1

OVERVIEW:

- Targeted Teaching
- The 'big ideas in number F - 10
- SNMY
- The Assessment for *Common Misunderstandings Tools**

* These are an extended version of the Probe Tasks referred to in *Talking Namba* (NT Department of Education)

2

Notion of targeted teaching that requires:

- **access to assessment techniques** that expose student's thinking;
- **a grounded knowledge of learning trajectories** (key steps in the development of big ideas and how to scaffold these);
- an **expanded repertoire of teaching approaches** which accommodate and nurture discourse, help uncover and explore student's ideas in constructive ways, and ensure all students can participate in and contribute to the enterprise;

- **sufficient time with students to develop trust and supportive relationships**; and
- **flexibility to spend time with the students who need it most**.

(MYNRP, Final Report, 2001)

3

This is particularly important in relation to a relatively small number of 'big' ideas and strategies in Number, without which students' progress in mathematics will be seriously impacted

(Siemon, 2006).

4

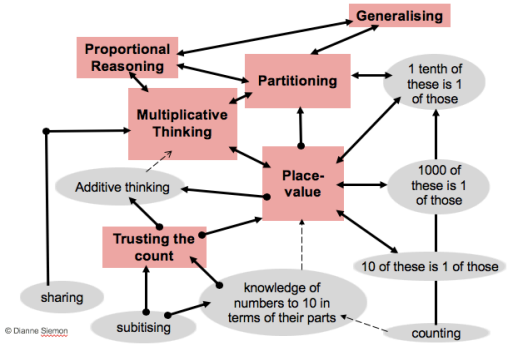
What is a 'big idea'?

- An idea, strategy, or way of thinking about some key aspect of mathematics, **without which students' progress in mathematics will be seriously impacted**
- Encompasses and **connects many other ideas** and strategies
- Provides an **organising structure** or a frame of reference that supports further learning and generalisations
- Cannot be clearly defined but can be **observed in activity** …

(Siemon, 2006)

5

The 'BIG IDEAS' in Number



6

The big ideas in Number F-10

By the end of:

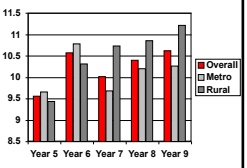
Year F/mid Yr 1	**Trusting the count** - developing flexible mental objects for the numbers 0 to 10, part-part-whole knowledge
Year 2	**Place-value** - the importance of moving beyond counting by ones, the structure of the base ten numeration system
Year 4	**Multiplicative thinking** (initial ideas) - the key to understanding rational number and developing efficient mental and written computation strategies in later years
Year 6	**Partitioning** (equal parts) - the missing link in building common fraction and decimal knowledge and confidence
Year 8	**Proportional reasoning** - extending what is known beyond rule-based procedures to represent and solve problems involving fractions, decimals, percent, ratio, rate and proportion
Year 10	**Generalising/Formalising** - skills and strategies to support equivalence, recognition of number properties and patterns, and the use of algebraic text

(Siemon, 2006; 2011)

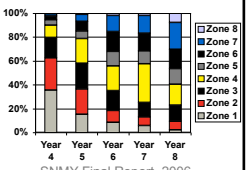
7

Evidence (MYNRP, SNMY & RMF Projects)

- As **much difference within Year levels as between Year levels**
- **Considerable within school variation**
- The **needs of many students, but particularly those 'at risk' or 'left behind', are not being met**.
- Differences in performance were largely due to an **inadequate understanding of fractions, decimals, and proportion, and a reluctance/inability to explain/justify solutions (i.e. Multiplicative Thinking)**
- **At least 30% and up to 55% of Year 8 students do not have access to multiplicative thinking**



MYNRP Final Report, 2001



SNMY Final Report, 2006

8

An 8 year range….

LAF Zone	1	2	3	4	5	6	7	8
Expected by	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8
Year 4	6	6	5	5	2	1	1	0
Year 5	3	5	5	5	2	3	2	0
Year 6	1	2	3	5	4	5	5	1
Year 7	1	2	2	6	3	3	6	1
Year 8	1	1	2	6	4	5	5	2

Implied class distribution by Year Level based on SNMY data (2004) – possible groupings

9

The 'shallow teaching syndrome'

The TIMSS Video Study of Year 8 mathematics classrooms in 7 countries found that while the Australian lessons had the second highest proportion of real-life contexts, they had the **highest proportion of similar problems and problems of low procedural complexity** … (Vincent & Stacey, 2008)

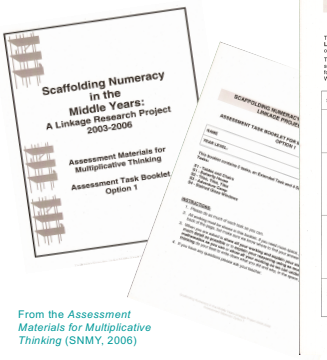
In her study of primary teachers beliefs and practices, Anderson (1998) found that while some primary teachers reported using open ended and unfamiliar mathematics problems on regular basis, the **majority used traditional exercises or application type problems** on a regular basis.

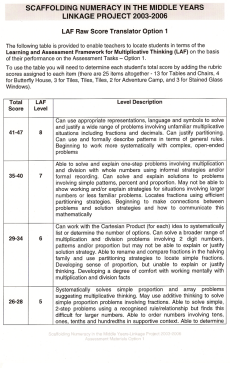
Types of questions	Often	Sometimes	Rarely
Exercises	66	29	5
Application Problems	68	28	4
Open-ended Problems	18	58	24
Unfamiliar Problems	10	52	38

Proportion (%) of teachers reporting frequency of use of different question/problem types (Anderson, 1998, p. 9)

10

Scaffolding Numeracy in the Middle Years – Class based tools





SCAFFOLDING NUMERACY IN THE MIDDLE YEARS LINKAGE PROJECT 2003-2006

LAF Raw Score Translator Option 1

The following table is provided to enable teachers to locate students in terms of the Learning and Assessment Framework for Multiplicative Thinking (LAF) on the basis of their performance on the Assessment Tasks – Option 1.

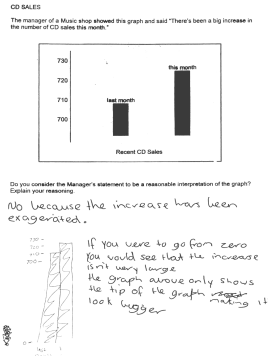
To use the table you will need to determine each student's total score by adding the rubric scores assigned to each item (there are 26 items altogether - 13 for Tables and Chairs, 4 for Butterfly House, 3 for Tiles, Tiles, Tiles, 2 for Adventure Camp, and 3 for Stained Glass Windows).

Total Score	LAF Level	Level Description
41-47	8	Can use appropriate representations, language and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals. Can justify partitioning. Can use and formally describe patterns in terms of general rules. Beginning to work more systematically with complex, open-ended problems
35-40	7	Able to solve and explain one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording. Can solve and explain solutions to problems involving simple patterns, percent and proportion. May not be able to show working and/or explain strategies for situations involving larger numbers or less familiar problems. Locates fractions using efficient partitioning strategies. Beginning to make connections between problems and solution strategies and how to communicate this mathematically.
29-34	6	Can work with the Cartesian Product (for each) idea to systematically list or determine the number of options. Can solve a broader range of multiplication and division problems involving 2 digit numbers, patterns and/or proportion but may not be able to explain or justify solution strategy. Able to rename and compare fractions in the halving family and use partitioning strategies to locate simple fractions. Developing sense of proportion, but unable to explain or justify thinking. Developing a degree of comfort with working mentally with multiplication and division facts
26-28	5	Systematically solves simple proportion and array problems suggesting multiplicative thinking. May use additive thinking to solve simple proportion problems involving fractions. Able to solve simple, 2-step problems using a recognised rule/relationship but finds this difficult for larger numbers. Able to order numbers involving tens, ones, tenths and hundredths in supportive context. Able to determine

From the *Assessment Materials for Multiplicative Thinking* (SNMY, 2006)

11

Rich tasks and scoring rubrics.

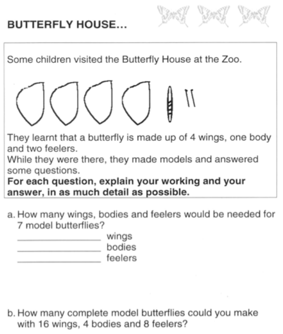


0. No response or 'yes' or 'no' without a reason.
1. Reasoning based on numbers alone, no recognition that 'big' is relative.
2. Reasoning shows some recognition that 'big' is relative to total sales, but unsupported conclusion, little/no explanation, eg, "it depends ...".
3. Reasoning concludes that increase is not 'big' relative to total sales, some attempt to relate this to proportion, eg, "15 out of 725 is not very big".
4. Correct conclusion, "not big", %, fractions, ratio used correctly to support well-reasoned explanation.

From the *Middle Years Numeracy Research Project* (Siemon, Virgona & Corneille, 2001)

12

An Extended Task


BUTTERFLY HOUSE...
Some children visited the Butterfly House at the Zoo.
They learnt that a butterfly is made up of 4 wings, one body and two feelers.
While they were there, they made models and answered some questions.
For each question, explain your working and your answer, in as much detail as possible.
a. How many wings, bodies and feelers would be needed for 7 model butterflies?
wings
bodies
feelers
b. How many complete model butterflies could you make with 16 wings, 4 bodies and 8 feelers?

Adapted from 'Butterflies and Caterpillars' (Kenney, Lindquist & Heffernan, 2002) for the SNMY Project (2003-2006)

This task had 9 items altogether including:

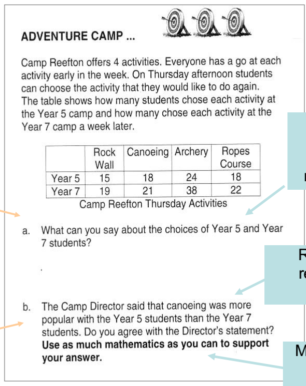
Items of **increasing complexity**, eg, "How many complete model butterflies could you make with 29 wings, 8 bodies and 13 feelers?"

Items involving **simple proportion and rate**, eg, "To feed 2 butterflies, the zoo needs 5 drops of nectar per day. How many drops would be needed per day to feed 12 butterflies?" …and

Items involving the **Cartesian product**, eg, given 3 different body colours, 2 types of feelers and 3 different wing colours, "How many different model butterflies could be made?"

13

A Short Task


ADVENTURE CAMP ...
Camp Reefton offers 4 activities. Everyone has a go at each activity early in the week. On Thursday afternoon students can choose the activity that they would like to do again.
The table shows how many students chose each activity at the Year 5 camp and how many chose each activity at the Year 7 camp a week later.
Rock Wall
Canoeing
Archery
Ropes Course
Year 5 15 18 24 18
Year 7 19 21 38 22
Camp Reefton Thursday Activities
a. What can you say about the choices of Year 5 and Year 7 students?
b. The Camp Director said that canoeing was more popular with the Year 5 students than the Year 7 students. Do you agree with the Director's statement?
Use as much mathematics as you can to support your answer.

Open-ended question

Solution strategy unclear, problem solving

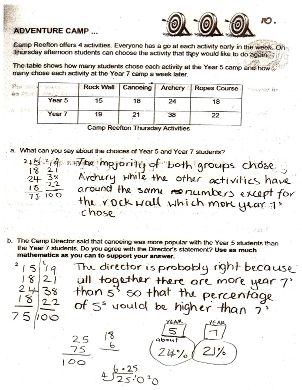
Reading and interpreting quantitative data relative to context

Recognising relevance of proportion

Mathematics used, eg, percent, fractions, ratio

SNMY Project (2003-2006)

14

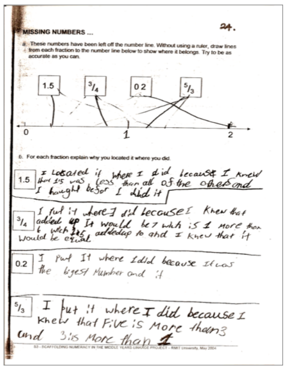


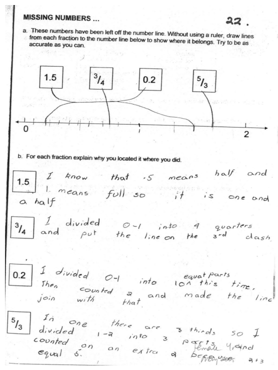
TASK:	RESPONSE:	SCORE
a.	No response or incorrect or irrelevant statement	0
	One or two relatively simple observations based on numbers alone, eg, "Archery was the most popular activity for both Year 5 and Year 7 students", "More Year 7 students liked the rock wall than Year 5 students"	1
	At least one observation which recognises the difference in total numbers, eg, "Although more Year 7s actually chose the ropes course than Year 5, there were less Year 5 students, so it is hard to say"	2
b.	No response	0
	Incorrect (No), argument based on numbers alone, eg, "There were 21 Year 7s and only 18 Year 5s"	1
	Correct (Yes), but little/no working or explanation to support conclusion	2
	Correct (Yes), working and/or explanation indicates that numbers need to be considered in relation to respective totals, eg, "18 out of 75 is more than 21 out of 100", but no formal use of fractions or percent or further argument to justify conclusion	3
	Correct (Yes), working and/or explanation uses comparable fractions or percents to justify conclusion, eg, "For Year 7 it is 21%. For Year 5s, it is 24% because 18/75 = 6/25 = 24/100 = 24%"	4

A Year 6 Student Response to *Adventure Camp* Short Task (SNMY, May 2004)

15

Allow all learners to make a start

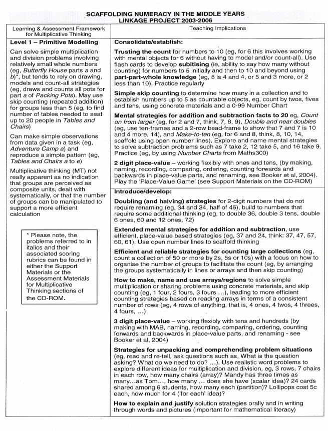




Two Year 4 Student Responses to *Missing Numbers* Short Task (SNMY, May 2004)

16

Advice to support targeted teaching



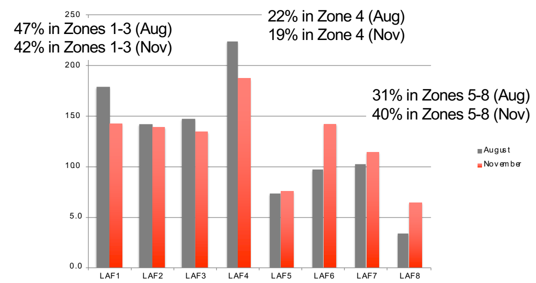
For example, 8 differentiated levels of multiplicative thinking determined on the basis of item analysis*

Teaching advice linked to levels (or zones)

* From the *Learning and Assessment Framework for Multiplicative Thinking* (SNMY, 2006)

17

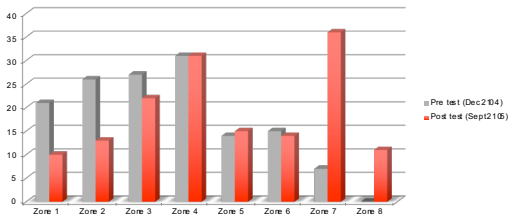
Targeting multiplicative thinking works!



Percentage of RMFI students at each LAF level, all students, August and November 2013 (N =1732)

18

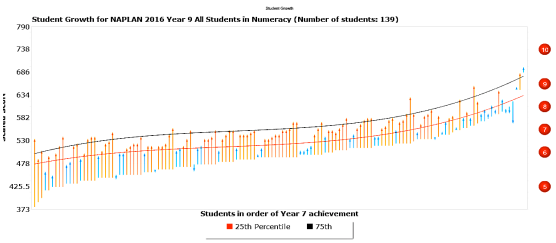
Results from an RMFII Project School who used SNMY data and the *Learning Assessment Framework for Multiplicative Thinking* to inform a targeted teaching approach in Year 8 in 2015



Percentage of students in each Zone of the SNMY LAF in August 2014 (n = 141) and September 2015 (n = 152) (Prasad, November 2016)

19

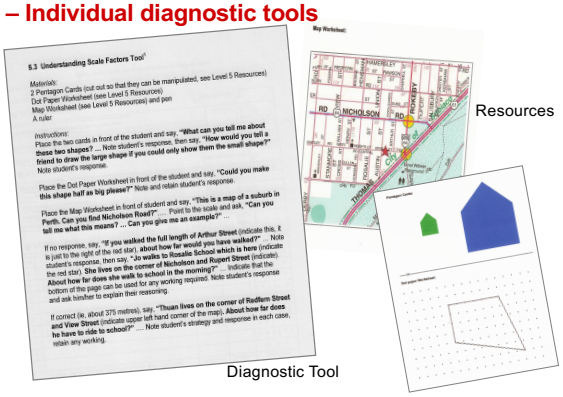
The Year 9 NAPLAN results for the same school in 2016 suggest that the growth in learning was sustained and transferable



Growth in student learning was demonstrated for all bar 5 students BUT 65% of students grew by more than the expected growth - well above State-wide expectations

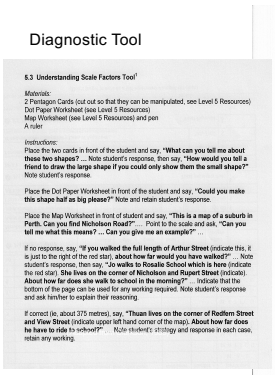
20

Assessment for Common Misunderstandings – Individual diagnostic tools

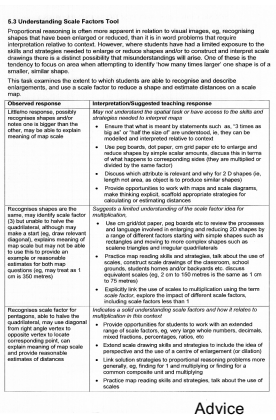


21

Diagnostic Tool



* From the *Assessment for Common Misunderstandings* (prepared by Dianne Siemon for DE&T, October 2006)



Advice

22

Trusting the Count …

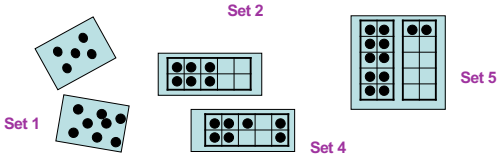
Trusting the count has a range of meanings:

- initially, children believe that if they counted the same collection again, they would get the same result and they recognise that counting is a strategy to determine how many*
- Ultimately, it is about having access to a range of **mental objects** for each of the numbers to ten, which can be used flexibly without having to make, count or see these collections physically.

* WA Department of Education, *First Steps in Mathematics* (2004)

23

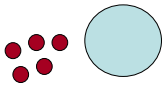
Subitising Tool (1.1)



Student:		Date:
Card Set:	**Pile A**	**Pile B**
1. Single Digit (2, 4, 5, 8, 10)		
2. Ten-Frame Doubles (1, 3, 4, 6, 9)		
3. Ten-Frame To Five (3, 6, 8, 0)		
4. Ten-Frames Random (2, 4, 5, 7, 10)		
5. Two Ten-Frames (12, 14, 17, 19)		

24

Mental Objects Tool (1.2)

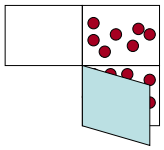


There are 5 here and 4 under the container …

How many altogether?

Three levels of response …

- perceptual,
- figural, and
- abstract Steffe, 1988)



This tool assesses students part-part-whole knowledge for the numbers to ten and beyond

Mental Objects Tool from the *Assessment for Common Misunderstandings*
http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm

25

Place-Value …

Children can be formally introduced to place-value as a system of recording numbers when they:

- can **comfortably count** to 20 and beyond;
- know the numbers 0 to 10 in terms of their parts (i.e., **part-part-whole knowledge**);
- can work flexibly with numerals to 10 without having to model the count (**trust the count**);
- interpret/visualise **numbers beyond ten** in terms of 1 ten and 4 more, "fourteen";
- recognise numbers to 10 as **countable units** for the purposes of counting, eg, 2, 4, 6, 8 ...)

26

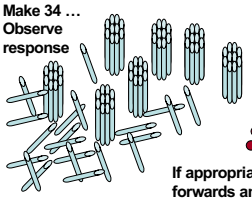
Number Naming Tool (2.1)

Ask student to count and record how many …

Circle the 6 and ask: What has this got to do with what you've got there?

Then circle the 2 and ask: What has this got to do with what you've got there?

Make 34 … Observe response

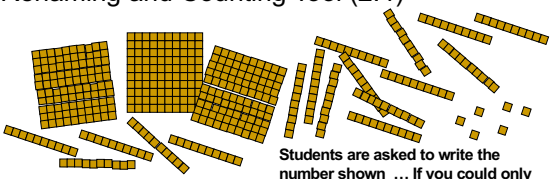


Then ask student to count counters, record and make groups of four … Repeat earlier questions …

If appropriate, ask students to count forwards and backwards using a 0-99 Number Chart and mask provided

27

Renaming and Counting Tool (2.4)



3 hundreds 17 tens 6 ones

Students are asked to write the number shown … If you could only use tens, how many needed?

Students are asked to write the number … name a number smaller than this but bigger than 50 … write the number 2 tens larger

63

673

Read the number … smaller than this but bigger than 517 … at least 1 ten larger but smaller than 968

Read and count on by ones … what number is 1 ten smaller than this?

5308

28

Multiplicative Thinking …

- a capacity to **work flexibly and efficiently with an extended range of numbers**;
- an ability to **recognise and solve a range of problems involving multiplication or division** including direct and indirect proportion; and
- the **means to communicate this effectively in a variety of ways** (e.g., words, diagrams, symbolic expressions, and written algorithms).

Siemon, 2004)

29

Note: Arrays and Regions (Year 2 to 3) support a critical shift in thinking

From counting equal groups:

1 three, 2 threes, 3 threes, 4 threes, ... ⇒ 1 x **3**, 2 x **3**, 3 x **3**, 4 x **3**, …

That is, the traditional focus on the **number in each group** and **how many groups**

To a focus on the **number of groups**:

3 ones, 3 twos, 3 threes, 3 fours, ... ⇒ **3** x 1, **3** x 2, **3** x 3, **3** x 4, …

and **generalising**:

3 groups of … is double the group and 1 more group.

This introduces the **factor** idea for multiplication

30

Additive Strategies Tool (3.2)

Do you agree that the sum of these numbers is 9?

3	2
4	9

Sum of 3, 2 and 4

Explore the thinking involved and identify strategies.
Stop as soon as student experiences difficulty.

6	8
9	

	5
7	24

34	72
58	

18	
22	87

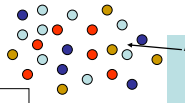
Assesses student's capacity to add and subtract mentally

From the *Assessment for Common Misunderstandings*
http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm

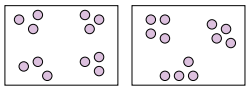
31

Sharing & Array and Regions Tools (3.3 & 3.4)

Can you share these among 6?



Assesses student's capacity to share equally, recognise commutativity, and work with the language and ideas of multiplication



Same or different?

Imagine you have 2 lollies and your sister has 3 times as many … How many lollies does your sister have?

Assesses extent to which student's can work with arrays and regions

How many dots altogether?

How many name-tags like this could be made from a sheet of paper this size?

From the *Assessment for Common Misunderstandings*
http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm

32

Cartesian Product Tool (3.5)

How many different T-shirts?

T-Shirts on Special
Only $9.89
3 colours (white, black, blue)
4 sizes (S, M, L, XL)

Representations and/or strategies used point to student's capacity to work with the 'for each' idea:

Additive if all possibilities listed and counted

Multiplicative if multiplication recognised as the appropriate operation

Claire ordered a peanut-butter role and a drink. What might she have ordered?

Claire's class has 23 students. If everyone ordered a roll with a filling and a drink, could they all have a different lunch order?

LUNCH ORDERS		
Rolls	**Fillings**	**Drinks**
White Multi-grain	Peanut butter Salad Cheese and vegemite Tuna and lettuce	Orange Juice Apple Juice Cola

33

Partitioning (equi*)

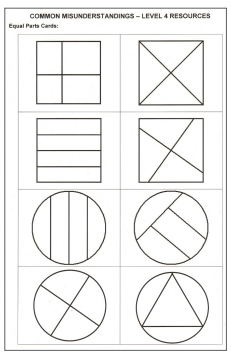
Is evident when students:

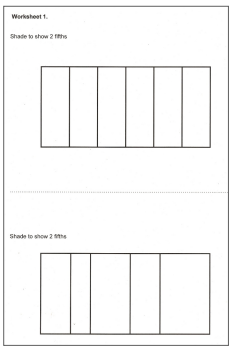
- use halving or related strategies to locate common fractions on a number line;
- estimate thirds and fifths by building on what is known (e.g., a third is smaller than a half … a fifth is smaller than a quarter);
- use fraction models to rename, compare and order fractions; and
- reason multiplicatively to construct decimal representations on an open number line.

*Confrey et al, 2009

34

Equal Parts Tool (4.1)



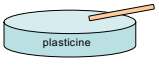


35

Fraction Making Tool (4.3)

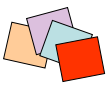
Assesses student's capacity to generate fraction models

(a) Can you give me half? … 1 third? … 5 eighths?



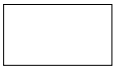
(b) Cut to make 8 equal pieces … If 3 quarters of the pizza was eaten, show how much was eaten.

(c) Can you use the ball of plasticine to show 5 thirds?



(d) Can you use these to show 2 and 5 sixths?

(e) Divide this rectangle into 3 equal parts … name of each part?



(f) Can you divide this line into 5 equal parts? Name of each part?

(g) If this is 2 thirds, what is 1?

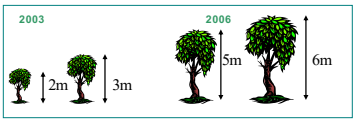
(h) Where is 1?

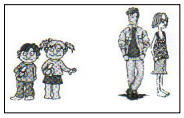
36

Proportional Reasoning

(a) … Who grew faster … Amy or Richard*?

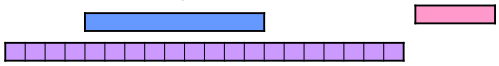
(b) .. Which tree grew more?





(c) … If the red rod is 1, what would the yellow rod be? … How many times longer is the yellow rod than the red rod? ..

(d) The purple rod is 2. What fraction name would you give to the blue rod? …What fraction is the pink rod of the blue rod?



* Adapted from Lamon (2001) for the ACM Tools

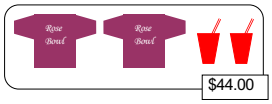
37

Generalising

(a) Some of these statements are true and some are false. Without calculating can you tell me which ones are true and which ones are false and why?

5 + 2 + 6 = 14 – 1
3.8 + 4.5 = 4.5 + 3.8
$\frac{3}{8} - \frac{2}{3} = \frac{2}{3} - \frac{3}{8}$
18 x 27 + 19 = 27 + 19 x 18
$4 \times 5\frac{3}{4} = 5 \times 4 + \frac{3}{4} \times 4$
83 ÷ 83 = 0

(b) How much does each item cost*?



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* Adapted from de Lange (1996)

38

Scaffolding Numeracy in the Middle Years

The Scaffolding Numeracy in the Middle Years 2003-2006 (SNMY) was an Australian Research Council Linkage Project awarded to RMIT University, the Victorian Department of Education and Training and the Tasmanian Education Department from July 2003 to June 2006.

https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/scaffoldnum.aspx

Assessment for common misunderstandings

https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx

39