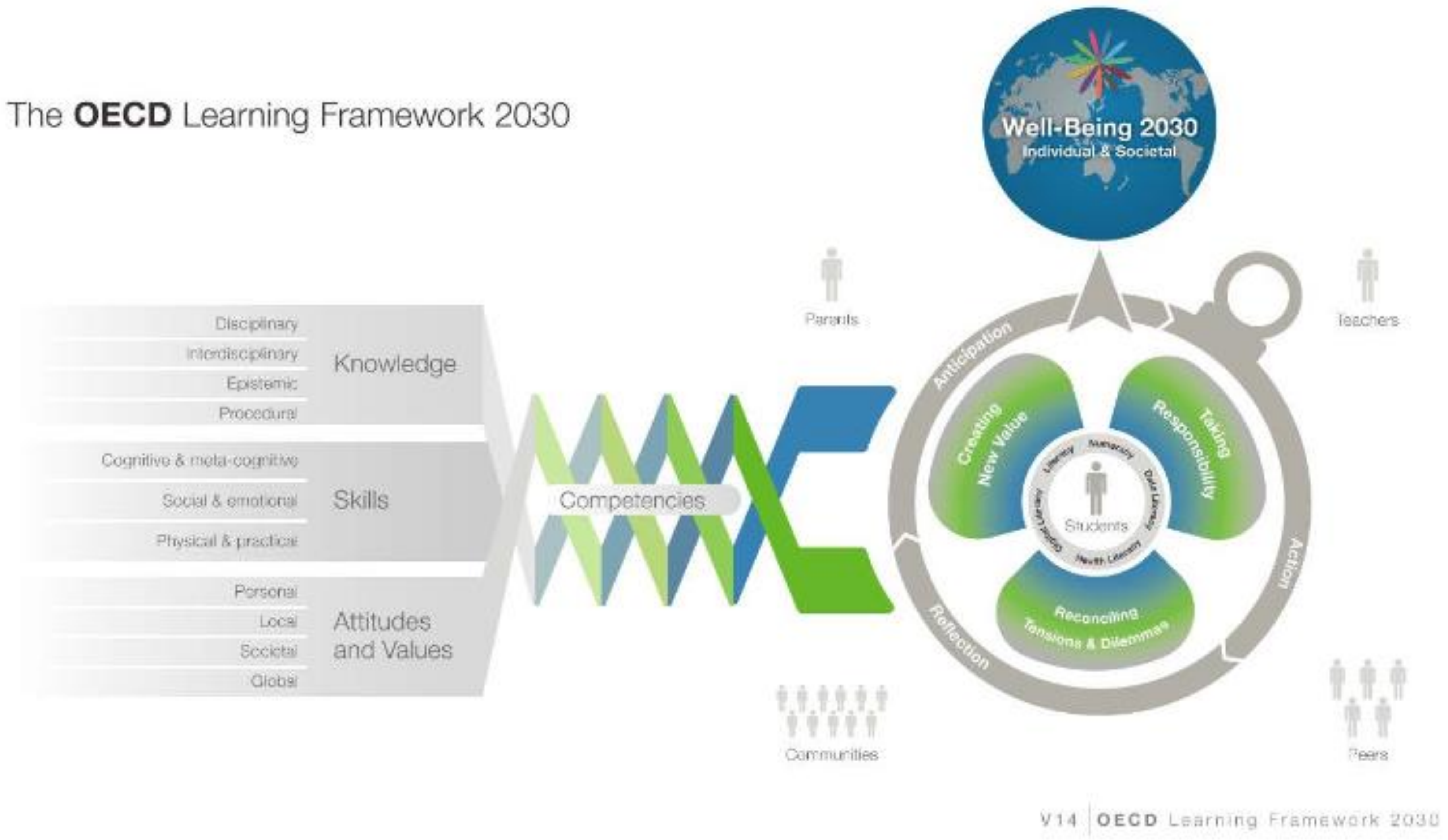
Inquiry based, student centred pedagogies can improve the mathematics learning of all students

Peter Sullivan

An overall approach to teaching that may be different from conventional approaches


The OECD Learning Framework 2030
Well-Being 2030
Individual & Societal
Disciplinary
Interdisciplinary
Epistemic
Procedural
Knowledge
Cognitive & meta-cognitive
Social & emotional
Physical & practical
Skills
Personal
Local
Societal
Global
Attitudes and Values
Competencies
Parents
Teachers
Communities
Peers
Anticipation
Action
Reflection
Creating New Value
Taking Responsibility
Reconciling Tensions & Dilemmas
Literacy
Numeracy
Data Literacy
Health Literacy
Digital Literacy
Students
V14 | OECD Learning Framework 2030

E2030 design principles (with my interpretation)

- Student agency (experience before instruction)
- Rigour (appropriately challenging)
- Focus (depth not breadth, do things well, not quickly)
- Coherence (sequence)
- Alignment (not only the curriculum but also assessment)
- Transferability (bit the same bit different)
- Choice (open middle or open ended)

Of course we do not want to initiate or exacerbate anxiety or fear, but which of these two approaches results in a sense of failure and which ends with students feeling they have learned?

- Moving from simple to complex
- Moving from confusion to clarity

Memory is the residue of thought

From Daniel Willingham, professor of psychology at the University of Virginia.

- Students remember what they have been thinking about, so if you make the learning too easy, students don't have to work hard to make sense of what they are learning and, as a result, forget it quickly.

… and high level student thinking is connected to student learning …

- National Council of Teachers of Mathematics (NCTM) (2014) noted:
  - Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature. (p. 17)

Some rules that guide my pedagogies

- Experience before instruction
- Give students time
- Do not tell them where the eggs are
- Let students read the question for themselves
- Start at the level that (nearly) all students do not know what to do
- Give them something to talk about
- It is at the end of the lesson we want them to know, not the start
- From active teacher and passive students to …
- Shut up

Cognitive activation tasks require students (prior to instruction) to

- plan their approach, especially sequencing more than one step;
- process multiple pieces of information, with an expectation that they make connections between those pieces, and see concepts in new ways;
- choose their own strategies, goals, and level of accessing the task;
- spend time on the task and record their thinking;
- explain their strategies and justify their thinking to the teacher and other students.

What are the characteristics of challenge?

- A task is challenging when students do not know how to solve the task and work on the task prior to teacher instruction.

Other characteristics of such tasks are that they:

- build on what students already know;
- take time;
- are engaging for students in that they are interested in, and see value persisting with a task;
- focus on important aspects of mathematics (hopefully as identified or implied in relevant curriculum documents);
- are simply posed using a relatable narrative;
- foster connections within mathematics and across domains; and
- can be undertaken when there is more than one correct answer and/or more than one solution pathway.

What score do you give yourself?

1 = Always give low achieving students tasks at which they are likely to be successful even if a long way below and quite different from what the bulk of the class are doing, and if they are stuck you tell them how to do it

10 = Always expect low achieving students to participate in the class activities, maybe slightly adapted, even if at a different level of expectation, giving minimal direct instruction

A Multiplication Sequence
Years 3/4

The rationale for the sequence

- The sequence provides examples of each form of multiplication, hopefully moving students along the path from additive to multiplicative thinking
- This sequence covers Years 3 to 4 but can be adjusted to other levels.

The big ideas

- Problems involving groups, arrays, times as many, for each (and ratios) require multiplicative thinking, and it is the commonality between the types that we want students to see
- Division can be done as reverse multiplication
- The first step is for students to recognise a composite unit i.e. a group of four is one four rather than four ones.

  Knowing this is an important step. It shifts student thinking from counting items within a group by ones to recognising that if they have more than one group of this size, it is the same. E.g., if they have three group of fours knowing that each group is one group of four and they have three of them.

Big ideas (continued)

- A big step is moving from repeated addition to multiplicative thinking (that is, 10×7 is easier than $7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7+ 7$)
- If we count the top row in an array, we do not need to count (one by one) the next row
- Recognising the relationship between the divisor and the quotient when exploring division
- The intention is to move students along the trajectory: concrete; pictorial; abstract.

The Australian Curriculum

- Year 2
  - Recognise and represent multiplication as repeated addition, groups and arrays (ACMNA031)
  - Recognise and represent division as grouping into equal sets and solve simple problems using these representations (ACMNA032)
- Year 3
  - Represent and solve problems involving multiplication using efficient mental and written strategies and appropriate digital technologies (ACMNA057)
- Year 4
  - Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder (ACMNA076)
- Year 5
  - Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies (ACMNA100)
  - Use efficient mental and written strategies and apply appropriate digital technologies to solve problems (ACMNA291)

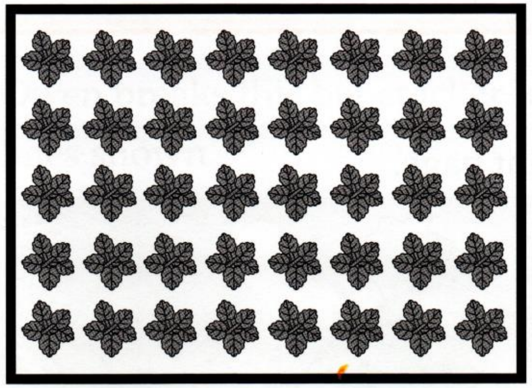
Necessary mathematical language

- Groups of, arrays (rows, columns), for each, every, per, times as many (twice as many, three times as many), equal groups

29 Avril buys some plants for her garden.

The plants are for sale at 5 for $2.00.

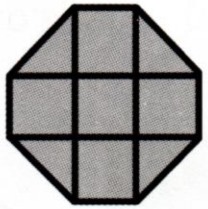
She buys enough of them to plant in her garden as shown.

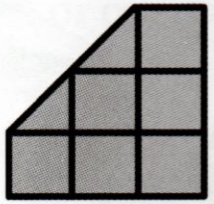


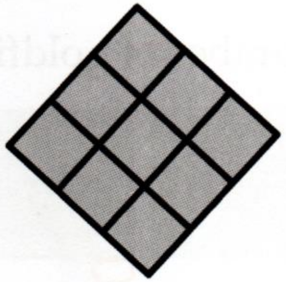
What is the total cost of the plants Avril buys?

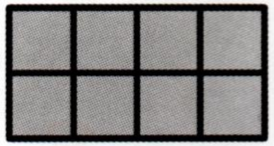
$8	$16	$40	$80
⬭	⬭	⬭	⬭

30 Select the **two** shapes that have the same area.









Nationally

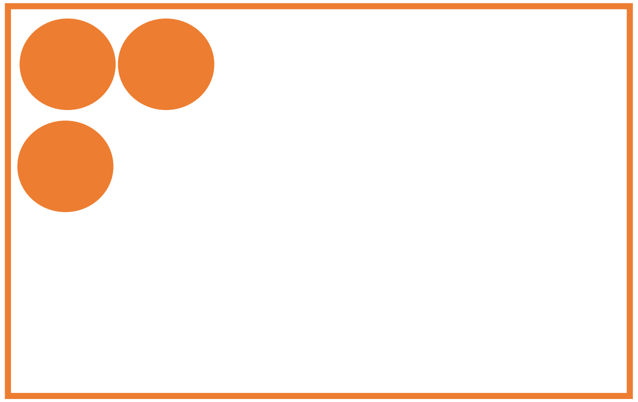
- Q 29 55%
- Q 30 26%

Suggestion 2 Arrays

- Learning focus
  - We can calculate how many objects altogether are in an array
- Pedagogical considerations
  - Arrays are perhaps the best model of multiplication
  - The goal is to move students beyond count all, and then to move them beyond repeated addition (although that is the first step)
  - Use “rows” and “columns” when describing arrays
  - A number of questions can prompt division equations as well as multiplication
  - In the first task, have counters and “boxes” that students can use.
  - Some questions ask students to find two solution strategies. This helps them to think flexibly.

How many chocolates?

- I had a full box of chocolates, but someone ate some of the chocolates.
- The box now looks like this.



- How can I work out the number of chocolates I started with?

ENABLING PROMPT

Make an array with 16 counters

EXTENDING PROMPT

If the box was twice as long and twice as wide,

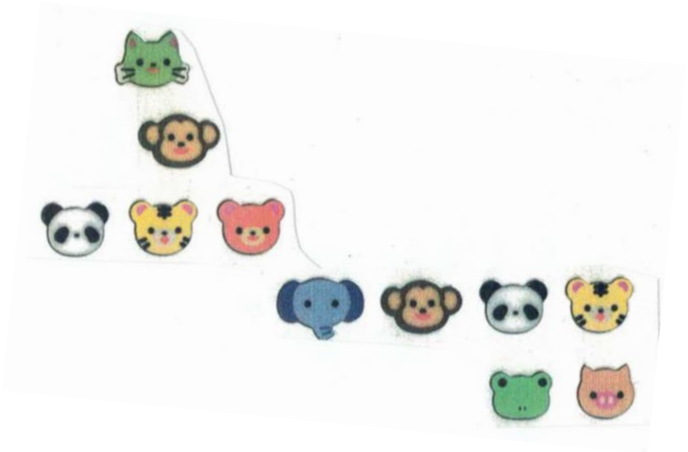
how many chocolates would you need to fill it?

Mel’s stickers

Mel collects stickers and she buys sheets of stickers in arrays.

Her younger sister Jess helped herself to some of the sheets!

How many stickers were on each sheet before Jess got to them?



Piggy Bank

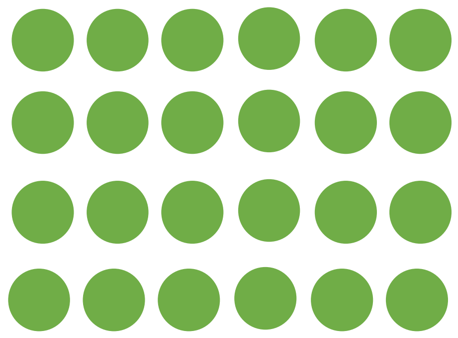
I emptied my piggy bank onto the table and arranged the $1 coins in an array without gaps. How much money is on the table?

Work this out two different ways.

Write some equations

This array has 4 row with 6 counters in each row.

Write down as many equations from this array as you can.



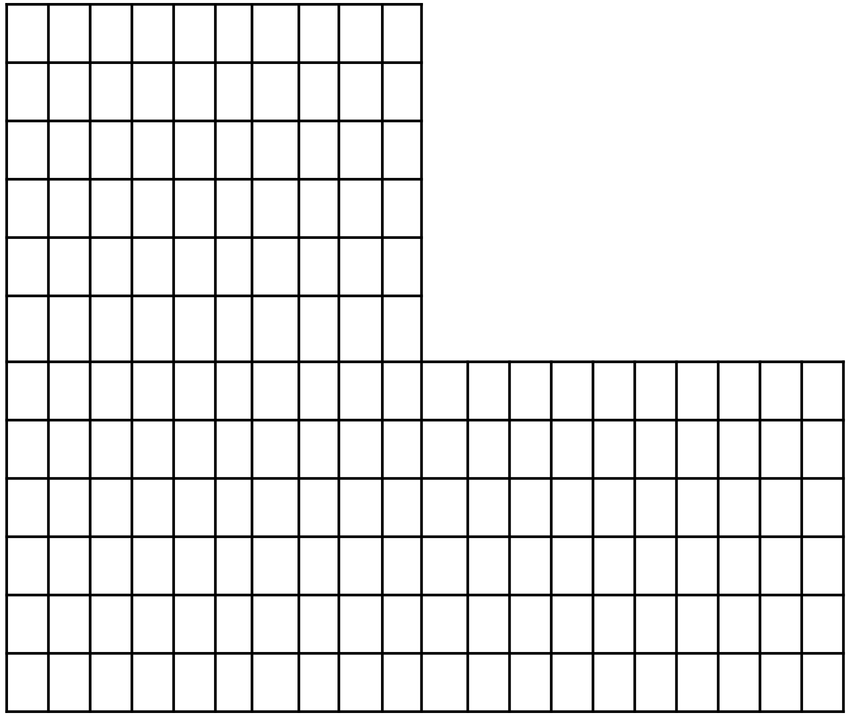
48 tiles

Imagine you are going to make an array with 48 tiles. What might it look like? Close your eyes and picture it. Open your eyes and make what you saw using the tiles.

How many different arrays can you make?

An ‘L’ shape

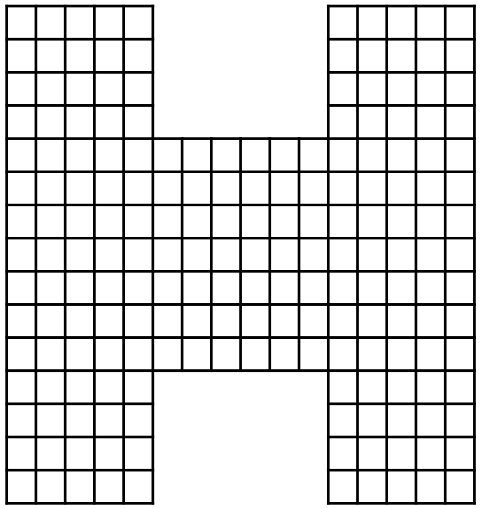
How many squares have been used to make this shape?



Find the answer two different ways.

An ‘H’ shape

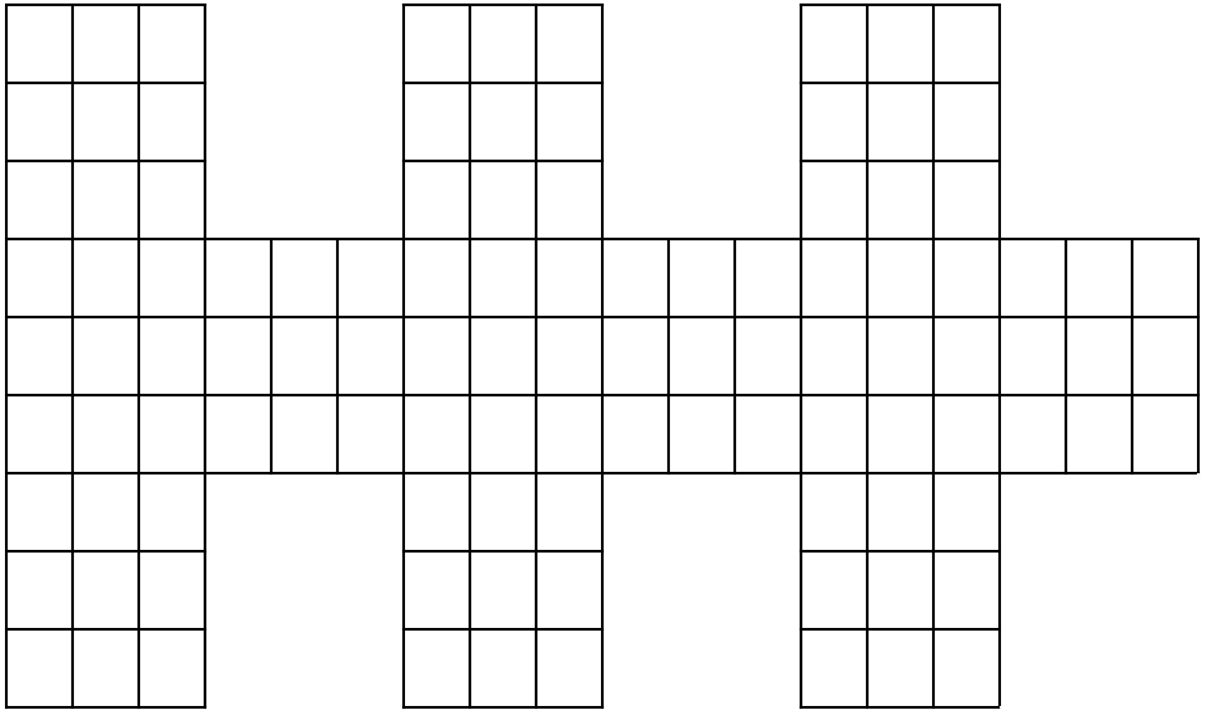
How many squares have been used to make this shape?



Find the answer two different ways.

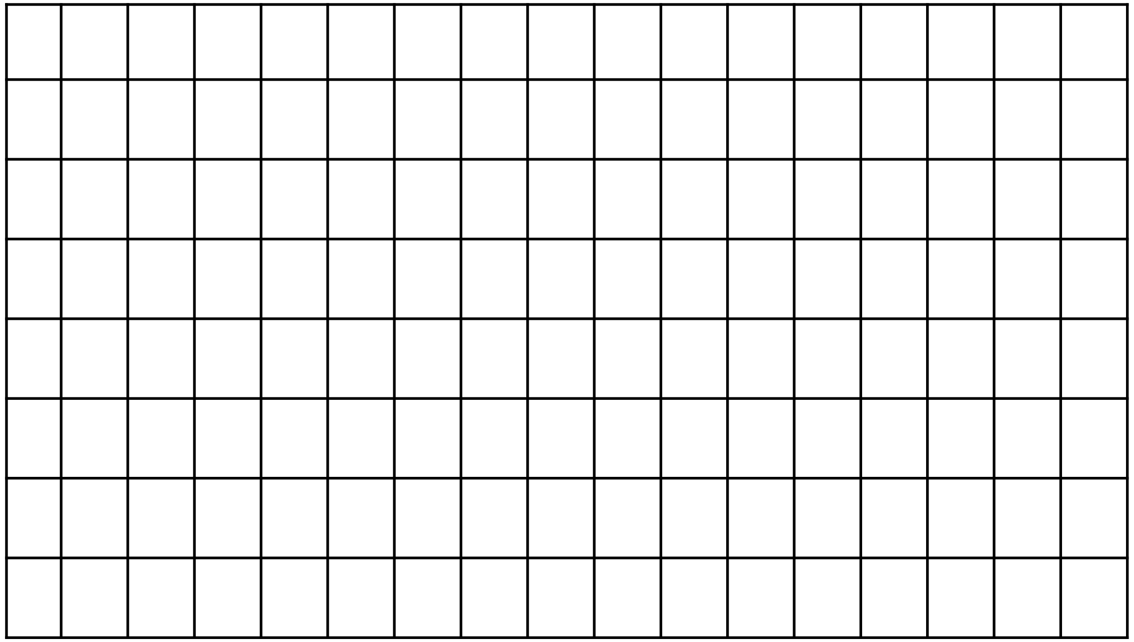
Arrays in patterns

What is an effective way of working out how many squares are in this diagram?



A T shape

What might a shape like a T made from 100 squares look like?



In what ways are those tasks a “sequence”?

The following slides are some arguments for the value of sequences

1. Seeing “light bulbs”

- Sequences of challenging tasks prompt “light bulb” moments.
- Many teachers claim to get satisfaction from seeing “light bulbs” in their students. But there are no light bulbs if students are told what to do.
- Students need to work on tasks that are challenging for them, and progressively see meaning by experiencing connected tasks with success developing progressively.

2. Reducing the sense of risk

- Sequences can reduce the sense of risk experienced by some students.
- Many teachers report that some students do not embrace challenges possibly fearing the risk of failure.
- One of the goals of sequences is for students to see that, even if they cannot do the current task, there is a similar task coming and that they can learn by engagement in the current task, even if not successful.

3. Thinking about the bigger picture

- Sequences can help students see the “bigger picture”.
- One of the disadvantages of conventional approaches to mathematics and numeracy is that mathematics is broken into sets of micro skills rather than coherent connections.
- Sequences may help students see connections by making the progression of learning more obvious.
- Note the implications for having larger learning intentions than just a single lesson

4. What it is and what it is not

- Students will understand both what the concept is and what it is not.
- Concepts are learned as much by what they are not as from what they are.
- So tasks that are carefully varied within sequences can emphasise what the central ideas are (and what they are not).

There are two types of variation …

… after the students solve a problem, you might pose a similar problem or inquiry in which you keep the context constant but change the mathematical focus in some way.

… after the students solve a problem, you might pose a similar problem in which you change the context but keep the mathematical focus of the questions substantially the same.

5. Teacher learning

- By proposing sequences with related professional learning, teachers can experience not only the notion of sequence but the ways that sequences enhance learning opportunities for students.