Process over product – It's more than an equation!

MAV – December, 2018

Lorraine Day & Derek Hurrell - University of Notre Dame Australia

THE UNIVERSITY OF NOTRE DAME AUSTRALIA

Noticing structure by removing numbers

- Abbey is 140 cm tall. Ben is 4 cm taller than Abbey and Abbey is 6 cm shorter than Charlie. How tall are Ben and Charlie?

- Ben is 4 cm taller than Abbey. Abbey is 6 cm shorter than Charlie.

Draw a picture showing Abbey, Ben and Charlie's heights. Explain what the 4 and the 6 represent. Try to express these height comparisons in other ways.

(Adapted from Carraher, Brizuela, & Schliemann, 2000)

Introduction of the ‘n’ number line

Generalisation

"Mathematicians see generalising as lying at the very heart of mathematics"

(Mason, Graham, & Johnston-Wilder, 2005, p. 283)

Generalisation involves:

- Noticing structure, rather than applying rules
- Sense-making
- Articulation and justification

Arithmetic Thinking vs Algebraic Thinking

Number and algebra are developed together, as each enriches the study of the other.

Australian Curriculum: Mathematics

Arithmetic thinking is about seeking answers (products).

Algebraic thinking is about noticing structure (processes).

(Malara & Navarra, 2003)

Generalisation, or the noticing of structure

- fundamental to mathematical success
- needs to be incorporated at all levels of the teaching and learning of mathematics.
- While generalising is quite innate, children need to practise, strengthen and extend this natural ability.

(Mason, Graham, & Johnston-Wilder, 2005)

- Teachers need to ask students explicit questions about what they notice, the patterns they can see, and how they are making sense of the mathematics. (Day, 2017)

Image from: https://www.educationnews.org/wp-content/uploads/2012/01/math_education.jpg

What is Multiplicative Thinking?

Multiplicative thinking is characterised by:

- **a capacity to work flexibly and efficiently with an extended range of numbers (for example, larger whole numbers, decimals, common fractions, ratio and percent)**
- **an ability to recognise and solve a range of problems involving multiplication or division including direct and indirect proportion**
- **the means to communicate this effectively in a variety of ways (for example, words, diagrams, symbolic expressions and written algorithms).**

Siemon et al., 2006

M/T is fundamental to the development of key concepts

- Division
- Measurement
- Place value
- Fractions
- Proportional reasoning
- Rates and ratios
- Statistical sampling
- Algebraic reasoning

40% of Year 7 and 8 students performed below curriculum expectations in multiplicative thinking and at least 25% were well below expected level.

Siemon, Breed, Dole, Izard, & Virgona (2006)

(Brown & Quinn, 2006; Mulligan & Watson, 1998; Siemon, Izard, Breed & Virgona, 2006).

This suggests that **up to 25% of Australian Year 8 and 9 students do not have the foundation knowledge and skills needed to participate effectively in further school mathematics**, or to access a wide range of post-compulsory training opportunities

(Siemon & Virgona, 2001; Thomson & Fleming, 2004; Siemon et al, 2006).

CHANGE IS NEEDED

The personal, social and economic costs of failing to address this issue are extremely high. It has been estimated that the cost of early school leaving, a direct consequence of underachievement in literacy and numeracy according to McIntyre and Melville (2005), is **$2.6 billion/year!**

Standard algorithm

$$\begin{array}{rrr} & 1 & 4 \\ \times & 1 & 2 \\ \hline & 2 & 8 \\ 1 & 4 & 0 \\ \hline 1 & 6 & 8 \end{array}$$

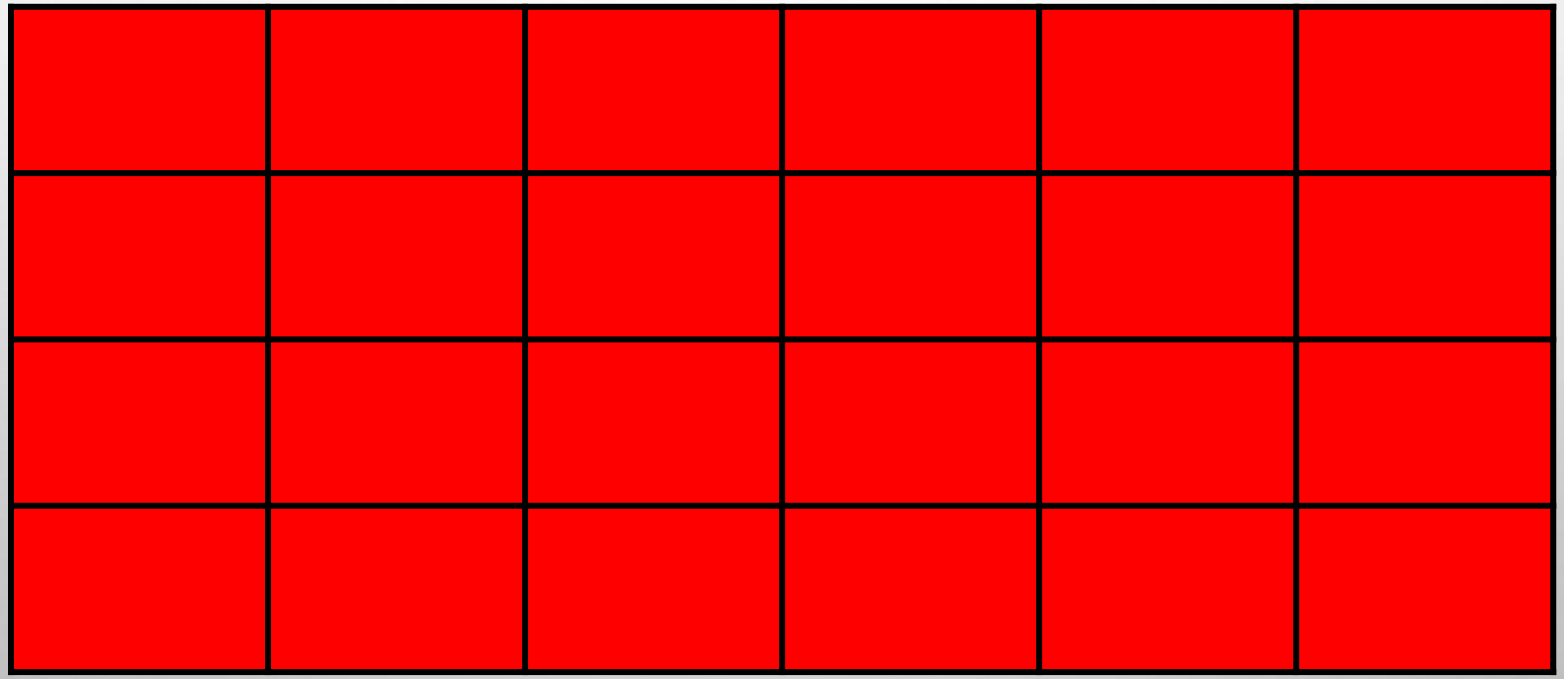
$$\begin{array}{rrr} & 1 & 4 \\ \times & 1 & 2 \\ \hline & 1 & 8 \\ \hline \end{array}$$

$$\begin{array}{rrr} & 1 & 4 \\ \times & 1 & 2 \\ \hline & 2 & 8 \\ 1 & 4 & \\ \hline 1 & 6 & 8 \end{array}$$

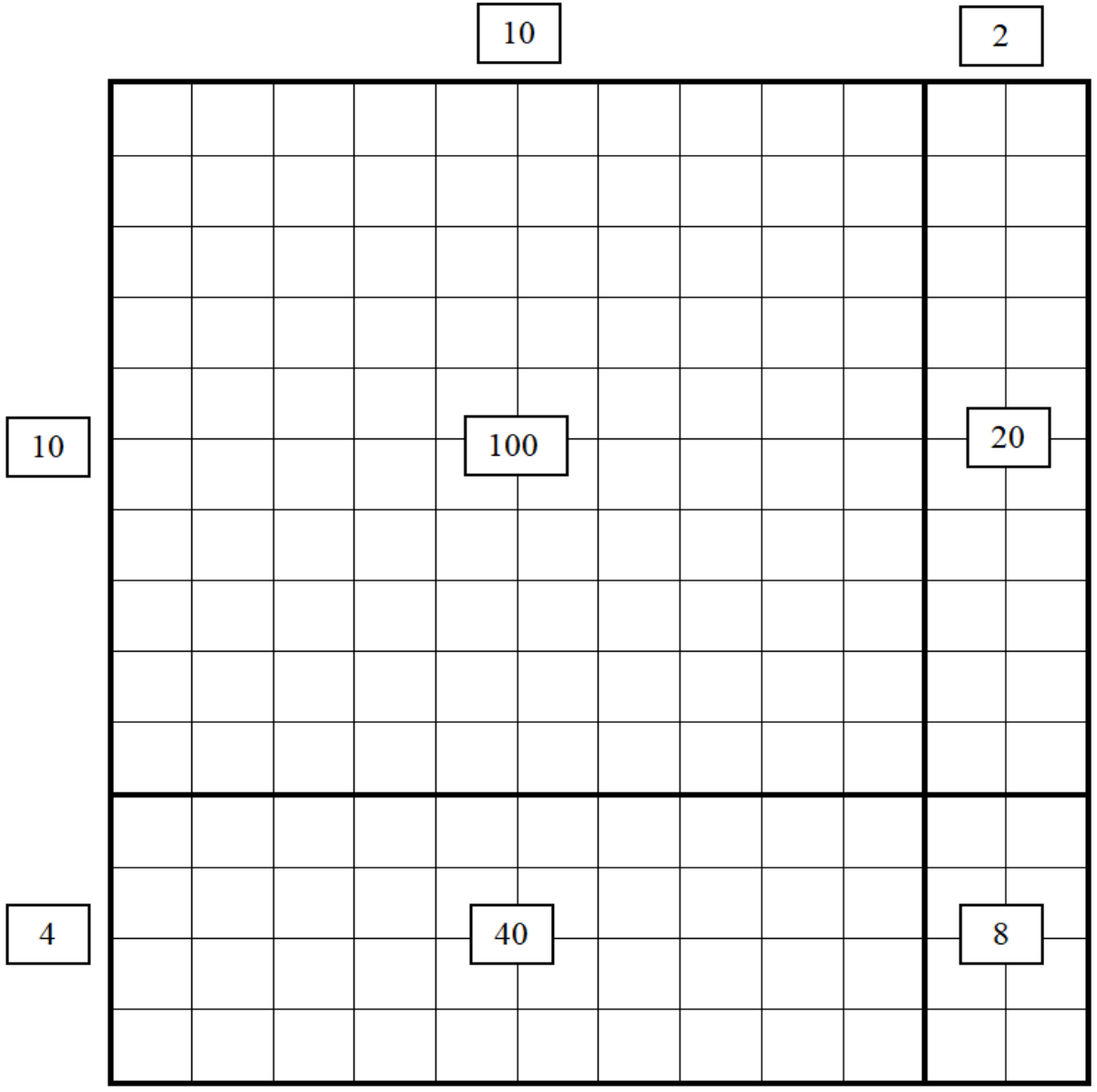
$$\begin{array}{rrr} & 1 & 4 \\ \times & 1 & 2 \\ \hline & 2 & 8 \\ \hline \end{array}$$

A bag of tiles…

let's make a 4 x 6 multiplicative array.

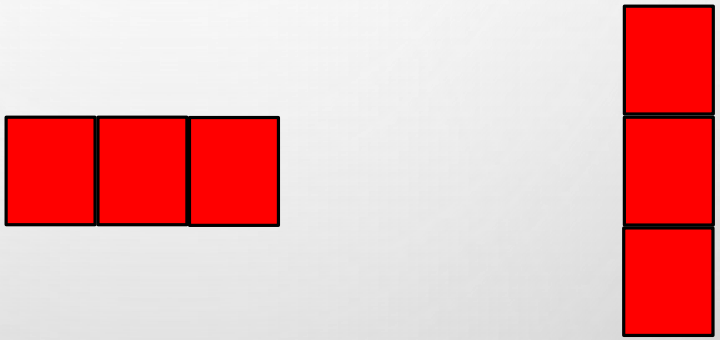


An extended array.



A bag of tiles…let's explore…

- For this activity you can't turn (rotate) the tiles and call the shape different. i.e. these are considered the same:



- Can we make a rectangle or square with two tiles in more than one way?

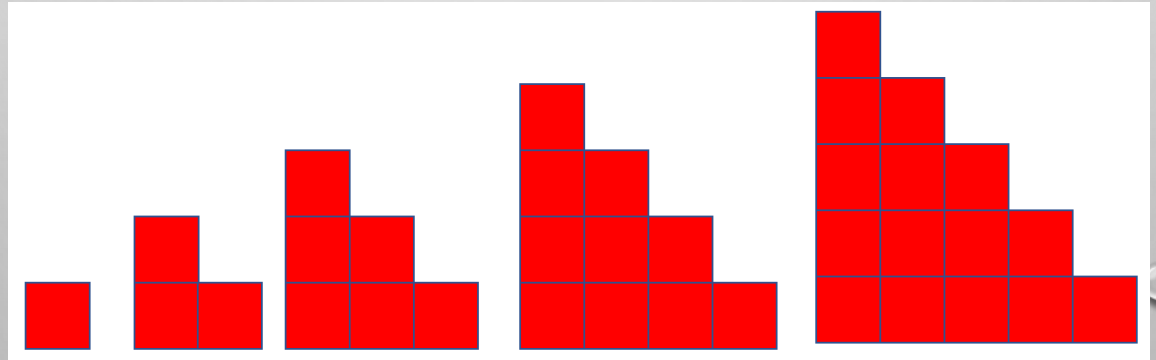
What do you notice? What do you wonder?

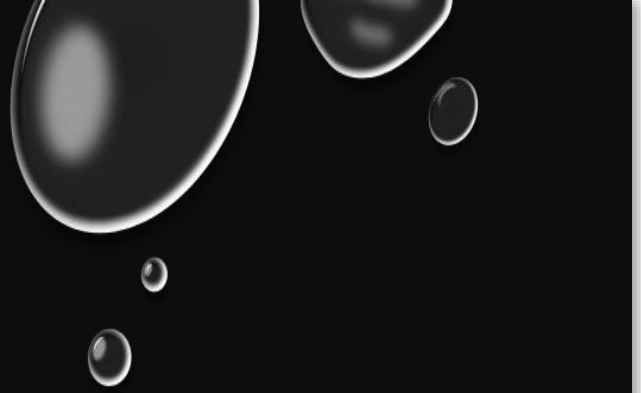
Noticing structure in number

- Investigate the property of commutativity.
- Investigate the sums of consecutive numbers.
- Investigate and describe patterns in the set of odd/even numbers
- What have you found out about adding odd and even numbers?

Algebraic Reasoning is about noticing structure.

St air cas es



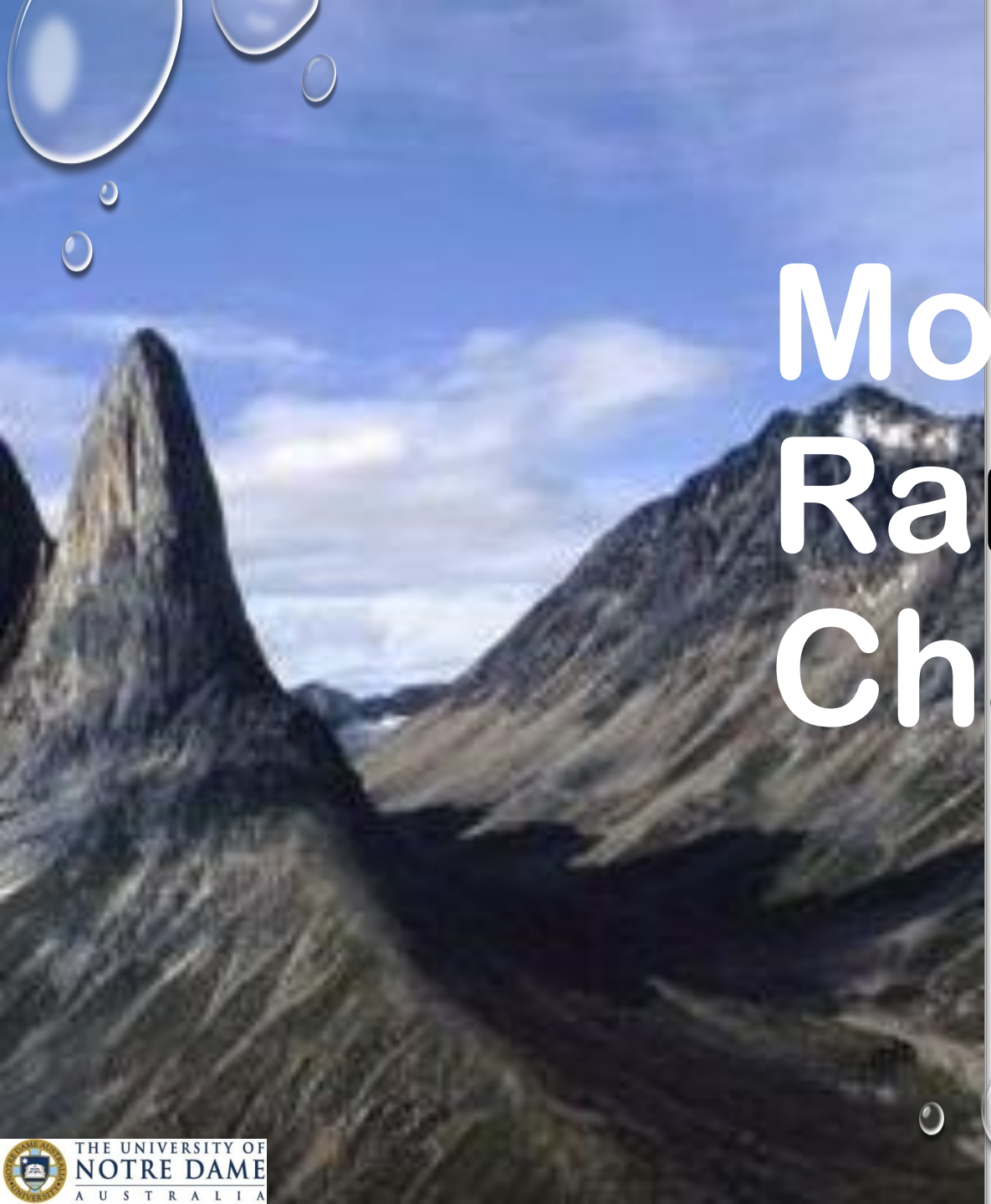


What changes (or grows) and what stays the same?

Noticing structure and algebraic reasoning

The noticing of structure assists students to make sense of the mathematics rather than just applying operations on numbers without necessarily understanding why they are doing so.

Understanding how our number system is structured greatly helps students to reason mathematically.



Mountain Range Challenge

THE UNIVERSITY OF NOTRE DAME AUSTRALIA

Thank you…

Lorraine Day

University of Notre Dame Australia

e-mail: Lorraine.Day@nd.edu.au

Derek Hurrell

University of Notre Dame Australia

e-mail: Derek.Hurrell@nd.edu.au

THE UNIVERSITY OF NOTRE DAME AUSTRALIA