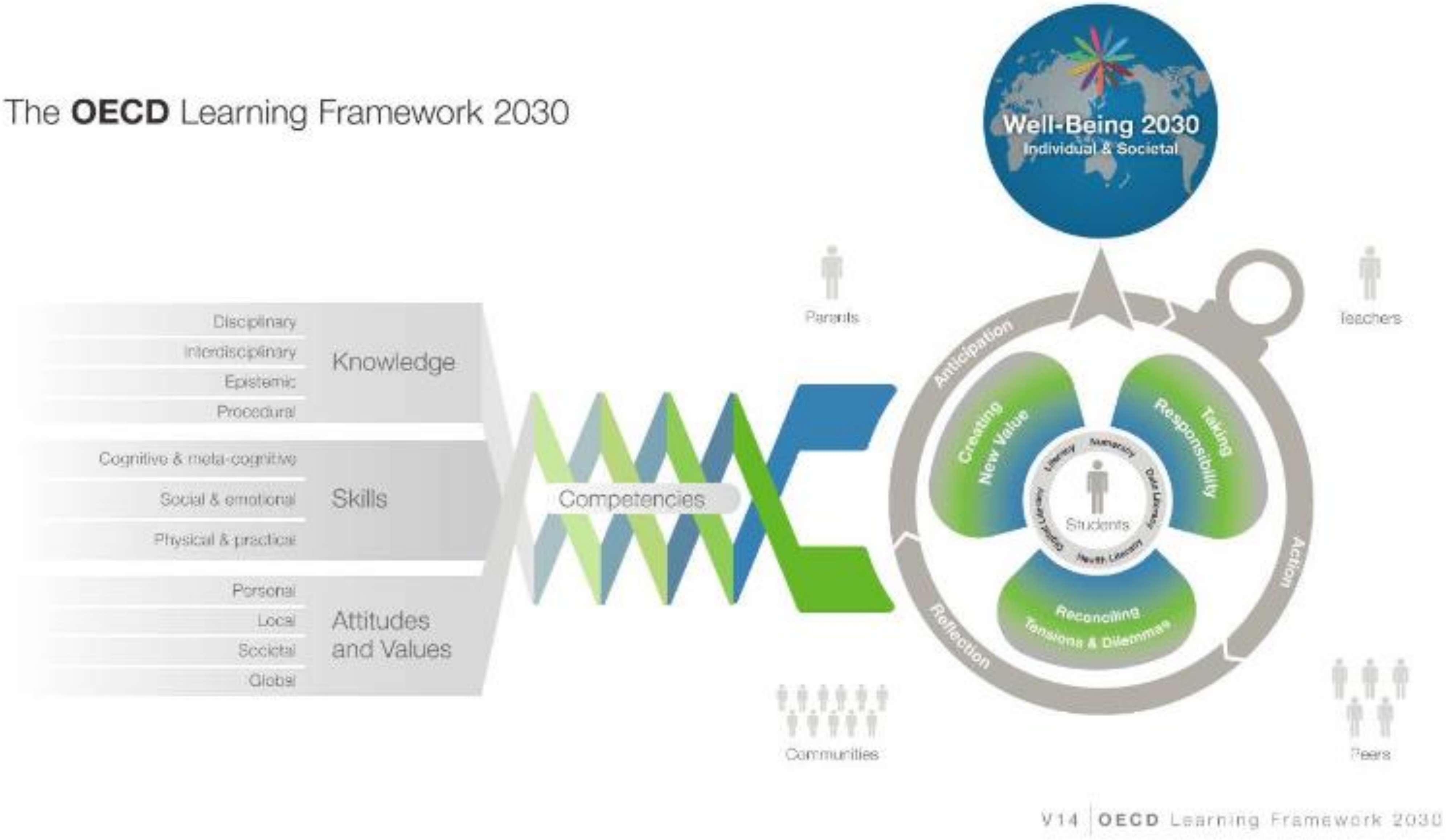
Inquiry based, student centred pedagogies can improve the mathematics learning of all students

Peter Sullivan

An overall approach to teaching that may be different from conventional approaches


The OECD Learning Framework 2030
Well-Being 2030
Individual & Societal
Disciplinary
Interdisciplinary
Epistemic
Procedural
Knowledge
Cognitive & meta-cognitive
Social & emotional
Physical & practical
Skills
Personal
Local
Societal
Global
Attitudes and Values
Competencies
Parents
Teachers
Communities
Peers
Anticipation
Action
Reflection
Creating New Value
Taking Responsibility
Reconciling Tensions & Dilemmas
Students
V14 OECD Learning Framework 2030

E2030 design principles (with my interpretation)

- Student agency (experience before instruction)
- Rigour (appropriately challenging)
- Focus (depth not breadth, do things well, not quickly)
- Coherence (sequence)
- Alignment (not only the curriculum but also assessment)
- Transferability (bit the same bit different)
- Choice (open middle or open ended)

Of course we do not want to initiate or exacerbate anxiety or fear, but which of these two approaches results in a sense of failure and which ends with students feeling they have learned?

- Moving from simple to complex
- Moving from confusion to clarity

Memory is the residue of thought

From Daniel Willingham, professor of psychology at the University of Virginia.

- Students remember what they have been thinking about, so if you make the learning too easy, students don't have to work hard to make sense of what they are learning and, as a result, forget it quickly.

… and high level student thinking is connected to student learning …

- National Council of Teachers of Mathematics (NCTM) (2014) noted:
  - Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature. (p. 17)

Some rules that guide my pedagogies

- Experience before instruction
- Give students time
- Do not tell them where the eggs are
- Let students read the question for themselves
- Start at the level that (nearly) all students do not know what to do
- Give them something to talk about
- It is at the end of the lesson we want them to know, not the start
- From active teacher and passive students to …
- Shut up

Cognitive activation tasks require students (prior to instruction) to

- plan their approach, especially sequencing more than one step;
- process multiple pieces of information, with an expectation that they make connections between those pieces, and see concepts in new ways;
- choose their own strategies, goals, and level of accessing the task;
- spend time on the task and record their thinking;
- explain their strategies and justify their thinking to the teacher and other students.

What are the characteristics of challenge?

- A task is challenging when students do not know how to solve the task and work on the task prior to teacher instruction.

Other characteristics of such tasks are that they:

- build on what students already know;
- take time;
- are engaging for students in that they are interested in, and see value persisting with a task;
- focus on important aspects of mathematics (hopefully as identified or implied in relevant curriculum documents);
- are simply posed using a relatable narrative;
- foster connections within mathematics and across domains; and
- can be undertaken when there is more than one correct answer and/or more than one solution pathway.

What score do you give yourself?

1 = Always give low achieving students tasks at which they are likely to be successful even if a long way below and quite different from what the bulk of the class are doing, and if they are stuck you tell them how to do it

10 = Always expect low achieving students to participate in the class activities, maybe slightly adapted, even if at a different level of expectation, giving minimal direct instruction

The suggestions

- Suggestion 1: Using equations to describe the relationships between variables
- Suggestion 2: Writing equations
- Suggestion 3: Finding possible values of variables in equations
- Suggestion 4: Using equations to describe situations and relationships
- Suggestion 5: Drawing linear equations on the Cartesian Plane

The suggestions

- Suggestion 1: Using equations to describe the relationships between variables
- Suggestion 2: Writing equations
- Suggestion 3: Finding possible values of variables in equations
- Suggestion 4: Using equations to describe situations and relationships
- Suggestion 5: Drawing linear equations on the Cartesian Plane

Suggestion 2: Writing equations

Learning focus

- Equations are useful for solving word problems.

Pedagogical considerations

- Students are asked to give two methods of solutions to the following: they can use an intuitive approach and an algebraic one. The intention is that they see the connections between the approaches.
- It can help to use linear visual representations (such as number lines or the Singapore bar method as the non algebraic method)
- Both the Singapore Bar method and algebra help the students see the importance of reading the question
- The emphasis is on students recording their solutions and explaining their thinking.
- They also learn to read the question carefully.
- Even though some are amenable to writing simultaneous equations, do not show this (as it comes at a later stage)

National: Year 5 58%

19 Peter has 68 stamps. Laura has 52 stamps.

Shade one bubble.

How many stamps should Peter give to Laura so they both have the same number of stamps?

8	16	60	120
○	○	○	○

Year 9 State 36%

18 The equation $2y + 1 = x$ is rearranged to have y as the subject.

Which of these correctly gives y as the subject?

$y = \dfrac{x-1}{2}$	$y = \dfrac{x+1}{2}$	$y = 2x - 1$	$y = 2x + 1$
○	○	○	○

Example 4

Solve:

a $3x + 7 = 2x + 13$ **b** $5a - 21 = 14 - 2a$

Solution

a
$$3x + 7 = 2x + 13$$
$$3x + 7 - 2x = 2x + 13 - 2x \quad \text{(Subtract } 2x \text{ from both sides.)}$$
$$x + 7 = 13$$
$$x = 6 \quad \text{(Subtract 7 from both sides.)}$$

b
$$5a - 21 = 14 - 2a$$
$$7a - 21 = 14 \quad \text{(Add } 2a \text{ to both sides.)}$$
$$7a = 35 \quad \text{(Add 21 to both sides.)}$$
$$a = 5 \quad \text{(Divide both sides by 7.)}$$

Exercise 5B

Example 3

1 Solve these equations.

a $a + 2 = 5$ **b** $b + 7 = 19$ **c** $c - 6 = 11$

d $d - 15 = 3$ **e** $2a = 6$ **f** $3b = 9$

g $5c = 35$ **h** $6d = 42$ **i** $-3m = 6$

j $-2n = 8$ **k** $8p = -24$ **l** $9q = -27$

m $b + 7 = 29\frac{1}{2}$ **n** $a + 3 = 15\frac{1}{4}$ **o** $3a = \frac{2}{3}$

p $x - 6 = 5\frac{1}{3}$ **q** $-4y = \frac{8}{9}$ **r** $2x = -\frac{5}{6}$

2 Solve these equations.

a $2a + 5 = 7$ **b** $3b + 4 = 19$ **c** $3c - 1 = 20$

d $5d - 7 = 23$ **e** $3e - 2 = 16$ **f** $4f - 3 = 13$

g $3g + 17 = 5$ **h** $5h + 21 = 11$ **i** $6a + 17 = -1$

j $4a + 23 = -9$ **k** $3a - 16 = -31$ **l** $7b - 17 = -66$

m $2b + 5 = 7\frac{1}{2}$ **n** $2x + 11 = 7\frac{1}{4}$ **o** $2x - 13 = 5\frac{2}{3}$

p $3a + 4 = -8\frac{1}{2}$ **q** $4m - 9 = -13\frac{4}{5}$ **r** $-2b + 4 = 9\frac{1}{4}$

3 Solve these equations.

a $2 - 3a = 8$ **b** $3 - 4b = 15$ **c** $5 - 2c = -13$

d $3 - 5d = -22$ **e** $-6 - 7e = 15$ **f** $-4 - 3f = 14$

Example 4

4 Solve these equations for x and check your answers.

a $5x + 5 = 3x + 1$ **b** $7x + 15 = 2x + 20$ **c** $8x + 5 = 4x + 21$

d $9x - 7 = 7x + 3$ **e** $5x - 6 = x - 2$ **f** $4x + 7 = x - 2$

g $3x + 1 = 9 - x$ **h** $4x - 3 = 18 - 3x$ **i** $2x - 3 = 7 - x$

j $8 - 3x = 2x - 7$ **k** $3x - 1 = 4 - x$ **l** $2x + 9 = 11 - 3x$

5C Equations with brackets

In this section, we look at solving equations in which brackets are involved. In previous work in this area, you have always expanded the brackets first. In the next two examples, we do not do this. A simpler procedure works.

Example 5

Solve:

a $3(x + 2) = 21$ **b** $2(3 - x) = 12$

Solution

a
$$3(x + 2) = 21$$
$$x + 2 = 7 \quad \text{(Divide both sides by 3.)}$$
$$x = 5 \quad \text{(Subtract 2 from both sides.)}$$

b
$$2(3 - x) = 12$$
$$3 - x = 6 \quad \text{(Divide both sides by 2.)}$$
$$-x = 3 \quad \text{(Subtract 3 from both sides.)}$$
$$x = -3$$

Pen and pencil

A pen and a pencil together cost $7.
The pen costs $6 more than the pencil.
How much does the pencil cost?

Represent your solution using two DIFFERENT methods only one of which uses algebra.

What does the task do?

The proficiencies work together

- The problem solving (working out how to decode and represent the task prior to instruction)
- Along with the reasoning (explaining their thinking verbally or in writing, seeing connections between their methods)
- Was intended to build understanding (of the power of algebra and other representations)
- Which would lead to fluency (at writing equations, reading the question)
- Which could be used for solving further problems
- And so on

An illustrative enabling prompt (that reduces the number of steps)

A drink costs $8.

The drink costs $2 more than the snack.

How much does the snack cost?

An illustrative enabling prompt (that reduces the number of steps)

A drink and a snack costs $10.

The drink costs $2 more than the snack.

How much does the drink cost?

An illustrative extending prompt (that increases the complexity)

A book and a ruler and an eraser costs \$20.

The book and the ruler costs \$16, the ruler and the eraser costs \$12.

What can you say about the cost of the book, the ruler and the eraser?

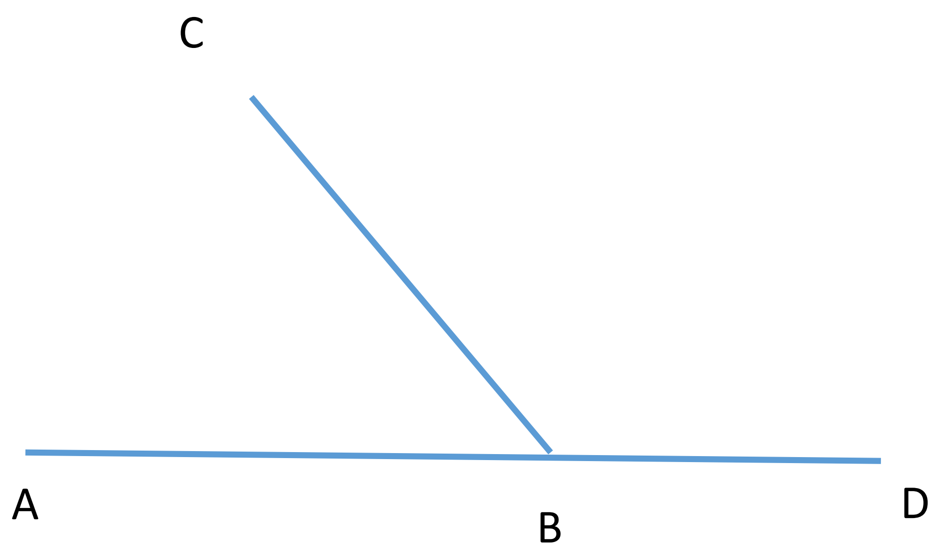
Hat and sunglasses

A hat and a pair of sunglasses together cost $110.
The sunglasses cost $100 more than the hat.
How much does the hat cost?

- Do this in two different ways, only one of which uses algebra.

$\angle ABC$

- $\angle$ ABC is 10 degrees less than $\angle$ DBC.
- What is the value of $\angle$ABC.
- Do this two different ways.



Travelling to Geelong

It takes $3\frac{1}{4}$ hours to ride from Melbourne to Geelong.

It takes $\frac{1}{2}$ hour longer to ride from Melbourne to Werribee than it is does to ride from Werribee to Geelong.

How long does it take to ride from Melbourne to Werribee?

How many adults?

At a party there are 230 people.

There are 100 more adults than children.

How many adults are there at the party?

- Do this in two different ways, only one of which uses algebra.
- Show your solution on a number line.

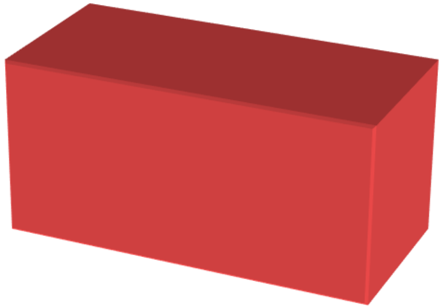
In what ways are those tasks a “sequence”?

The brick

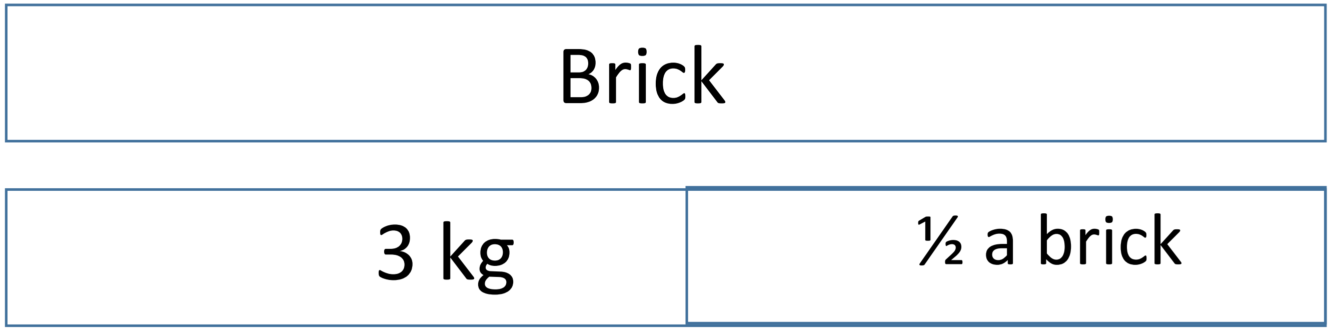
A brick weighs the same as 3 kg plus half a brick.

What does the brick weigh?

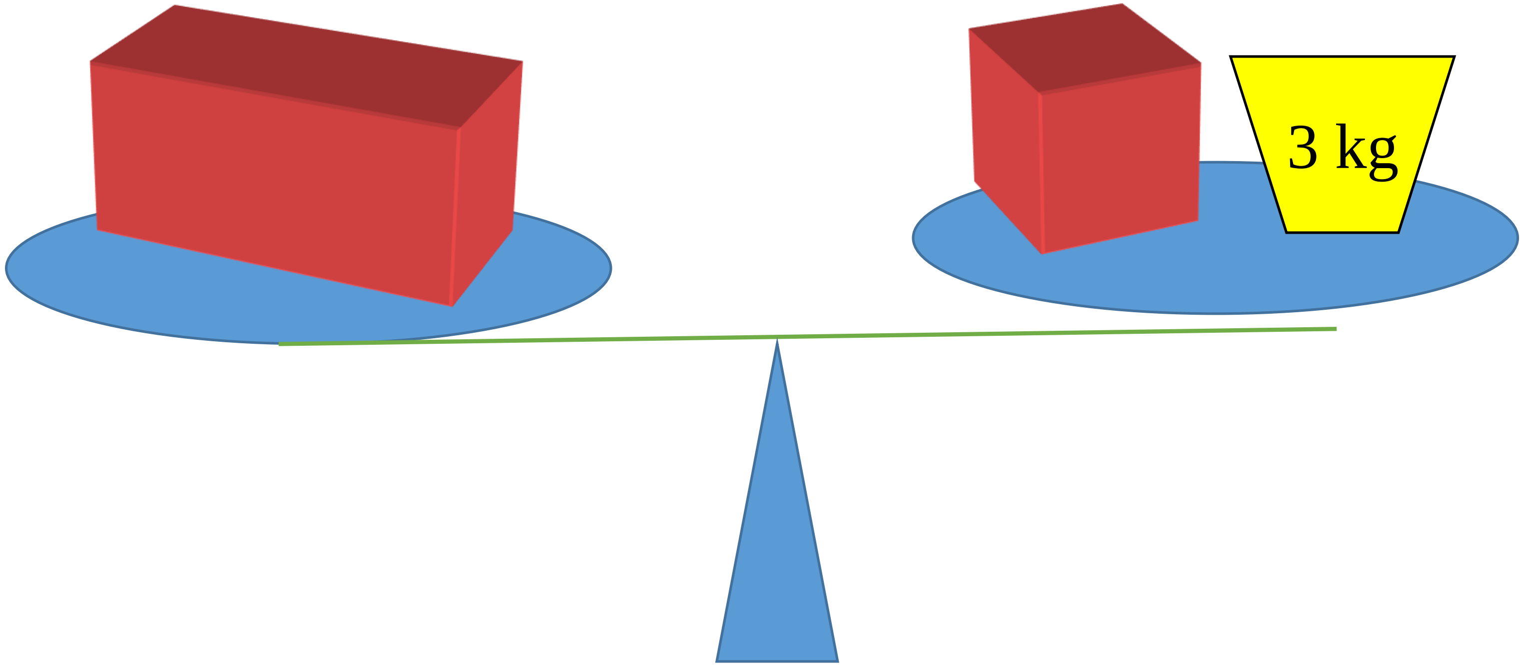
(Represent your solution in two different ways)



A visual method



A brick weighs the same as 3 kg plus half a brick. What does the brick weigh?



An illustrative enabling prompt (that reduces the number of steps)

One half of a brick weighs 10 kg.

How much does the brick weigh?

An illustrative extending prompt (that increases the complexity)

One half of a brick weighs the same as 750 g and one third of a brick.

How much does the brick weigh?

A box

A box weighs the same as 12 kg plus a quarter of a box.

What does the box weigh?

(do this in two different ways)

A pizza

A pizza costs the same as $10 plus a sixth of a pizza.

How much does the pizza cost?

(do this in two different ways)

Mowing the lawn

- It takes me 1 ½ hour less to mow three quarters of the lawn than it takes to mow the whole lawn.
- How long does it take to mow the whole lawn?

In what ways are those tasks a “sequence”?

The following slides are some arguments for the value of sequences

1. Seeing “light bulbs”

- Sequences of challenging tasks prompt “light bulb” moments.
- Many teachers claim to get satisfaction from seeing “light bulbs” in their students. But there are no light bulbs if students are told what to do.
- Students need to work on tasks that are challenging for them, and progressively see meaning by experiencing connected tasks with success developing progressively.

2. Reducing the sense of risk

- Sequences can reduce the sense of risk experienced by some students.
- Many teachers report that some students do not embrace challenges possibly fearing the risk of failure.
- One of the goals of sequences is for students to see that, even if they cannot do the current task, there is a similar task coming and that they can learn by engagement in the current task, even if not successful.

3. Thinking about the bigger picture

- Sequences can help students see the “bigger picture”.
- One of the disadvantages of conventional approaches to mathematics and numeracy is that mathematics is broken into sets of micro skills rather than coherent connections.
- Sequences may help students see connections by making the progression of learning more obvious.
- Note the implications for having larger learning intentions than just a single lesson

4. What it is and what it is not

- Students will understand both what the concept is and what it is not.
- Concepts are learned as much by what they are not as from what they are.
- So tasks that are carefully varied within sequences can emphasise what the central ideas are (and what they are not).

Varying the examples to make the concepts or contexts clearer

- Variation Theory offers a process that can guide the planning of these subsequent consolidating tasks.
- Kullberg, Runesson, and Mårtensson (2013), for example, argued:
  - In order to understand or see a phenomenon or a situation in a particular way one must discern all the critical aspects of the object in question simultaneously.
  - Since an aspect *is noticeable only if it varies against a background in invariance* (emphasis in original), the experience of variation is a necessary condition for learning something in a specific way. (p. 611)

There are two types of variation …

… after the students solve a problem, you might pose a similar problem or inquiry in which you keep the context constant but change the mathematical focus in some way.

… after the students solve a problem, you might pose a similar problem in which you change the context but keep the mathematical focus of the questions substantially the same.

5. Teacher learning

- By proposing sequences with related professional learning, teachers can experience not only the notion of sequence but the ways that sequences enhance learning opportunities for students.

Parallel lines

Line A goes through the point (1,2). Line B goes through (-2,3).

- What might be the equations of line A and line B if they are parallel to each other?
- Give three different possibilities.

Perpendicular lines

Line A goes through the point (1,2). Line B goes through (-2,3).

- What might be the equations of line A and line B if they are perpendicular to each other?
- Give three different possibilities.