Reasoning as a mathematical habit of mind

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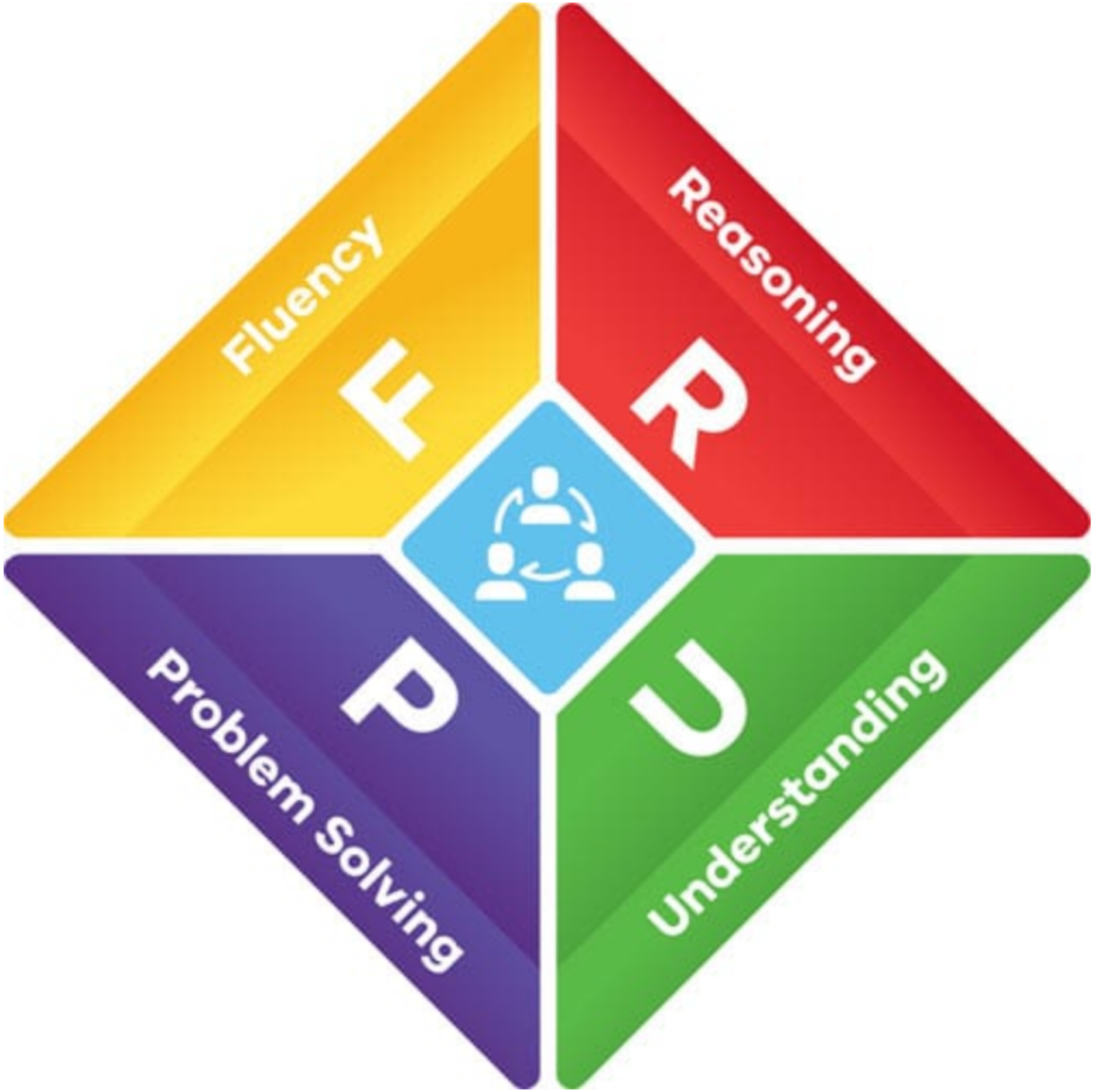
Fluency
F
Reasoning
R
Problem Solving
P
Understanding
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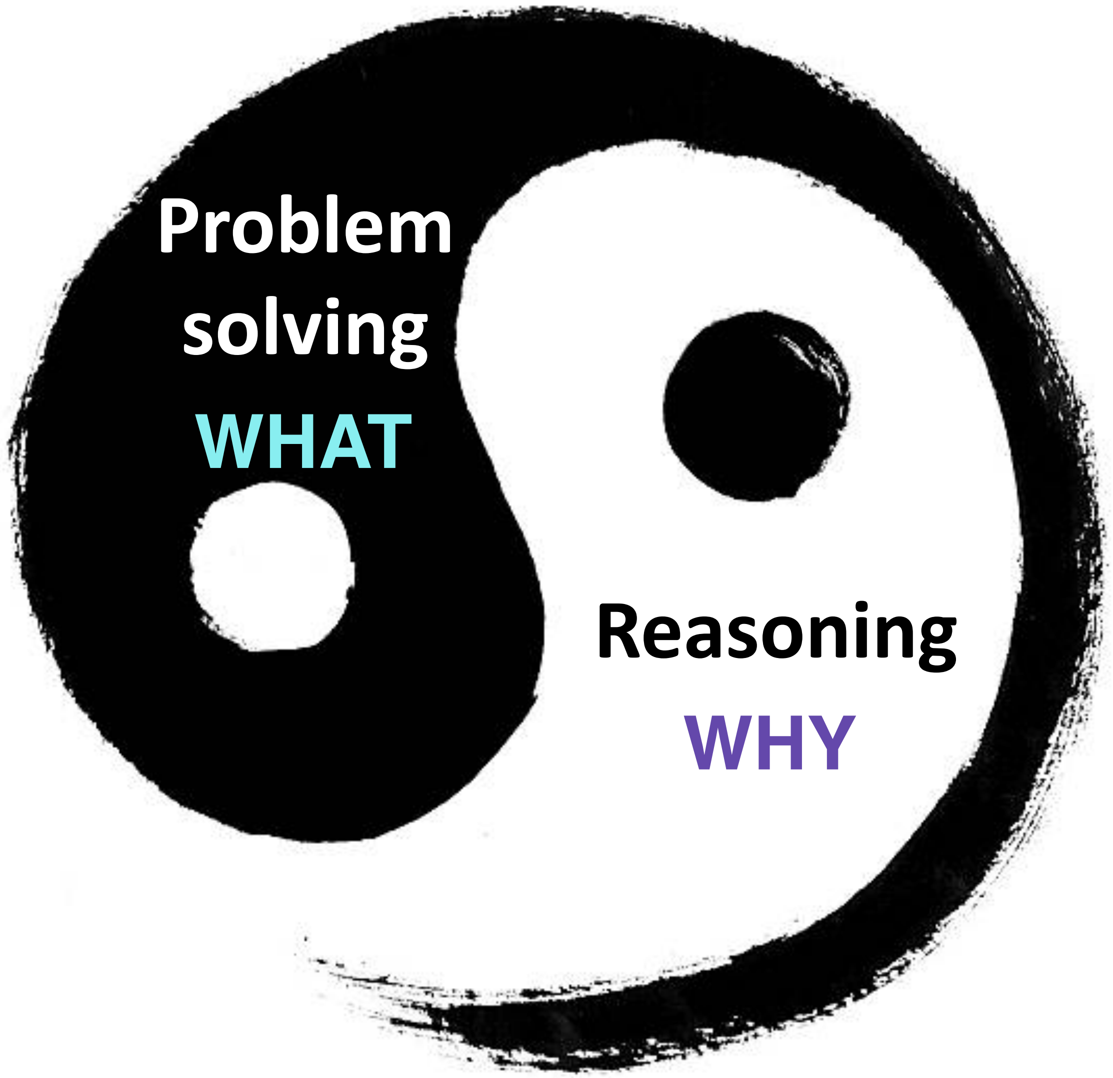

Problem solving
WHAT
Reasoning
WHY

What is Teaching?

Creating a
common experience
to reflect on
and so bring about learning.

Mathematics
is not a
spectator sport

True or false?

$39 \times 46 = 39 \times 45 + 46$

$39 \times 46 = 39 \times 45 + 39$

True or false?

$3 \times 5 = 3 \times 4 + 3$

$39 \times 46 = 39 \times 45 + 39$

$326 \times 18 = 326 \times 17 + 326$

$4.6 \times 3.9 = 4.6 \times 4 - 0.46$

$(x + 2)(3y + 7) = (x + 2)(3y + 5) + (2x + 4)$

Much mathematical reasoning is independent of arithmetical fluency.

True or false?

$3 \times 5 = 3 \times 4 + 3$

$39 \times 46 = 39 \times 45 + 39$

$\mathbf{326 \times 18 = 326 \times 17 + 326}$

$4.6 \times 3.9 = 4.6 \times 4 - 0.46$

$(x + 2)(3y + 7) = (x + 2)(3y + 5) + (2x + 4)$

England’s National Tests for 11-year-olds

- Given $5543 \div 17 = 326$
- Could they use this to show how to answer 18×326

- 77% attempted an answer
- 23% answered it correctly

Research Report DCSF-RR118

Development of Maths Capabilities and Confidence in Primary School

Terezinha Nunes, Peter Bryant, Kathy Sylva and Rossana Barros
Department of Education,
University of Oxford
In collaboration with ALSPAC, University of Bristol

department for children, schools and families

DCSF-RR118

Mathematical reasoning, even more so than children's knowledge of arithmetic, is important for children's later achievement in mathematics.

Nunes et al. DSFC RR-118

Provoking reasoning

Teaching for reasoning

- Cannot directly teach reasoning

- Can set up tasks that encourage reasoning

$5 \times 6 = 30$

$7 \times 8 = 56$

$4 \times 5 = 20$

$6 \times 7 = 42$

$4 \times 5 = 20$

$5 \times 6 = 30$

$6 \times 7 = 42$

$7 \times 8 = 56$

What do you notice?

What do you wonder?

VARIETY	VARIATION
5 x 6 = 30	4 x 5 = 20
7 x 8 = 56	5 x 6 = 30
4 x 5 = 20	6 x 7 = 42
6 x 7 = 42	7 x 8 = 56

Variation

Careful choice and ordering of examples that provoking reasoning not just repetition.

The relationship between examples is as important (if not more so) than the examples on their own.

Variation

Key question:

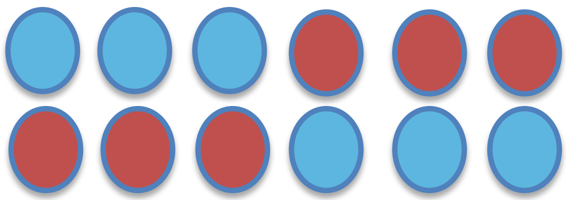
What is the same?

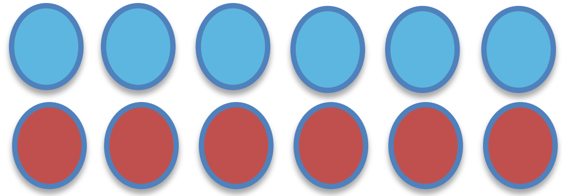
What is different?

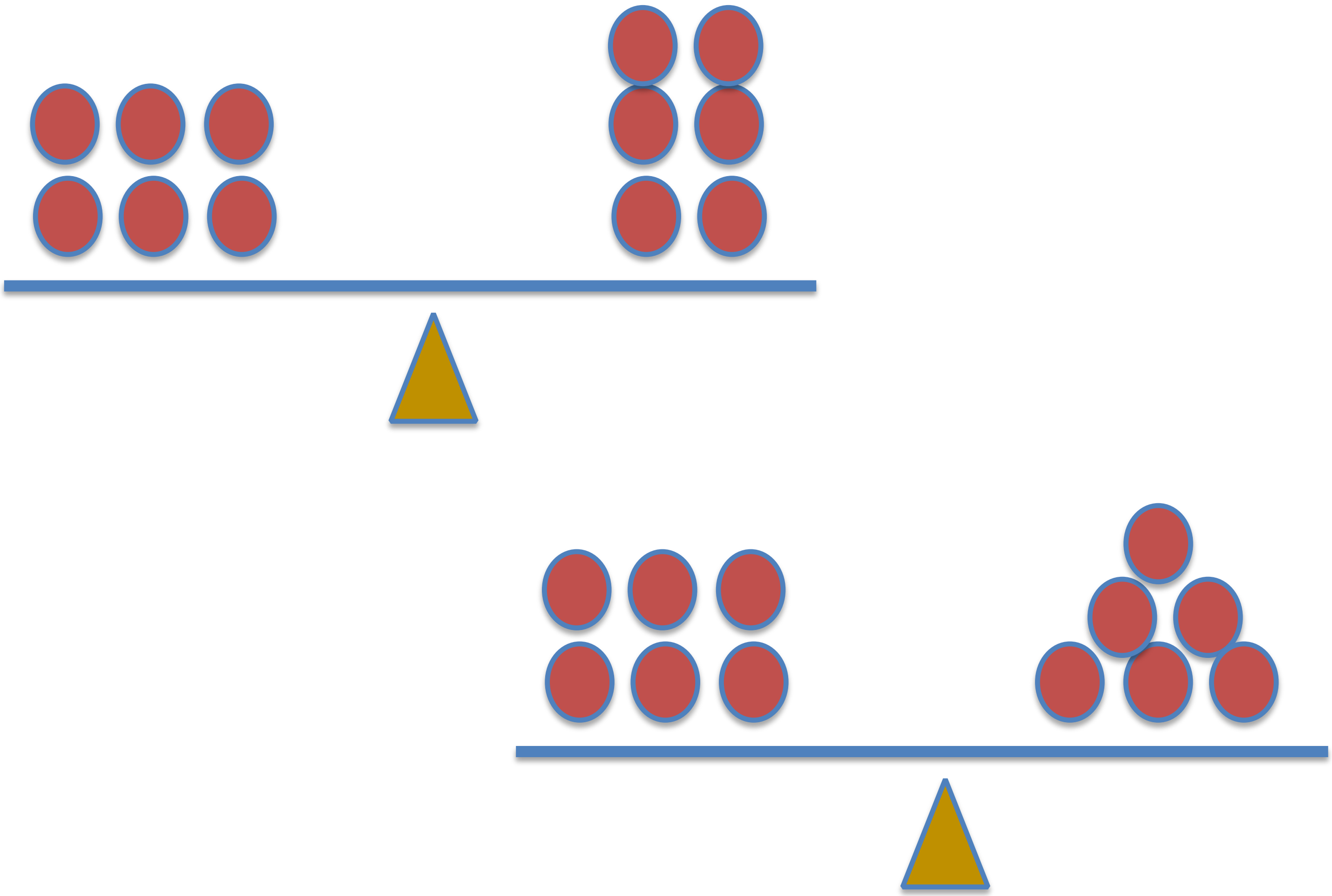
Key to keep in mind

When planning

When directing learners' attention





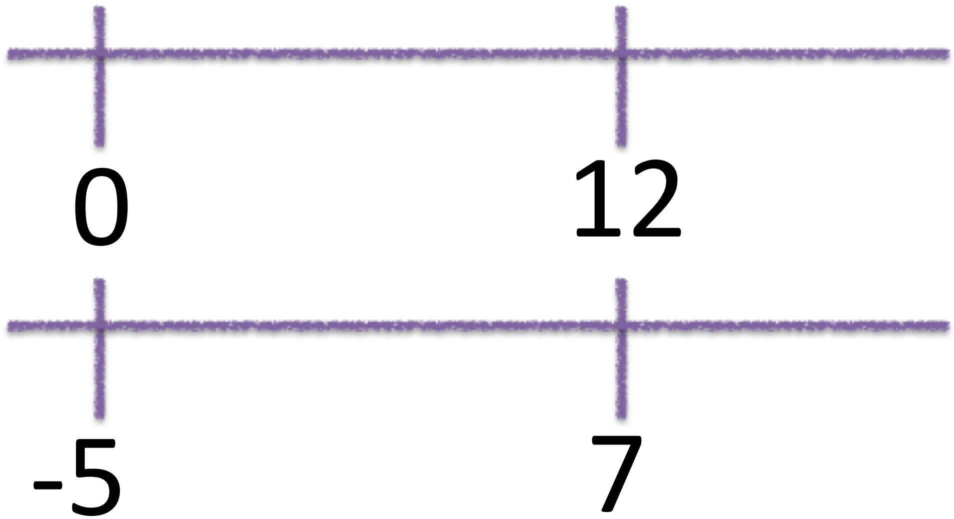


Power of variation

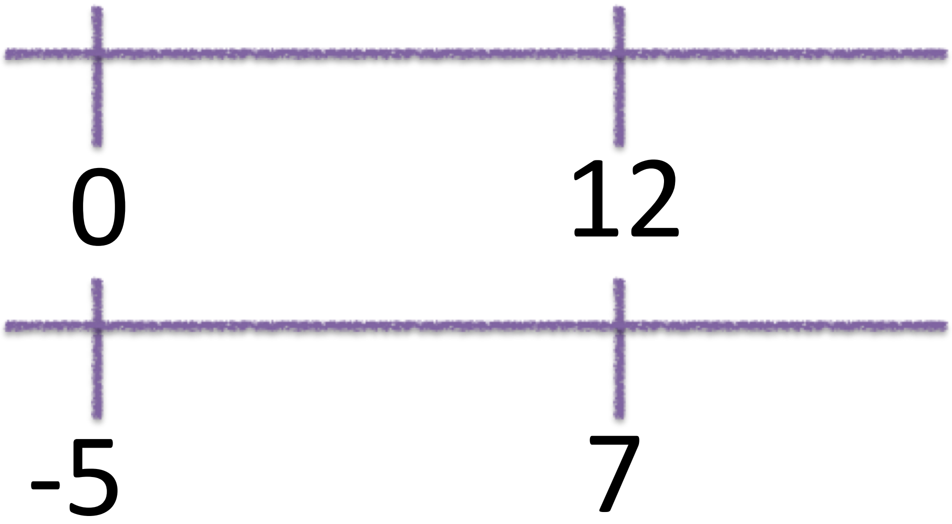
Our conclusions after three years of work in a range of natural settings are that control of dimensions of variation and ranges of change is a powerful design strategy for producing exercises that encourage learners to engage with mathematical structure, to generalize and to conceptualize even when doing apparently mundane questions.

(Watson and Mason, 2006)

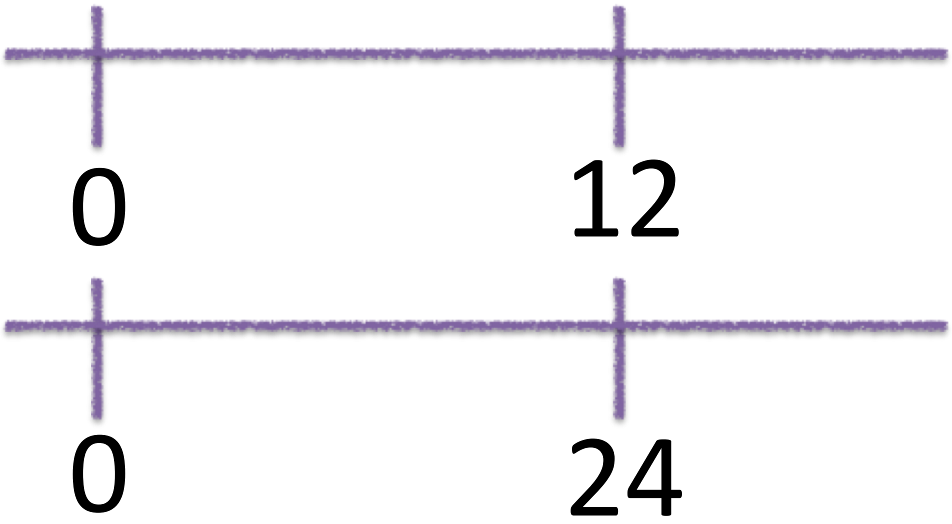
Same and different?



Variation



Variation



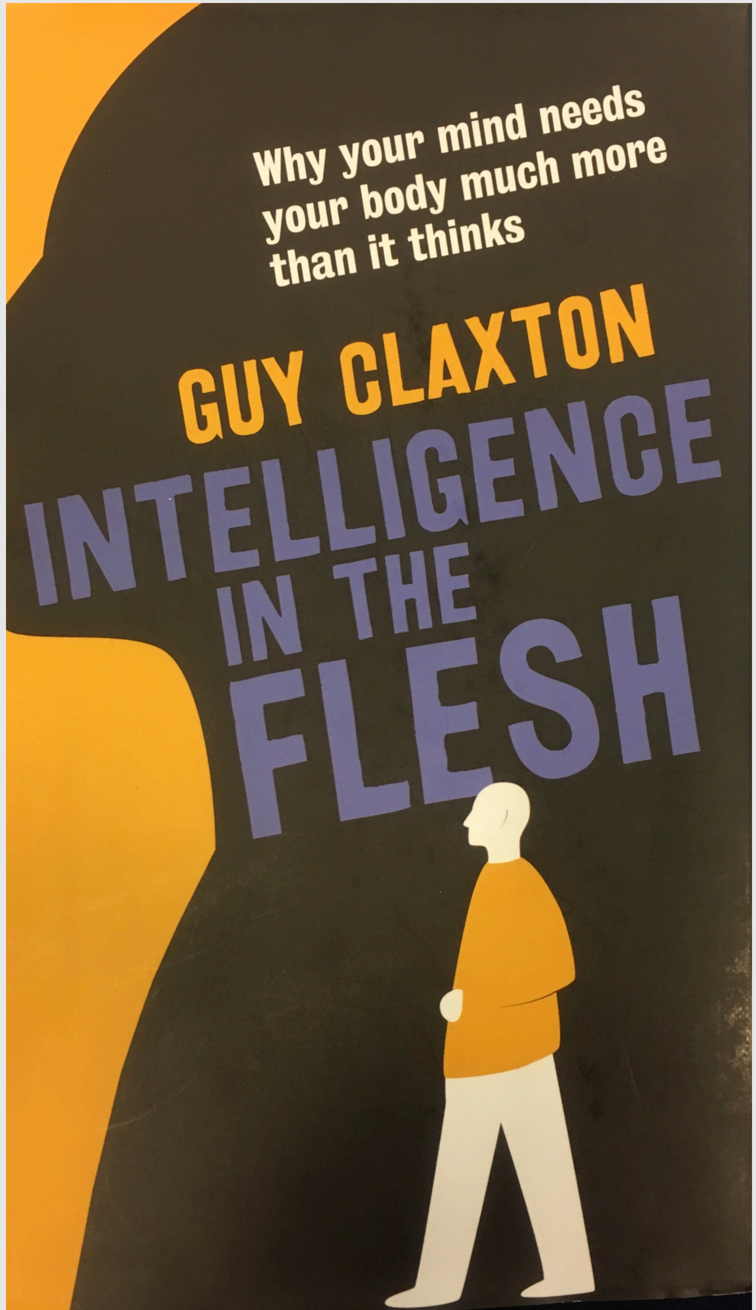
CPA

- Concrete
- Pictorial
- Abstract

CPA

- NOT stages to move through
- Repertoire of ways of working

We need to select and work with representations in ways that encourage insight not just answers


Why your mind needs
your body much more
than it thinks
GUY CLAXTON
INTELLIGENCE
IN THE
FLESH

One of the major errors of twentieth-century psychology was to suppose that there are childish ways of knowing which are outgrown, and ought to be transcended, as one grows up. The childish ones are the bodily ones, and are to do with concrete action and experience. The grown-up ones are abstract, logical and propositional. But it is a Cartesian mistake to think that, once you have mastered logic you don't need the body anymore. … We should think of the developing mind as a tree that grows new branches, not as a spaceship whose booster rockets fall away for ever one they have done their job and are spent.

Claxton, 2015, p 165-66

How we reason mathematically does not change substantially over the years.

What we reason about mathematically can change substantially over the years.

Relational

Sue and Julie are cycling equally fast around a track.

Sue started first.

When Sue had cycled 9 laps,
Julie had cycled 3 laps.

When Julie completed 15 laps, how many laps had Sue cycled?

Sue	9	45
Julie	3	15

Sue	9	21
Julie	3	15

The numbers in a problem can never reveal the relationship between them.

Analogical

A mathematical answer to 27 divided by 6 is 4 remainder 3.

Make up a real world problem that involves 27 divided by 6 but where it makes sense to round the answer up to 5.

Expert problem solvers seek analogical problems rather than treat each problem anew.

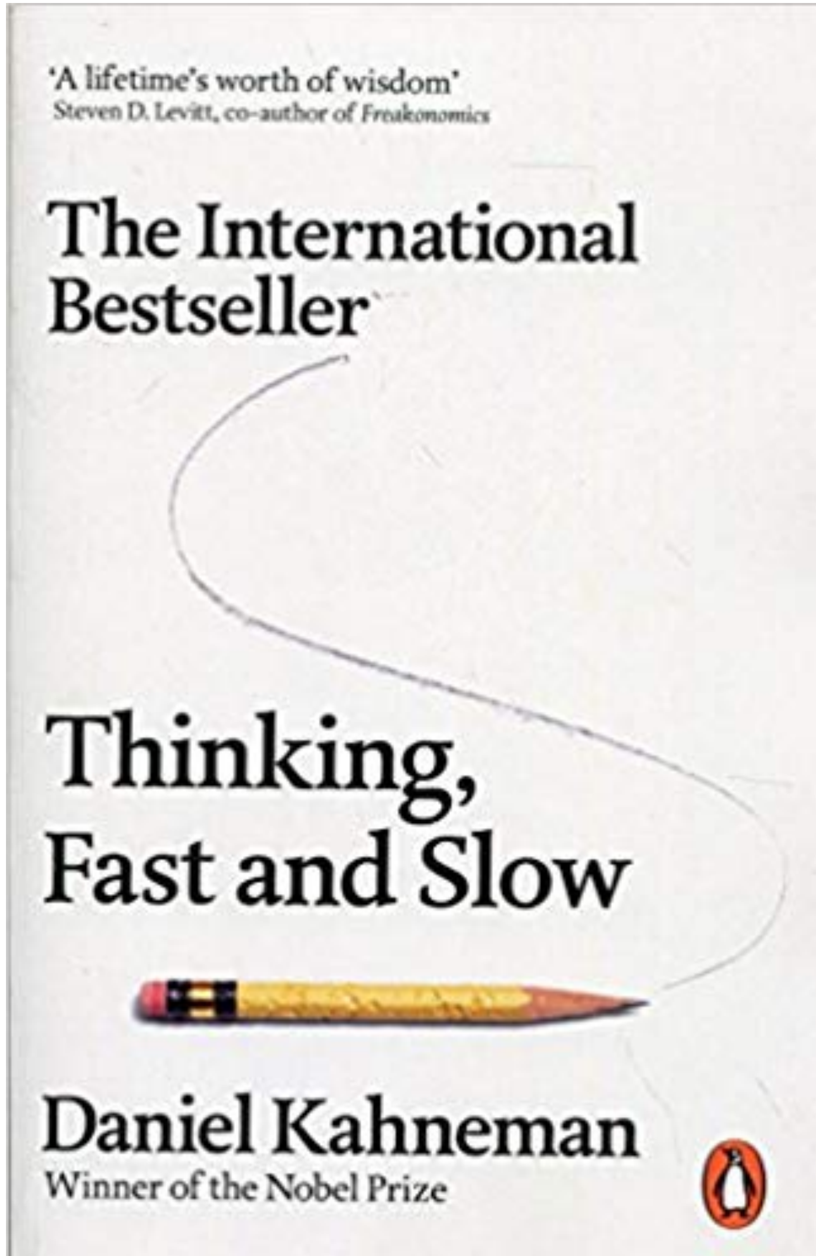
Reasoning – Why is it difficult to enact?

Toast and butter

At my local café, a slice of toast and a pat of butter cost £1.10.

The toast costs £1.00 more than the butter.

How much is the butter alone?


'A lifetime's worth of wisdom'
Steven D. Levitt, co-author of Freakonomics
The International Bestseller
Thinking, Fast and Slow
Daniel Kahneman
Winner of the Nobel Prize

Moving from Fast to Slow thinking …

… results in a slight, but noticeable,
feeling of depression

Teaching demands

Requires making sense of learners' methods

AND

aligning these with accepted mathematical knowledge.

TALK central to both of these

Reasoning – A mathematical habit of mind

Mathematical power

Mathematical power is best described by a set of *habits of mind*. People with mathematical power perform thought experiments; tinker with real and imagined machines; invent things; look for invariants (patterns); make reasonable conjectures; describe things both casually and formally (and play other language games); think about methods, strategies, algorithms, and processes; visualize things (even when the "things" are not inherently visual); seek to explain *why* things are as they seem them; and argue passionately about intellectual phenomena.

Cuoco, Goldenberg & Mark (1996)

References

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