302 for n=2 t = 2.4302

For t = 0.4697 for n=2 t = 2.4697

As can be seen , from , decreasing translates the graph in the positive direction of the x axis. This is consistent with A(t) =sin(t).

where the term has the dilation 1/ away from the y axis, thus as decreases , 1/ increases away from the y axis.

(c)Summarize the change in the graph of sin(t) resulting from including the term in the model A(t) =sin(t). **( 3 marks)**

This term diminishes the amplitude of the graph sin(t) as t increases. As decreases the amplitude approaches 2.

The term does not affect the period of sin(t).

The term also affects the position of the stationary points through a dilation from the y axis for .

Component Two Non calculator section. ( 15 marks)

*Consideration of general features of the context*

1. Find the period of t cos(2t) . Show working. ( 2 marks)

Period =2

1. Let =1, and choosing suitable values of investigate using calculus the position of the first four stationary points t cos(2t) using exact values. Show all working . ( 3marks)

t cos(2t)

 =(2t) - (2t)

=0

(2t) - (2t) =0

(2t) - (2t) =0

tan(t)= -/

 =tan(t)= - , t = 3 , 7, 11 , 15

 t= 3 , 7, 11 , 15

 =tan(t)= - , t= 5 , 11 , 17 , 23

 t= 5 , 11 , 17 , 23

 =, tan(t)= -, t = 2, 5, 8, 11

t = 2, 5, 8, 11

Component 3 Calculator Required

*Variation or further specification of assumption or conditions involved in the context to focus on a particular feature or aspect related to the context.*

The vibrations in buildings caused by Earthquakes can be modelled by the differential equation where (the acceleration) is the double derivative w.r.t. to x ,

 (the velocity) is the single derivative w.r.t. to x .

 is the rigidity and w represents the elasticity of a building to vibrations.

We will be investigating three different equations which all satisfy the above differential equation.

Case 3 is the equation x = (A+Bt) A=B = 1

Using a calculator find , and show x = (1+1t) satisfies the above differential equation when **( 1+1+1+1+1 =5marks)**

1. The Differential equation describing vibrations. produces two other solutions. **( 1+1 = 2marks)**

Case 1 Overdamped motion

 where *x* = and

Case 2 Underdamped motion where

where *x* = and

(i)For case 1, briefly state why the condition is consistent for for solutions which are real numbers.

(ii)Similarly for case 2 , briefly state why the condition is consistent with

 for non-real solutions

c. Investigate which of the three cases is least destructive on a building by determining the average value of the size of vibration of for the first 6 seconds while varying vary . From your answers make a recommendation about the value which provides the least vibrations .

Case 1 Overdamped motion

 Where *x* = and

for 0

Calculator commands

Analysis/gsolve/integral/integraldx/ lower limit and upper limit chosen with execute

Case 1 ,initially dies of quickly as but after time actually so average value increases

|  |  |  |
| --- | --- | --- |
|  | Case 1 average = dt |  |
| w |  |  | roots | Average x | Equation for average |
| 1 | 2 |  | none | 0.542 |  dt |
| 1 | 3 |  |  | 0.653 |  dt |
| 1 | 4 |  |  | 0.721 |  |
|  |  Case 2 Underdamped motion where Where *x* = and  average =  |  |
| w |  |  | roots | Average x |  |
| 2 | 1 |  | 1.36034953.1741489 | 0.84012976+0.1827324=1.001/6 =0.16 |  |
| 3 | 1 |  | 0.833, 1.9443.0544 | 0.6186+0.257+0.08458=0.3202.32/6=0.05 |  |
| Case 3 , x = (1+t)  |  |
| w |  |  | none |  |  |
| 1 | 1 |  |  | 1.98/6=0.33 |  |
| 2 | 2 |  |  | 0.75/6=0.125 |  |
| 3 | 3 |  |  | 0.44/6=0.073 |  |

**D Make and justify your recommendation about the values of and w to give a building the best chance of surviving an earthquake. ( 2marks)**