Problem Solving in Primary Mathematics

Dr John West

The Mathematical Association of Western Australia Inc.

AUSTRALIA ECU EDITH COWAN UNIVERSITY

About me

I have worked as a:

- Mathematics teacher, tutor, mentor and G&T coordinator;
- Lecturer in mathematics education/ pedagogy/assessment/special ed and research methods at ECU and UWA (incl. Singapore/Hong Kong/Vietnam);
- Consultant for CEO, AIS, GATCA (WA);
- Researcher and author;

and

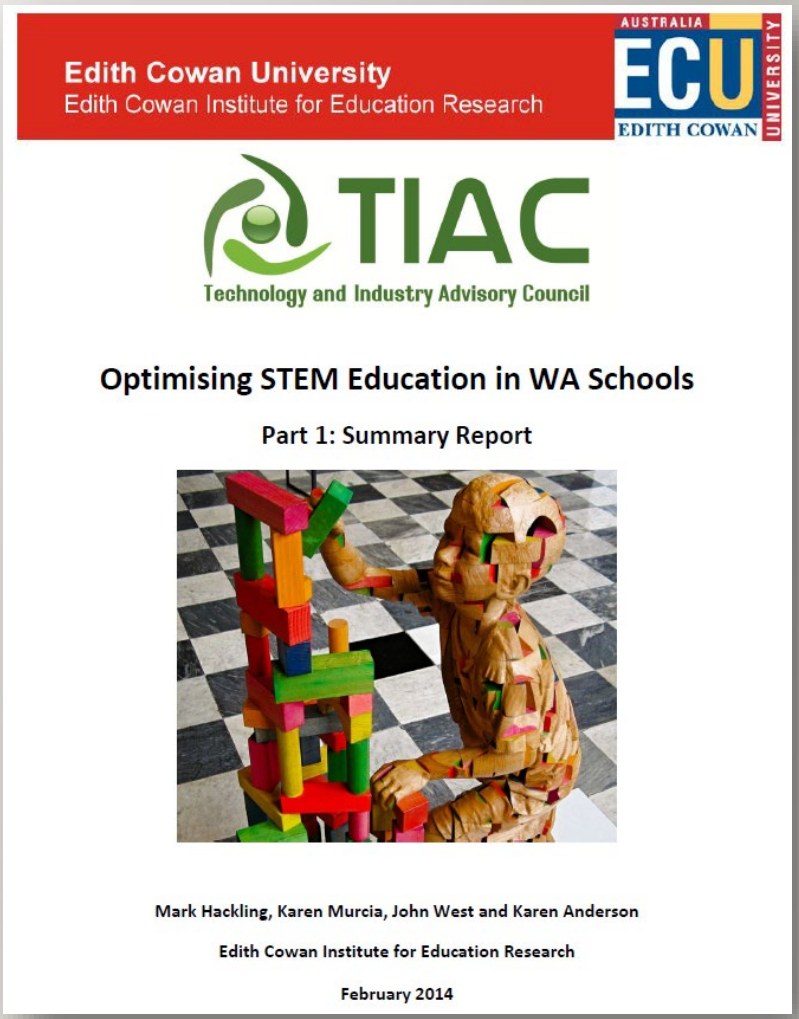
- MAWA Committee member (President elect)

Background

In 2013 a team of ECU researchers were commissioned to report on the status of *STEM Education in WA Schools*.

The results revealed:

- declining performance in STEM subjects against our international competitors;
- increasingly negative attitudes towards STEM subjects (internationally; and between Year 4 and Year 8);
- declining participation in academically demanding STEM subjects; and
- concern about the low proportion of Australian students reaching the advanced achievement benchmarks.



Background

Challenges specific to WA include:

- a strong negative impact of social disadvantage, geolocation and gender on achievement, attitudes and participation (i.e., the gap);
- concerns about teacher qualifications and out-of-field teaching; and
- the need to ensure fair and even delivery of support across the state (shown to scale)

The Good News

On the positive side, the research revealed:

- widespread industry support for a diverse range of initiatives to support STEM education; and
- a willingness to work with educators who provide specialist expertise e.g., understanding of curriculum.

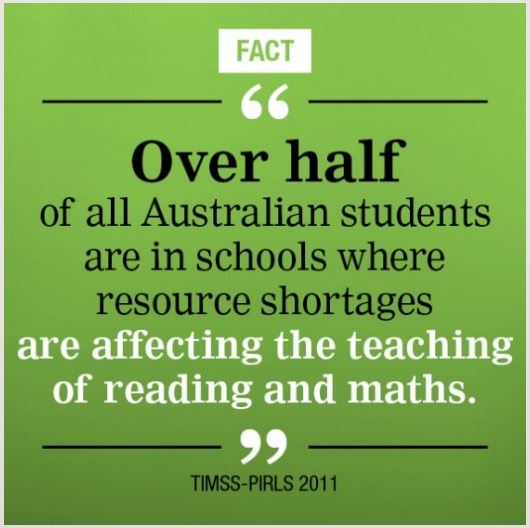
Companies just wanted **evidence** to justify their investment.

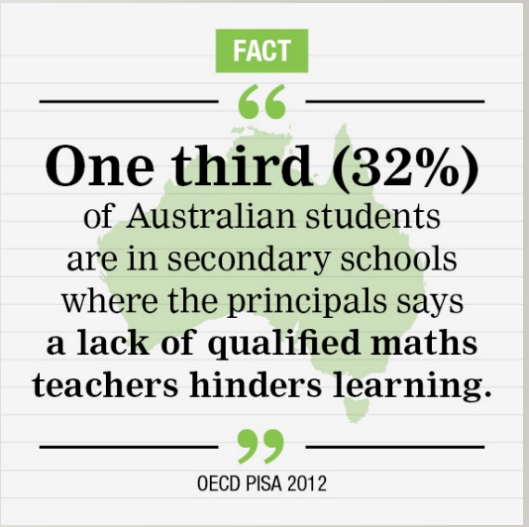
International Benchmarking Studies

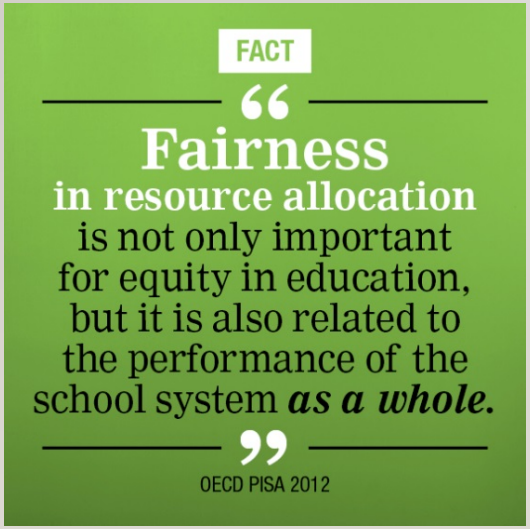
- The **T**rends in **I**nternational **M**aths and **S**cience **S**tudy is conducted every four years to collect data on maths and science achievement in Year 4 and Year 8.
  - TIMSS compares performance of students in approximately 57 countries

- The **P**rogramme for **I**nternational **S**tudent **A**ssessment is conducted by the OECD every three years to collect data on reading literacy and mathematical literacy of 15 year-old students.
  - PISA compares the performance of students in approximately 65 countries

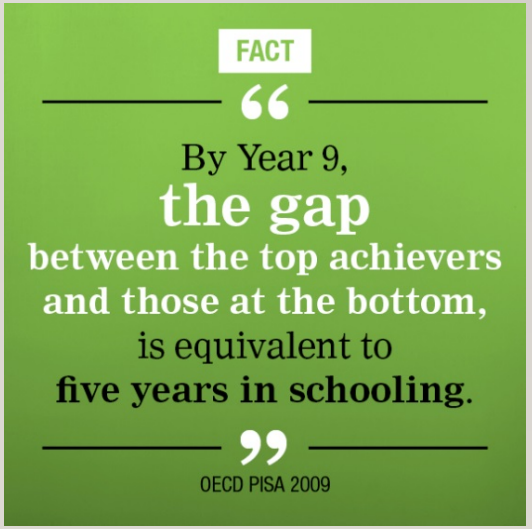

Australia's Year 4 students in both maths and science had the fifth widest gap between high and low achievers in OECD countries.
Former Education Minister Peter Garrett


FACT
Over half
of all Australian students are in schools where resource shortages are affecting the teaching of reading and maths.
TIMSS-PIRLS 2011


FACT
One third (32%)
of Australian students are in secondary schools where the principals says a lack of qualified maths teachers hinders learning.
OECD PISA 2012


FACT
Fairness
in resource allocation is not only important for equity in education, but it is also related to the performance of the school system as a whole.
OECD PISA 2012


FACT
High performing countries and economies tend to allocate resources more equitably across socio-economically advantaged and disadvantaged schools.
OECD PISA 2012


FACT
By Year 9,
the gap
between the top achievers and those at the bottom, is equivalent to five years in schooling.
OECD PISA 2009

So what can we do?

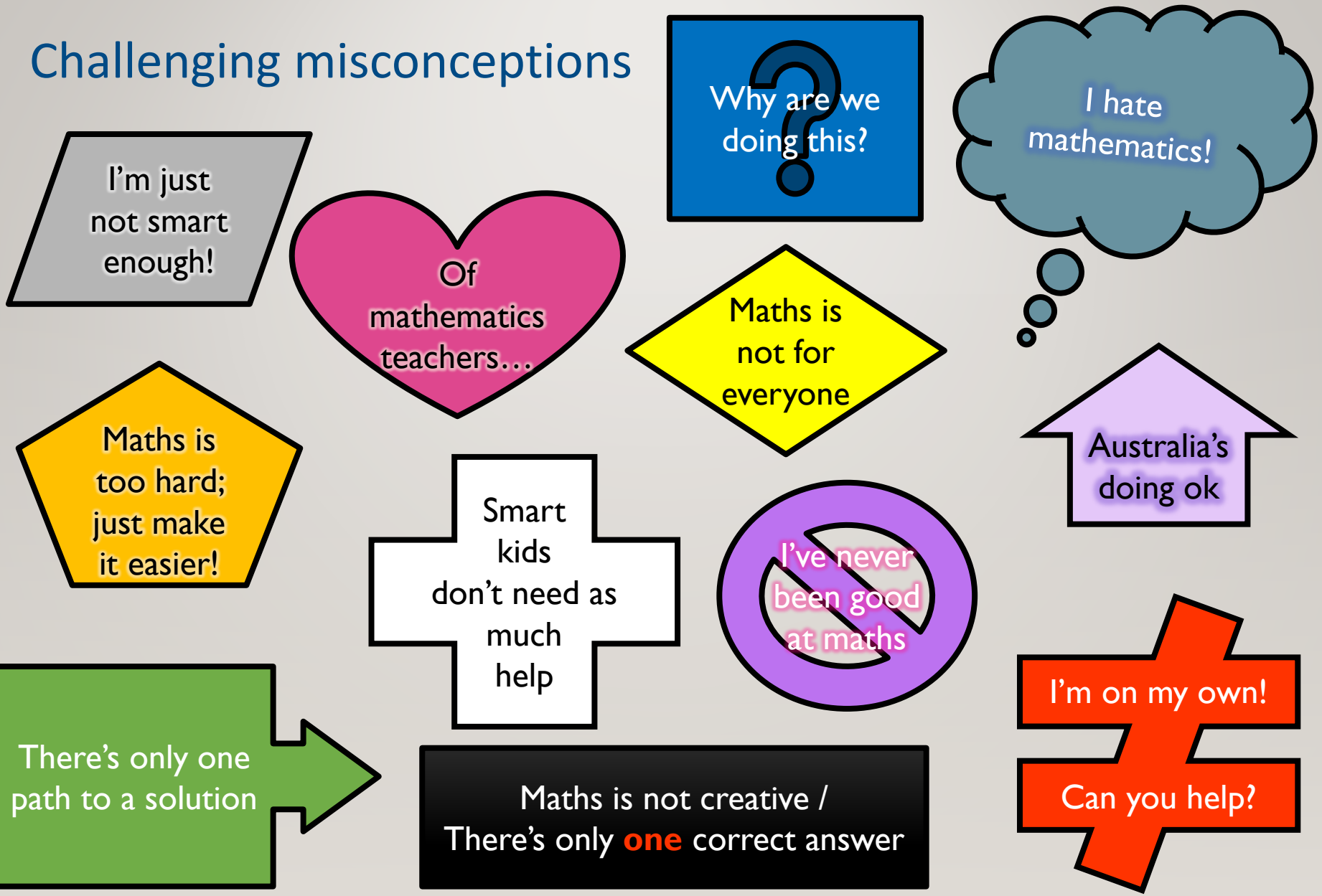
Let's look at the facts:

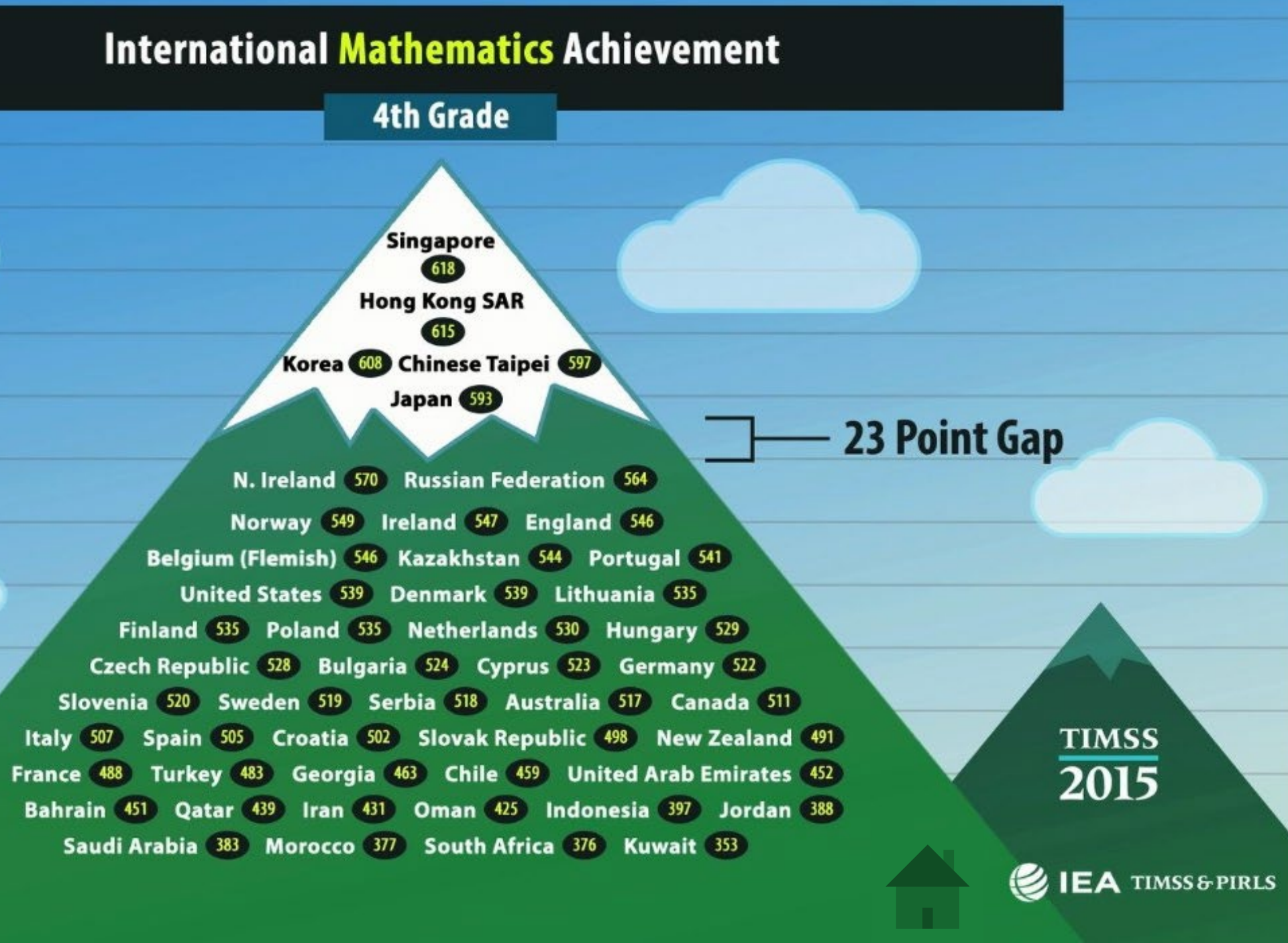
- Education is under increasing scrutiny
- Teachers are working harder than ever
- Teachers are burning out and leaving the profession
- Students lack confidence in mathematics
- Huge emphasis on summative assessment

The solution…?

- We need to trust our educators more
- We need to help our teachers to work smarter not harder
- Burnout is a bad outcome for everyone
- We need to increase student responsibility and confidence
- We need to increase the emphasis on assessment for learning

Challenging misconceptions




International Mathematics Achievement
4th Grade
Singapore 618
Hong Kong SAR 615
Korea 608 Chinese Taipei 597
Japan 593
23 Point Gap
N. Ireland 570 Russian Federation 564
Norway 549 Ireland 547 England 546
Belgium (Flemish) 546 Kazakhstan 544 Portugal 541
United States 539 Denmark 539 Lithuania 535
Finland 535 Poland 535 Netherlands 530 Hungary 529
Czech Republic 528 Bulgaria 524 Cyprus 523 Germany 522
Slovenia 520 Sweden 519 Serbia 518 Australia 517 Canada 511
Italy 507 Spain 505 Croatia 502 Slovak Republic 498 New Zealand 491
France 488 Turkey 483 Georgia 463 Chile 459 United Arab Emirates 452
Bahrain 451 Qatar 439 Iran 431 Oman 425 Indonesia 397 Jordan 388
Saudi Arabia 383 Morocco 377 South Africa 376 Kuwait 353
TIMSS 2015
IEA TIMSS & PIRLS

The Score Card (TIMSS 2015)

- International Ranking (Year 4 Mathematics)
  - 11th place TIMSS 1995
  - 28th place TIMSS 2015
- International Ranking (Year 8 Mathematics)
  - 13th place TIMSS 1999
  - 17th place TIMSS 2015

Thomson, Wernert, O'Grady & Rodrigues (2016)

Sample Year 4 TIMSS items

Tom ate $\frac{1}{2}$ of a cake, and Jane ate $\frac{1}{4}$ of the cake. How much of the cake did they eat altogether?

Education system	Percent correct
Singapore	84
Northern Ireland-GBR	68
Chinese Taipei-CHN	54
Ireland	53
Hong Kong-CHN	53
England-GBR	51
Finland	46
Germany	41
Australia	37
Korea, Rep. of	36
United States	**35**
New Zealand	33
Denmark	32
Belgium (Flemish)-BEL	30
Netherlands	28
Japan	28
Austria	28
Malta	24
International average	23

Sample Year 4 TIMSS items

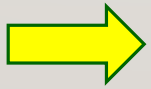
If the pattern 3, 6, 9, 12 was continued, which of these numbers would be one of the numbers in the pattern?

A. 26

B. 27

C. 28

D. 29



Education system	Percent correct
Singapore	93
Korea, Rep. of	89
Northern Ireland-GBR	84
United States	**83**
Czech Republic	82
Ireland	81
Chinese Taipei-CHN	81
Denmark	80
Hong Kong-CHN	80
England-GBR	80
Finland	80
Japan	79
Slovenia	79
Germany	78
Portugal	78
Norway	77
Italy	75
Australia	74
Slovak Republic	72
Malta	71
Lithuania	70
Turkey	68
New Zealand	68
Sweden	68
Serbia	68
Belgium (Flemish)-BEL	67
Austria	66
Croatia	65
Russian Federation	64
International average	62

International average	62
Spain	62
Netherlands	62
Hungary	60
Azerbaijan	57
Kazakhstan	56
Chile	55
Poland	53
Romania	52
Georgia	52
United Arab Emirates	48
Qatar	46
Bahrain	42
Armenia	41
Iran, Islamic Rep. of	34
Saudi Arabia	34
Oman	34
Kuwait	29
Thailand	28
Tunisia	27
Morocco	24
Yemen	20

Sample Year 4 TIMSS items

Which shows a correct method for finding $\frac{1}{3} - \frac{1}{4}$?

A. $\frac{1-1}{4-3}$

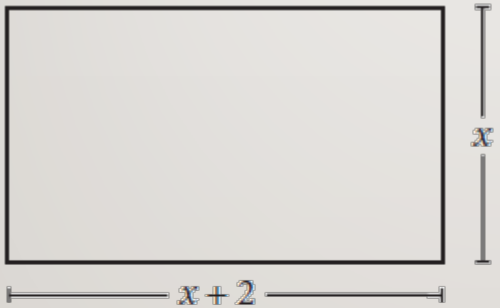
B. $\frac{1}{4-3}$

C. $\frac{3-4}{3\times4}$

D. $\frac{4-3}{3\times4}$

Education system	Percent correct
Korea, Rep. of	86
Singapore	83
Chinese Taipei-CHN	82
Hong Kong-CHN	77
Japan	65
Russian Federation	63
Armenia	56
Italy	51
Kazakhstan	50
Lebanon	49
Malaysia	46
Israel	45
Ukraine	41
United Arab Emirates	38
Turkey	37
International average	37
Romania	36
Palestinian Nat'l Auth.	35
Australia	34
Jordan	33
Hungary	33
Georgia	33
Slovenia	30
Thailand	29
United States	**29**
Lithuania	28
England-GBR	28
Saudi Arabia	28
Iran, Islamic Rep. of	27
New Zealand	26
Macedonia, Rep. of	25
Oman	24
Qatar	24
Bahrain	24
Indonesia	21
Tunisia	21
Morocco	21
Ghana	19

Sample Year 4 TIMSS items



What is the area of this rectangle?

A. $x^2 + 2$

B. $x^2 + 2x$

C. $2x + 2$

D. $4x + 4$

Of course, chance effects explain 25% in this case!

Education system	Percent correct
Chinese Taipei-CHN	83
Singapore	79
Russian Federation	72
Hong Kong-CHN	69
Korea, Rep. of	67
Kazakhstan	55
Armenia	55
Ukraine	52
Israel	49
Japan	48
Romania	48
Georgia	45
Turkey	42
Lithuania	42
Thailand	41
International average	40
Macedonia, Rep. of	39
Lebanon	39
United States	**37**
Indonesia	36
England-GBR	35
Syrian Arab Republic	34
Palestinian Nat'l Auth.	34
Qatar	33
Iran, Islamic Rep. of	33
Finland	32
Slovenia	32
Italy	31
Tunisia	30
United Arab Emirates	30
Hungary	30
Bahrain	30
Norway	28
New Zealand	28
Morocco	28
Ghana	27
Australia	26
Malaysia	25

The Score Card (TIMSS 2015)

- While our 2015 scale score is **no different** from 1995, our international competitors have increased and we have fallen in the rankings
- What is it that they are doing that we aren't?
- Not the gap between high and low achievers
  - Australia – 275 points
  - Singapore – 280 points

TIMSS Advanced Achievement Benchmarks

- Just **9%** of Australian Year 4 students reached the advanced achievement benchmark in mathematics, compared to **50%** of students in Singapore

- Just **7%** of Australian Year 8 students reached the advanced achievement benchmark, compared to **54%** of students in Singapore

Thomson, Wernert, O'Grady & Rodrigues (2016)

Stimulating Reasoning using open-ended tasks

Australian Primary Mathematics Classroom, 23(1), 32-35

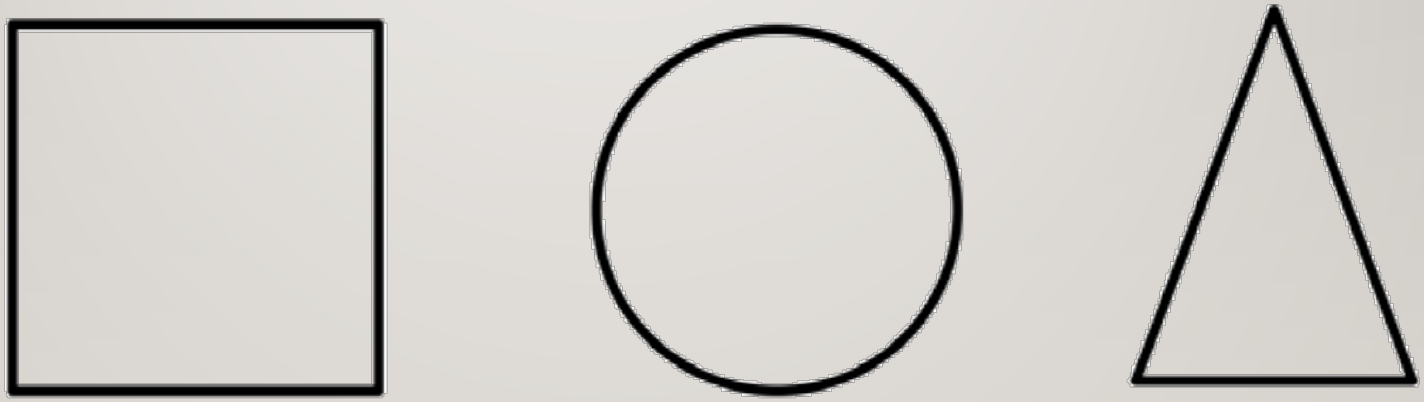


Figure 1. Which of these shapes is the odd one out?

Stimulating Reasoning using open-ended tasks

Australian Primary Mathematics Classroom, 23(1), 32-35

4 9 3

Which one is the ‘odd’ one out? Explain your reasoning.

Stimulating Reasoning using open-ended tasks

Australian Primary Mathematics Classroom, 23(1), 32-35

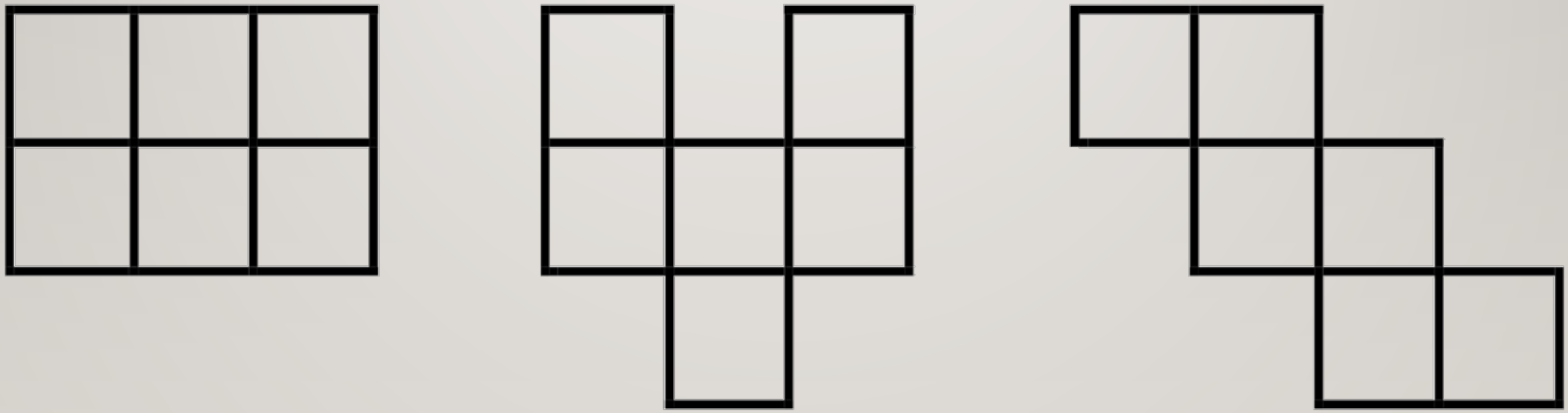
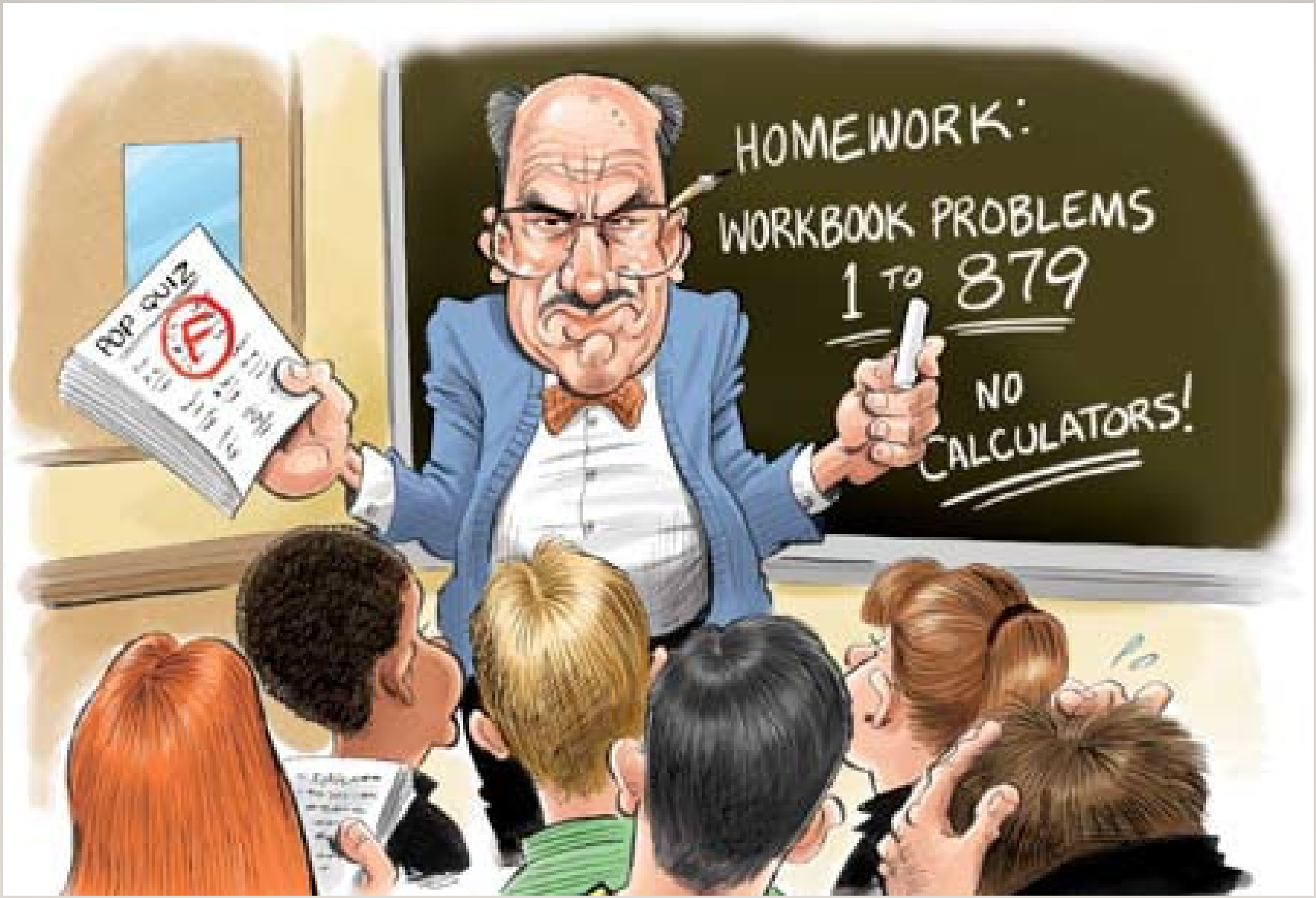
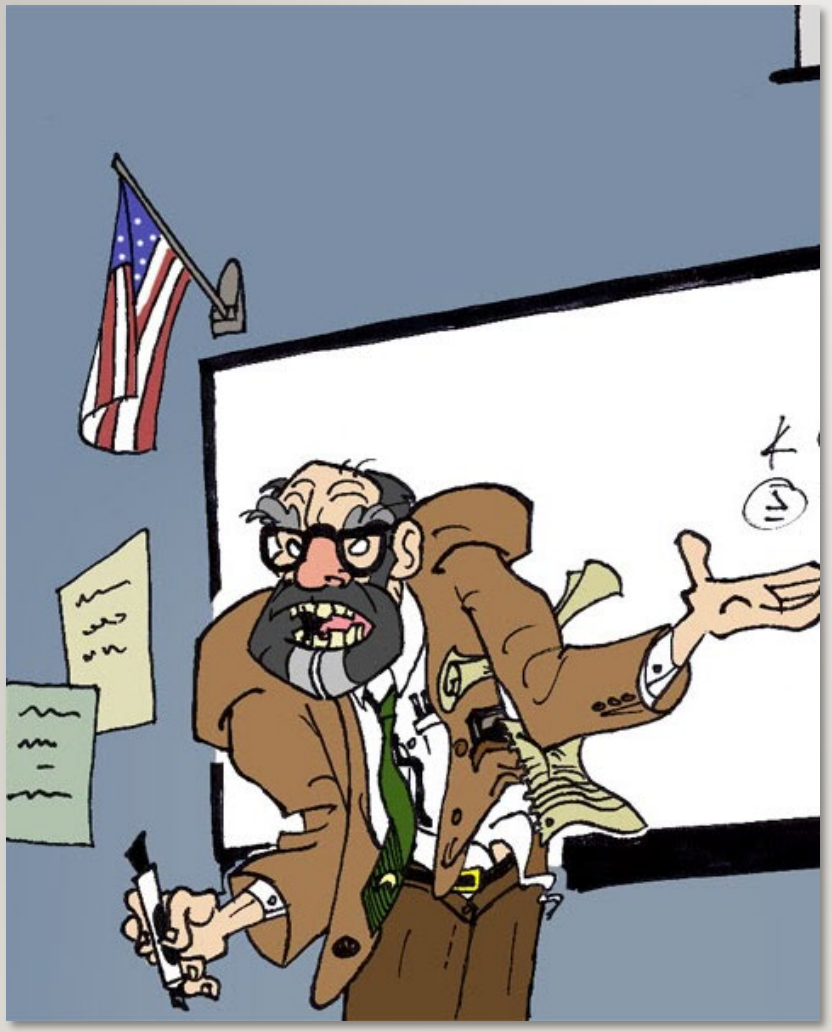
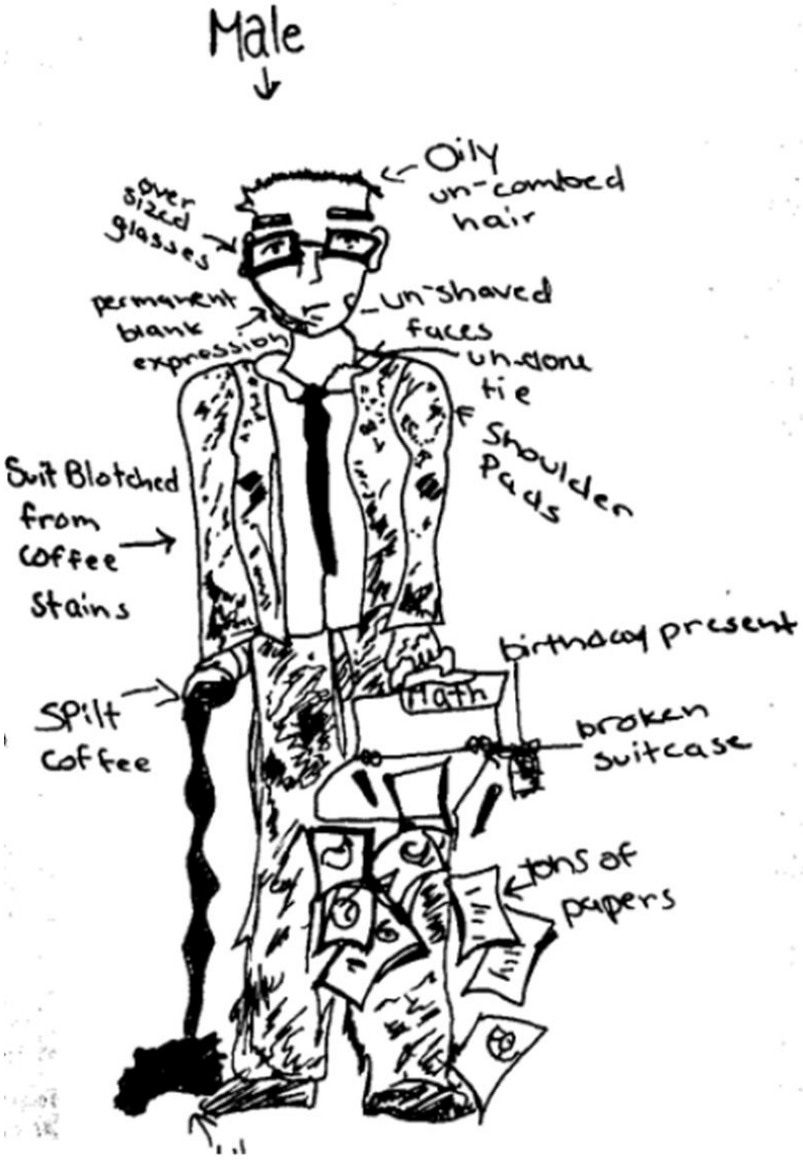


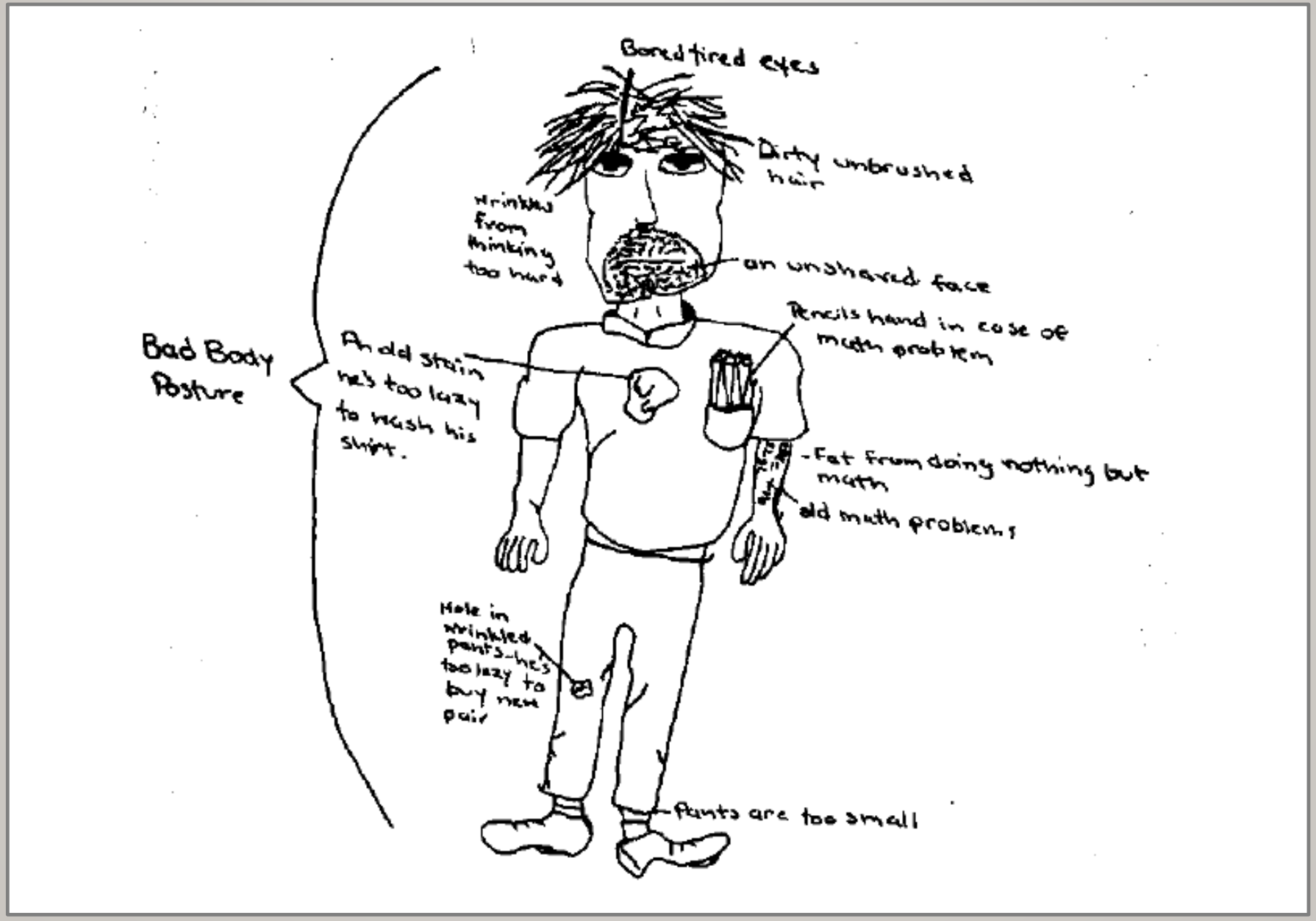
Figure 4. Which of these shapes is the odd one out?


HOMEWORK:
WORKBOOK PROBLEMS
1 TO 879
NO
CALCULATORS!
POP QUIZ
F



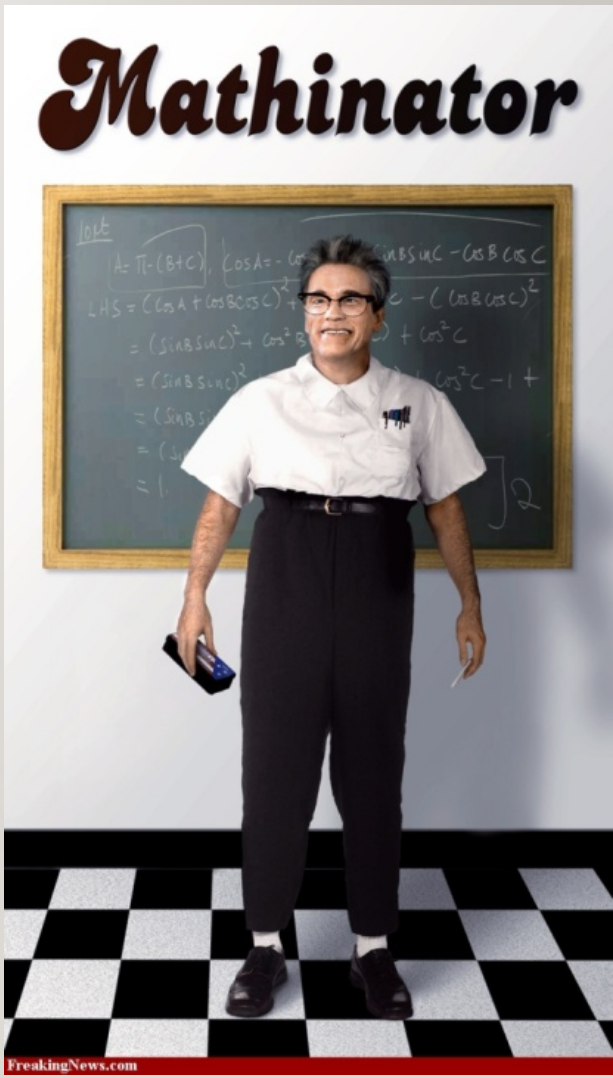

MY EVIL
MATH
TEACHER


Male
Oily un-combed hair
over sized glasses
permanent blank expression
un-shaved faces
un-done tie
Shoulder Pads
Suit Blotched from Coffee Stains
birthday present
Math
Spilt Coffee
broken suitcase
tons of papers


Bored tired eyes
Dirty unbrushed hair
Wrinkles from thinking too hard
an unshaved face
Pencils hand in case of math problem
Bad Body Posture
An old stain he's too lazy to wash his shirt.
-Fat from doing nothing but math
old math problems
Hole in wrinkled pants-he's too lazy to buy new pair
-Pants are too small

Teachers of mathematics:

- ☹ have no friends (except other maths teachers);
- ☹ are not married or seeing anyone;
- ☹ are usually fat;
- ☹ are very unstylish;
- ☹ have wrinkles from thinking so hard;
- ☹ have no social life whatsoever;
- ☹ are 30 years old; and
- ☹ have a short temper.

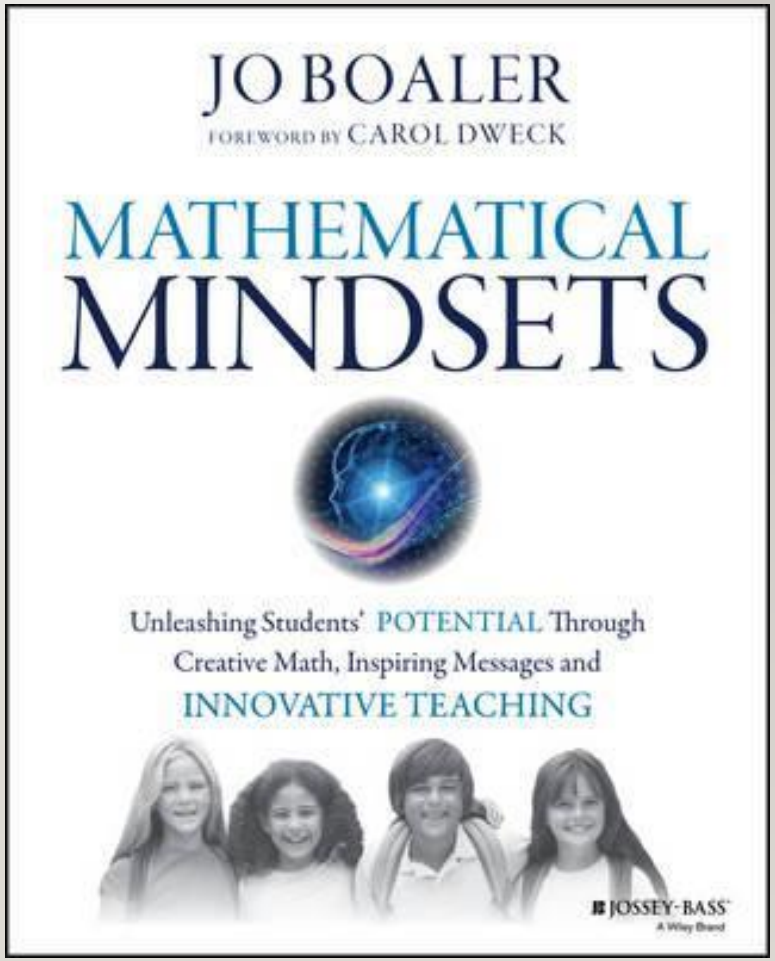


Growth Mindset

- Professor Jo Boaler (Stanford/Cambridge)
  - Reframing students' and teachers' perceptions of mathematics
  - Developing a growth-oriented mindsets
  - Focusing on developing understanding

- Challenging many myths about the brain and learning maths

- Free resources are available:
  - www.youcubed.org



youcubed at Stanford University

Building a Mathematical Mindset Community

Teachers and students believe *everyone* can learn maths at HIGH LEVELS.

- Students are not tracked or grouped by achievement
- All students are offered high level work
- "I know you can do this" "I believe in you"
- Praise effort and ideas, not the person
- Students vocalize self-belief and confidence

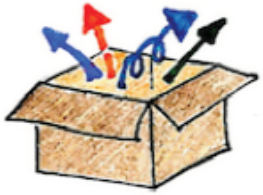
Communication and *connections* are valued.

- Students work in groups sharing ideas and visuals.
- Students relate ideas to previous lessons or topics
- Students connect their ideas to their peers' ideas, visuals, and representations.
- Teachers create opportunities for students to see connections.
- Students relate ideas to events in their lives and the world.

The maths is VISUAL.

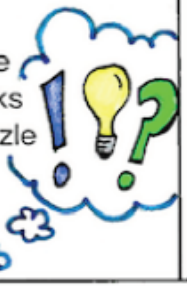
- Teachers ask students to draw their ideas
- Tasks are posed with a visual component
- Students draw for each other when they explain
- Students gesture to illustrate their thinking

The maths is OPEN.



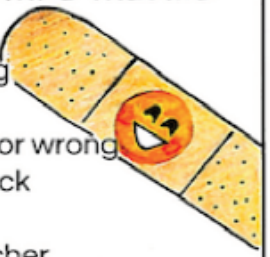
- Students are invited to see maths differently
- Students are encouraged to use and share different ideas, methods, and perspectives
- Creativity is valued and modeled.
- Students' work looks different from each other
- Students use ownership words - "my method", "my idea"

The environment is filled with *WONDER* and *CURIOSITY.*



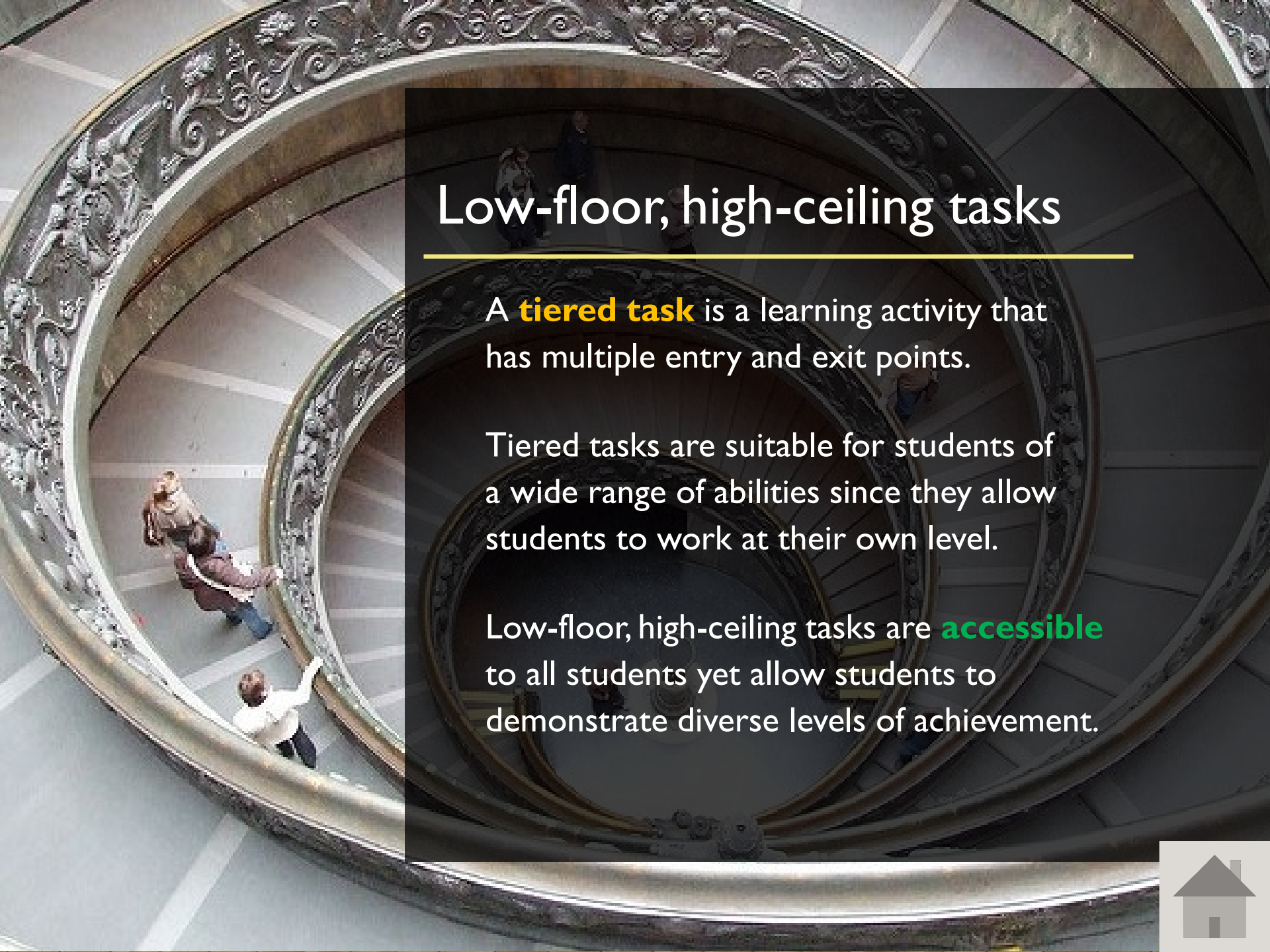
- Students extend their work and investigate
- Teacher invites curiosity when posing tasks
- Students see maths as an unexplored puzzle
- Students freely ask and pose questions
- Students seek important information
- "I've never thought of it like that before."

The classroom is a risk-taking, *MISTAKE VALUING* environment



- Students share ideas even when they are wrong
- Peers seek to understand rather than correct
- Students feel comfortable when they are stuck or wrong
- Teachers and students work together when stuck
- Tasks are low floor/high ceiling
- Students disagree with each other and the teacher

Developed by Jo Boaler/Youcubed.org and Tulare County Office of Education



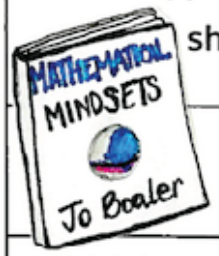
Low-floor, high-ceiling tasks

A **tiered task** is a learning activity that has multiple entry and exit points.

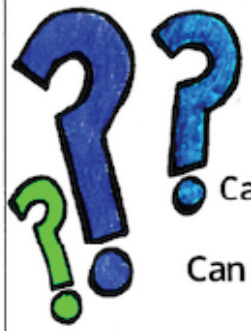
Tiered tasks are suitable for students of a wide range of abilities since they allow students to work at their own level.

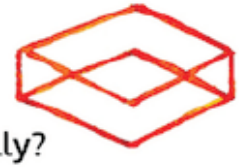
Low-floor, high-ceiling tasks are **accessible** to all students yet allow students to demonstrate diverse levels of achievement.

Recommendations for Task/Lesson Design
Open the task to encourage multiple methods, pathways and representations.
Pose a problem before teaching the method.
Design a task that allows all learners to contribute to the learning and have room for extension.
Make opportunities for students to authentically share their thinking with peers.
Add a visual component.
Add the requirement to convince and reason, be skeptical.



Powerful Questions to develop a deep level of understanding
How do you see that idea?
Why does that answer make sense?
Why does that method work?
How is that method connected to others?
How can that idea be represented in different ways?
Can you prove it?
Can you prove it visually?
Can you justify your thinking?
Can you predict what would happen if....?
Did you make any interesting mistakes?





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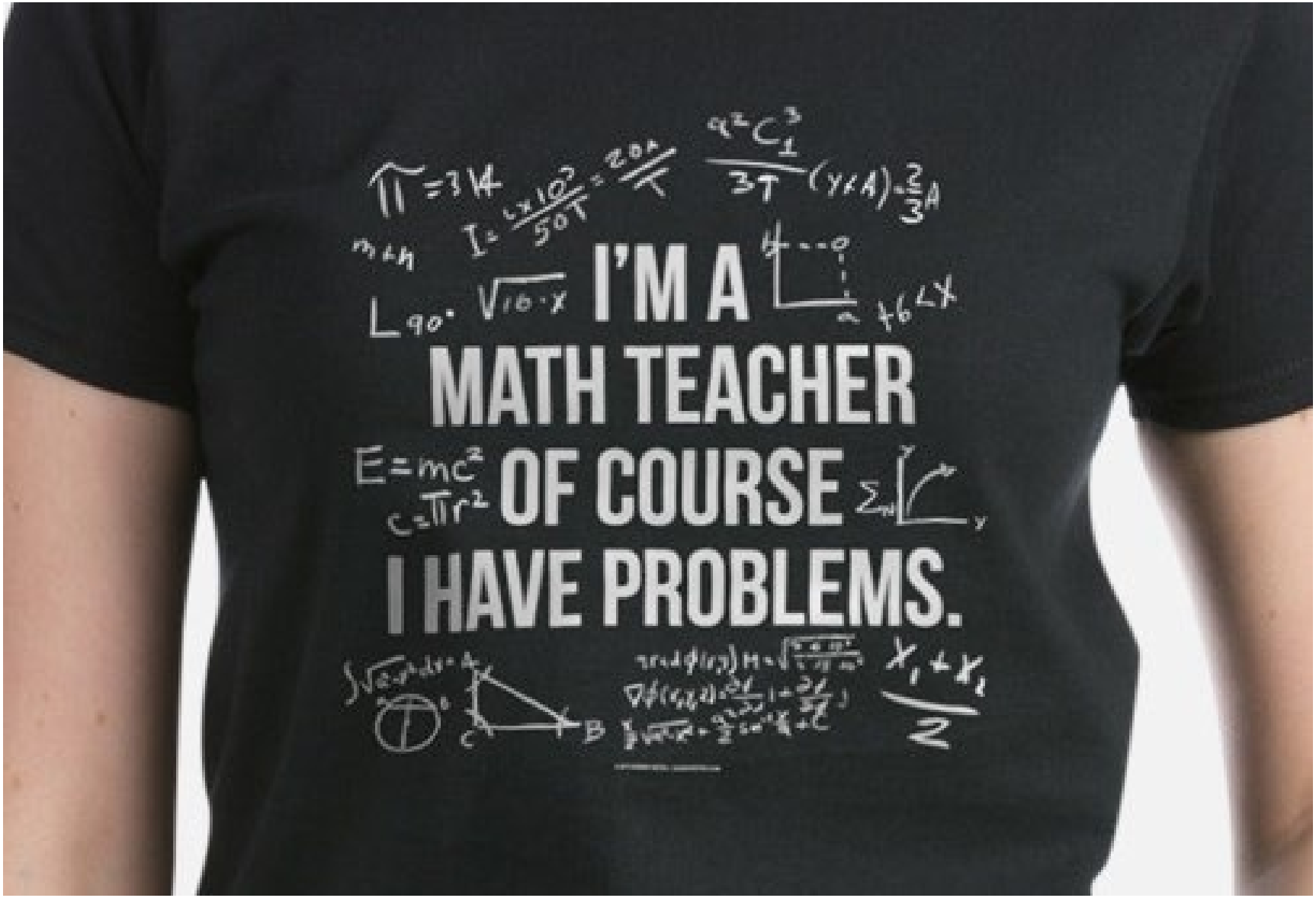
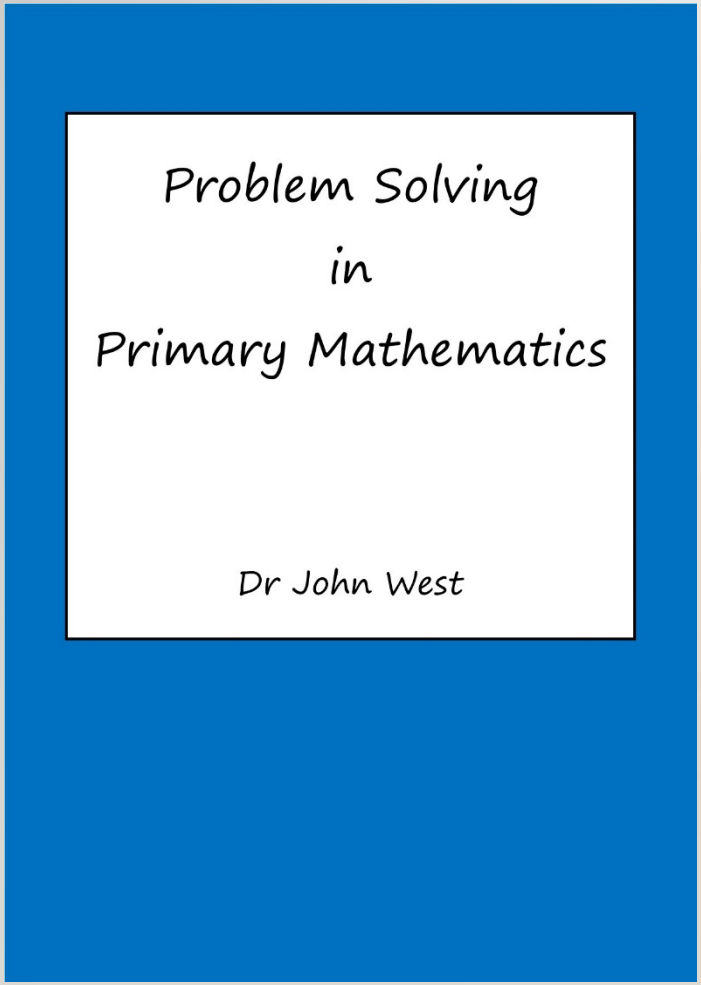

I'M A
MATH TEACHER
OF COURSE
I HAVE PROBLEMS.



Finding problems

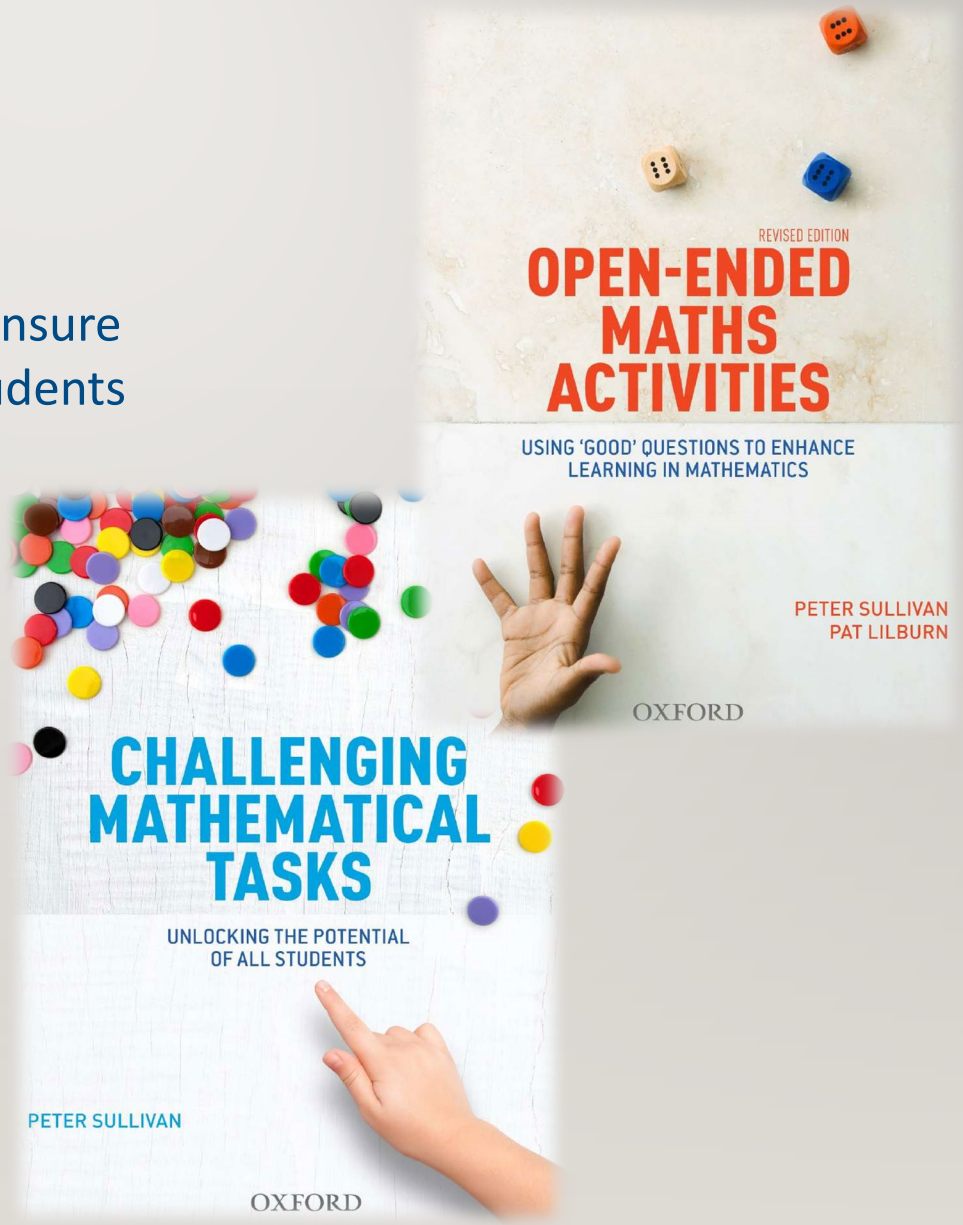
- 45 worked examples and 200 problems with solutions drawn from across the curriculum
- Each chapter focuses on a particular PS strategy
- Transfer of learning
- Purposefully teach PS strategies.

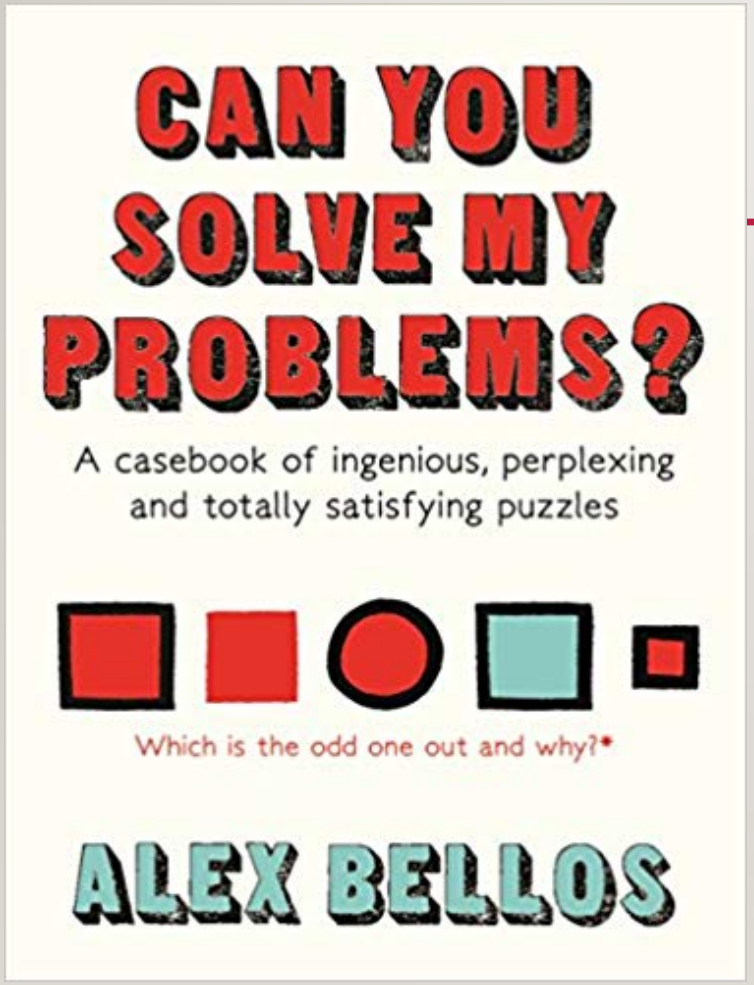
Resources for Collaborative Problem Solving

Dr Paul Swan & David Dunstan

Professor Peter Sullivan

- Lead author, AC:M
- **Extending** and **enabling** prompts ensure tasks are suitable for a range of students



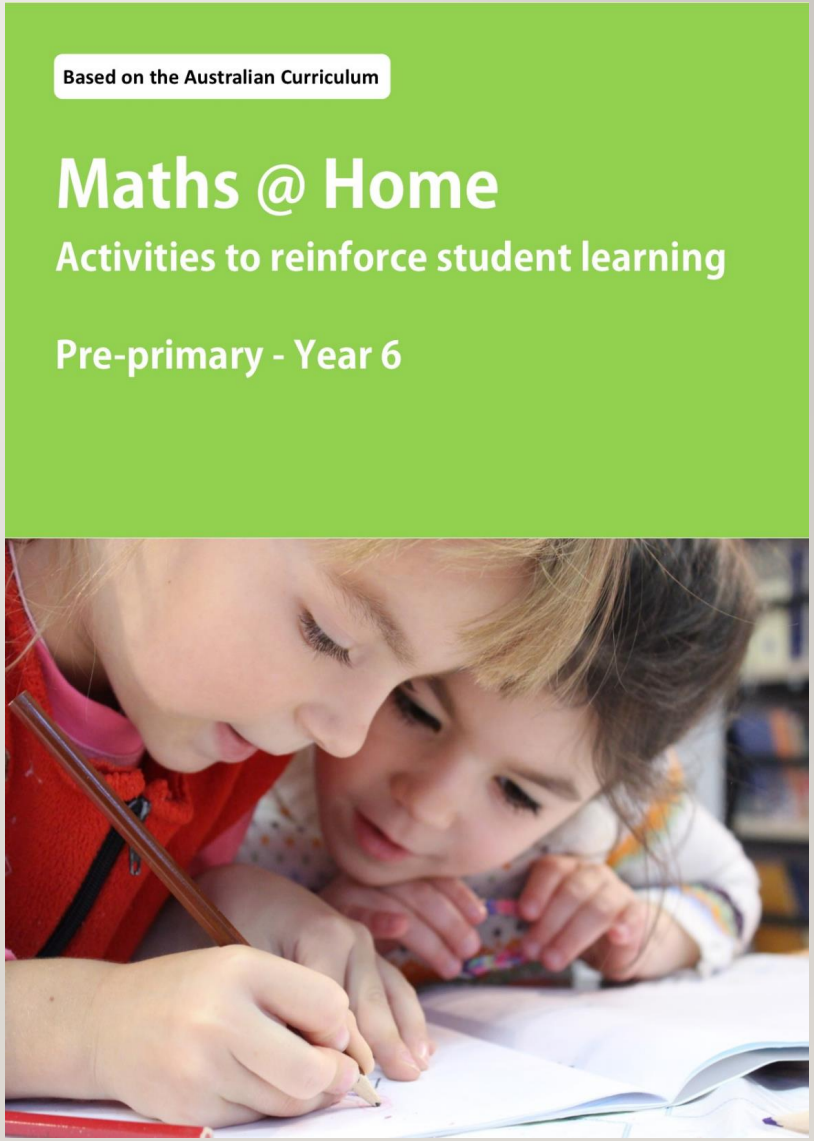


Alex Bellos

- Author of
  Alex's Adventures in Numberland
- Well-written
- Easy to read

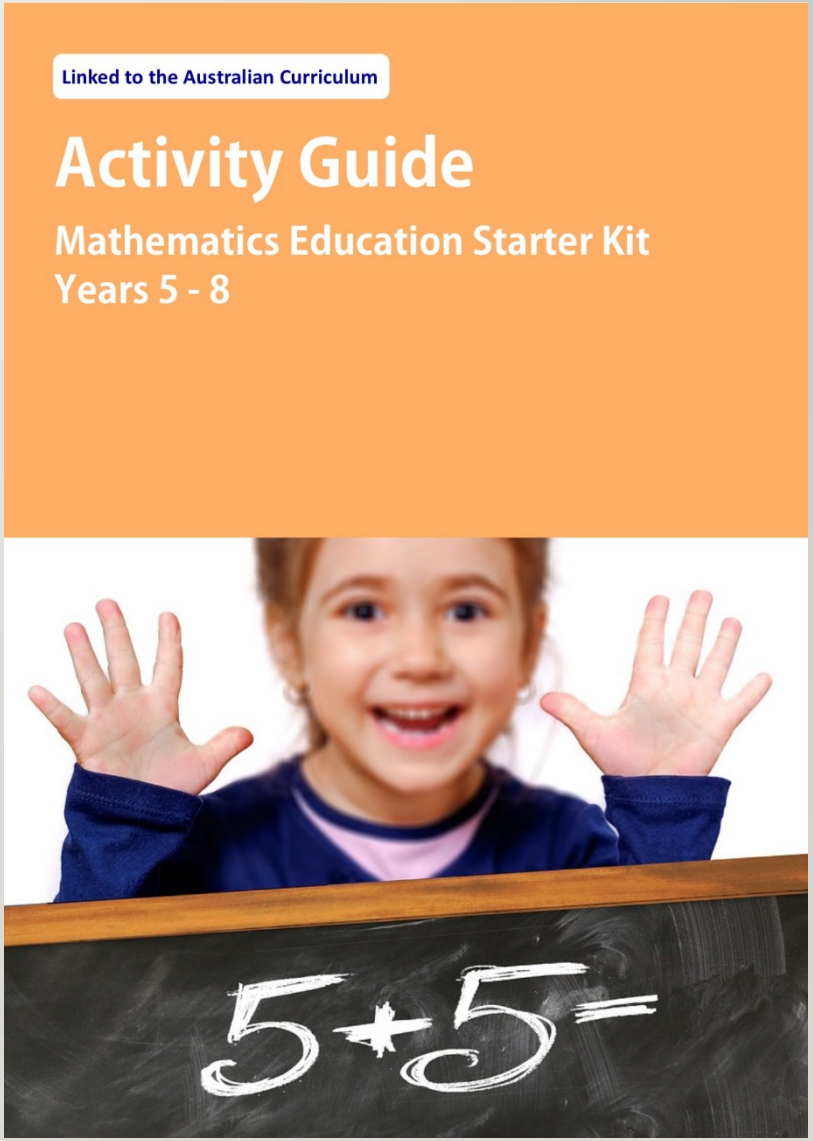
MAWA

- 48 pages
- Affordable ($15)



MAWA

- 48 pages
- Affordable ($10)



Inspiring Generation STEM

There is a mathemaTIcian and scienTIst in *every* student.

TAKE THE #GenSTEM QUIZ

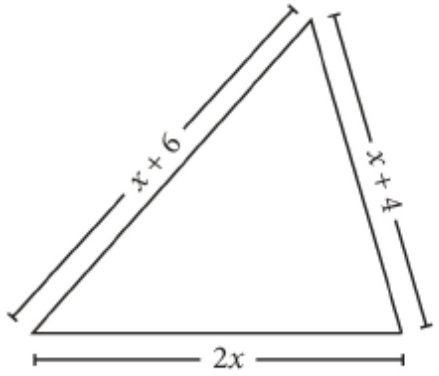
Year 8 Sample Question TIMSS Advanced Achievement Benchmark

Country	Percent Full Credit
Korea, Rep. of	69 (1.8) ▲
Chinese Taipei	67 (2.0) ▲
2 Singapore	65 (1.8) ▲
Hong Kong SAR	62 (2.7) ▲
Russian Federation	47 (2.6) ▲
3 Israel	43 (2.2) ▲
Kazakhstan	41 (3.2) ▲
Japan	41 (2.2) ▲
Hungary	38 (2.4) ▲
2 Lithuania	32 (2.8) ▲
1 2 Georgia	29 (2.3) ▲
Sweden	27 (2.3) ▲
Malta	26 (1.6) ▲
† United States	25 (1.5)
1 † Canada	24 (1.4)
Ireland	23 (1.9)
International Avg.	22 (0.3)
Turkey	18 (1.8) ▼
Iran, Islamic Rep. of	16 (1.7) ▼
Qatar	15 (1.6) ▼
England	15 (1.8) ▼
Lebanon	15 (1.9) ▼
United Arab Emirates	13 (0.7) ▼
Norway (9)	13 (1.4) ▼
Oman	11 (1.1) ▼
Australia	11 (1.1) ▼

Content Domain: Algebra

Cognitive Domain: Applying

Description: Constructs and uses the solution of a linear equation to solve a word problem



The sum of the lengths of the sides of this triangle is 30 cm.

A. Write an equation that would enable you to find the value of x.

Equation: $4x + 10 = 30$

B. What is the length of the LONGEST side of the triangle in centimeters?

Answer: 11 cm

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS 2015

Year 8 Sample Question TIMSS Advanced Achievement Benchmark

Country	Percent Full Credit
[2] Singapore	64 (1.8)
Hong Kong SAR	59 (2.6)
[2] Lithuania	59 (2.4)
Korea, Rep. of	59 (2.0)
Chinese Taipei	55 (1.7)
Japan	45 (2.0)
Norway (9)	43 (2.4)
Ireland	39 (2.1)
Hungary	39 (2.3)
[2] Italy	38 (2.3)
[3] Israel	38 (1.9)
Slovenia	37 (2.0)
Turkey	35 (2.4)
[1] † Canada	34 (1.8)
Russian Federation	27 (2.6)
Kazakhstan	27 (2.1)
† United States	26 (1.4)
International Avg.	25 (0.3)
England	25 (2.0)
Australia	23 (1.5)
[1] [2] Georgia	23 (2.0)
Sweden	22 (2.0)
† New Zealand	19 (1.9)

Content Domain: Data and Chance

Cognitive Domain: Reasoning

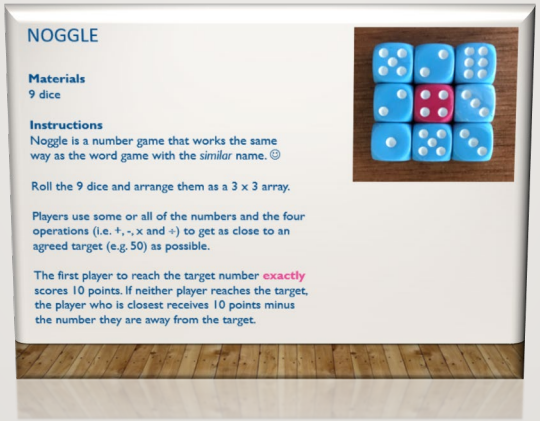
Description: Uses understanding of average to solve a problem

Ahmed had the following scores out of 10 on his first 4 mathematics tests: 9, 7, 8, 8. Ahmed has 1 more test with a maximum of 10 points and says he wants to get an overall average of 9. Is it possible for him to do this?

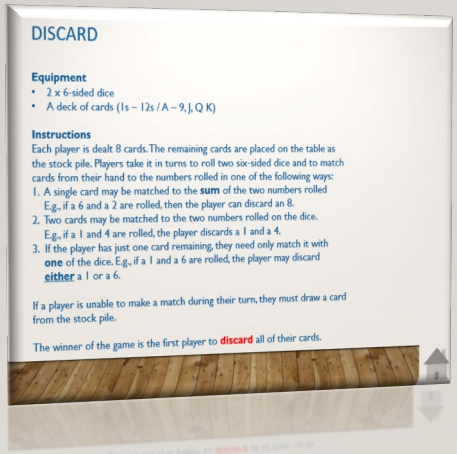
Explain your answer.

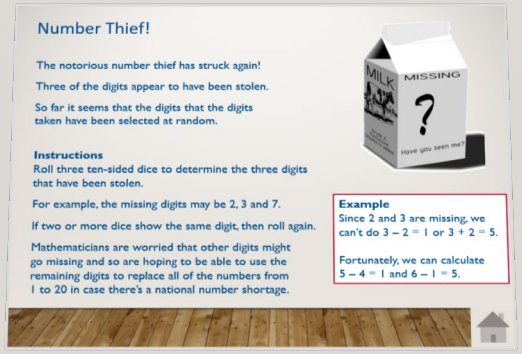
No, Ahmed would have to score 13 to do this.

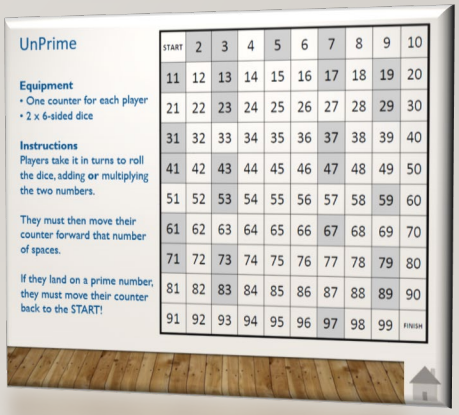
SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS 201



ACTIVITY SPLASH PAGE







POLYNGO

Materials
A set of polyhedral dice
A BINGO card

Instructions
Roll all of the polyhedral dice.

Use some (or all) of the numbers and the four operations to make numbers on the BINGO card.

The winner is the first player to make all of the numbers in a row, column or diagonal of their card. They need to record their calculations.

45	17	42	3
14	35	11	31
26	7	22	38
41	29	6	20

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14	35	~~11~~	~~31~~
26	7	~~22~~	38
41	29	~~6~~	20

$7 - 1 = 6$

$8 + 8 + 7 - 1 = 22$

$8 + 2 + 1 = 11$

$(8 + 8) \times 2 - 1 = 31$

NOGGLE

Materials
9 dice

Instructions
Noggle is a number game that works the same way as the word game with the *similar* name. ☺

Roll the 9 dice and arrange them as a 3 x 3 array.

Players use some or all of the numbers and the four operations (i.e. +, -, x and ÷) to get as close to an agreed target (e.g. 50) as possible.

The first player to reach the target number **exactly** scores 10 points. If neither player reaches the target, the player who is closest receives 10 points minus the number they are away from the target.

POWERBALL NOGGLE

Materials
9 dice

Instructions
Noggle is a number game that works the same way as the word game with the *similar* name. ☺

Roll the 9 dice and arrange them as a 3 x 3 array.

Players use some or all of the numbers and the four operations (i.e. +, -, x and ÷) to get as close to an agreed target (e.g. 50) as possible.

In this variation, players **must** use the number on the central die. **Hint:** Use a different colour die so players cannot simply move any number to the centre.

Chicken Nugget Problem

The chicken nugget problem caused mathematicians a real stomach ache until they worked out that there is a maximum number of nuggets that cannot be purchased in a single transaction!

Say nuggets can only be bought in packs of 4 and/or 7.

What is the **largest** number it is impossible to buy?

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Say nuggets can only be bought in packs of 4 and/or 7.

What is the **largest** number it is impossible to buy?

Clearly it is **not** possible to purchase 1, 2 or 3 nuggets.
It is also **not** possible to purchase 5 or 6.
It is possible to purchase 8 nuggets but **not** 9 or 10.
It is possible to purchase 11 or 12 nuggets, but **not** 13.
It is possible to purchase 14 nuggets, but **not** 15.
It is possible to purchase 16 nuggets, but **not** 17.
It is possible to purchase 18, 19, 20 or 21 nuggets…

DISCARD

Equipment

- 2 x 6-sided dice
- A deck of cards (1s – 12s / A – 9, J, Q K)

Instructions

Each player is dealt 8 cards. The remaining cards are placed on the table as the stock pile. Players take it in turns to roll two six-sided dice and to match cards from their hand to the numbers rolled in one of the following ways:

1. A single card may be matched to the **sum** of the two numbers rolled
   E.g., if a 6 and a 2 are rolled, then the player can discard an 8.
2. Two cards may be matched to the two numbers rolled on the dice.
   E.g., if a 1 and 4 are rolled, the player discards a 1 and a 4.
3. If the player has just one card remaining, they need only match it with **one** of the dice. E.g., if a 1 and a 6 are rolled, the player may discard **either** a 1 or a 6.

If a player is unable to make a match during their turn, they must draw a card from the stock pile.

The winner of the game is the first player to **discard** all of their cards.

Number Thief!

The notorious number thief has struck again!

Three of the digits appear to have been stolen.

So far it seems that the digits that the digits taken have been selected at random.

Instructions
Roll three ten-sided dice to determine the three digits that have been stolen.

For example, the missing digits may be 2, 3 and 7.

If two or more dice show the same digit, then roll again.

Mathematicians are worried that other digits might go missing and so are hoping to be able to use the remaining digits to replace all of the numbers from 1 to 20 in case there's a national number shortage.

Example
Since 2 and 3 are missing, we can't do $3 - 2 = 1$ or $3 + 2 = 5$.

Fortunately, we can calculate $5 - 4 = 1$ and $6 - 1 = 5$.

UnPrime

Equipment

- One counter for each player
- 2 x 6-sided dice

Instructions

Players take it in turns to roll the dice, adding **or** multiplying the two numbers.

They must then move their counter forward that number of spaces.

If they land on a prime number, they must move their counter back to the START!

START	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	FINISH