

Numeracy Learning Specialists:

Building excellence in teaching and learning.

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The major purpose of
assessment is to
support learning and to
inform teaching

Learning Log – Algebra Equivalence

Year _____ Level _____ Task _____ Date _____

Balance scales	Introducing number sentences	Number sentences	Number properties	Order of operations
<p>The student:</p> <ul style="list-style-type: none"> recognises the equals sign is a balance point (uses scales to see what is on one side is equal to the other) 	<p>The student:</p> <ul style="list-style-type: none"> recognises the equals sign as meaning 'is equivalent to' or 'is the same as' not just 'makes' (recognises that $5 + 3 = 6 + 2$) They find missing values in a number sentence ($5 + \underline{\quad} = 6 + 2$). 	<p>The student:</p> <ul style="list-style-type: none"> uses equivalent number sentences involving addition or subtraction to find an unknown ($527 + 96 = ?$ is the same as $527 + 100 - 4 = ?$) 	<p>The student:</p> <ul style="list-style-type: none"> creates and interprets number sentences demonstrating the inverse relationship between multiplication and division balances number sentences involving one or more operations following conventions of order of operations ($5 \times 2 + 4 = 4 \times 2 + ?$, $5 + 2 \times 3 = 11$) 	<p>The student:</p> <ul style="list-style-type: none"> uses equivalent number sentences involving multiplication and division to find unknown quantities explores the use of brackets and order of operations to write number sentences

Details of progression provide nuanced and detailed descriptions of student learning – what students can say, do, make or write. Examples of student learning in each step are not hierarchical.

Numeracy Learning Progression Structure

Numeracy Learning Progression map

To support teachers to use the Numeracy Learning Progressions within Victorian schools, each Numeracy Learning Progression has been mapped against the levels of the Victorian Curriculum

F-10: Mathematics. For Numeracy, the subheading of each step has also been included to support teacher use. Teachers are advised to familiarise themselves with this map.

[📄 The Numeracy Learning Progressions map \(docx - 99.34kb\)](#)

How are the Numeracy Learning Progressions structured?

The Numeracy Learning Progressions are organised in the three strands of the Victorian Curriculum F-10: Mathematics. These files are designed to be used as A3 documents.

Number and Algebra	Measurement and Geometry	Statistics and Probability
Quantifying numbers (PART A) (docx - 113.59kb) Quantifying numbers (PART B) (docx - 113.46kb)	Understanding geometric properties (docx - 108.35kb)	Understanding chance (docx - 107.85kb)
Additive strategies (docx - 110.03kb)	Positioning and locating (docx - 107.28kb)	Interpreting and representing data (docx - 107.09kb)
Multiplicative strategies (docx - 111.34kb)	Measuring time (docx - 107.86kb)	
Operating with decimals (docx -	Understanding units of	

➤ National Numeracy Learning Progression



Understand how the Numeracy Progression works

Explore the learning progression elements

Number
sense and
algebra

Measurement
and geometry

Statistics and
probability

Incredible Equations

On March 16th, some of the students in Mrs. Daniels' class wrote equations equal to 16. During recess, Mrs. Daniels erased parts of each equation. Find the missing parts.

16

$$7 + \underline{\quad} = 16$$

$$\underline{\quad} + 8 + \underline{\quad} = 16$$

$$16 = 3 + \underline{\quad}$$

$$27 - \underline{\quad} = 16$$

$$100 - \underline{\quad} = 16$$

$$16 = \underline{\quad} + 10$$

$$\underline{\quad} - 4 = 16$$

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

$$11 + 5 = \underline{\quad} + 8$$

Show how you know your answer is correct.

What mathematical understandings do these work samples display?

$$7 + \underline{9} = 16$$

$$\underline{3} + 8 + \underline{5} = 16$$

Student A

$$16 = 3 + \underline{13}$$

$$27 - \underline{11} = 16$$

$$100 - \underline{84} = 16$$

$$16 = \underline{6} + 10$$

$$\underline{20} - 4 = 16$$

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

$$11 + 5 = \overset{16}{\underline{10}} + 8$$

Show how you know your answer is correct.

$$10 + 5 = 15 + 8 + 1 = 16$$

$$7 + \underline{9} = 16$$

$$\underline{4} + 8 + \underline{4} = 16$$

Student B

$$16 = 3 + \underline{14}$$

$$27 - \underline{11} = 16$$

$$100 - \underline{94} = 16$$

$$16 = \underline{6} + 10$$

$$\underline{20} - 4 = 16$$

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

$$11 + 5 = \underline{24} + 8$$

Show how you know your answer is correct.

$$8 + 11 + 5 = 24$$

$$7 + \underline{9} = 16$$

$$\underline{4} + 8 + \underline{4} = 16$$

Student C

$$16 = 3 + \underline{13}$$

$$27 - \underline{10} = 16$$

$$100 - \underline{84} = 16$$

$$16 = \underline{6} + 10$$

$$\underline{20} - 4 = 16$$

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

$$11 + 5 = \underline{16} + 8 \quad 24$$

Show how you know your answer is correct.

$$10 + 6 = 16$$

$$7 + \underline{9} = 16$$

$$\underline{4} + 8 + \underline{4} = 16$$

Student D

$$16 = 3 + \underline{13}$$

$$27 - \underline{11} = 16$$

$$100 - \underline{84} = 16$$

$$16 = \underline{6} + 10$$

$$\underline{20} - 4 = 16$$

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

$$11 + 5 = \underline{8} + 8$$

Show how you know your answer is correct.

$8 + 8$ is = to $11 + 5$ they both
= 16

Learning Log – Algebra Equivalence

Give me Post 40

Year	Level	Task	Date
Luke (16)	Jade (24)		
Mara	Georgina		
Spencer			
Jordan			
Claudia →			
Max →			
Till			
Cam.			
Tea			
Ruby.			
Jesse.			
Eric.			

Balance scales	Introducing number sentences	Number sentences	Number properties	Order of operations
<p>The student:</p> <ul style="list-style-type: none"> recognises the equals sign is a balance point (uses scales to see what is on one side is equal to the other) 	<p>The student:</p> <ul style="list-style-type: none"> recognises the equals sign as meaning 'is equivalent to' or 'is the same as' not just 'makes' (recognises that $5 + 3 = 6 + 2$) They find missing values in a number sentence ($5 + ? = 6 + 2$). 	<p>The student:</p> <ul style="list-style-type: none"> uses equivalent number sentences involving addition or subtraction to find an unknown ($527 + 96 = ?$ is the same as $527 + 100 - 4 = ?$) 	<p>The student:</p> <ul style="list-style-type: none"> creates and interprets number sentences demonstrating the inverse relationship between multiplication and division balances number sentences involving one or more operations following conventions of order of operations ($5 \times 2 + 4 = 4 \times 2 + 2$, $5 + 2 \times 3 = 11$) 	<p>The student:</p> <ul style="list-style-type: none"> uses equivalent number sentences involving multiplication and division to find unknown quantities explores the use of brackets and order of operations to write number sentences

Details of progression provide nuanced and detailed descriptions of student learning – what students can say, do, make or write. Examples of student learning in each step are not hierarchical.

PLC Learning Log:

Task _____

Date _____

What level are the students currently working at?

What goals are set for the students' next level of learning?

-
-

Next stage of learning. Once you have evidence of the students' current level of understanding and thinking. You then decide what the very next stage for them is, your teaching is based on the evidence you collect around student needs, not necessarily based on the grade level curriculum statement.

What teaching strategies/tasks could be used to achieve the goals?

-
-
-
-

What resources are needed?

-
-
-
-

What evidence would show that the goals are met?

Number Balance

$$\boxed{2} \boxed{+} \boxed{3} \boxed{=} \boxed{7} \boxed{-} \boxed{2}$$

Student A

Make your own number balances like the one shown above

$$\boxed{4} \boxed{+} \boxed{5} \boxed{=} \boxed{3} \boxed{+} \boxed{6}$$

$$\boxed{9} \boxed{+} \boxed{9} \boxed{=} \boxed{20} \boxed{-} \boxed{2}$$

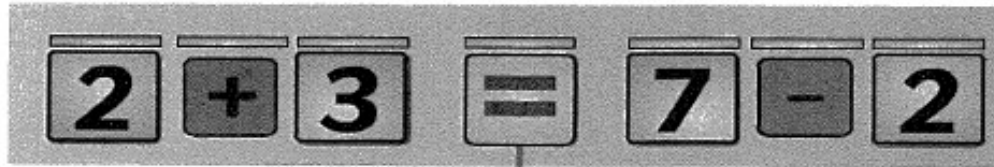
$$\boxed{\cancel{8}} \boxed{-} \boxed{2} \boxed{=} \boxed{3} \boxed{+} \boxed{5}$$

$$\boxed{9} \boxed{+} \boxed{3} \boxed{=} \boxed{20} \boxed{-} \boxed{8}$$

Number Balance

Name

Student C



Make your own number balances like the one shown above

$$6 + 6 = 100 - 88$$

$$7 + 5 = 5 + 7$$

$$16 - 4 = 4 + 8$$

$$9 + 3 = 20 - 8$$

$$13 + 5 = 20 - 2$$

$$14 + 4 = 24 - 6$$

$$~~9 + 9~~ \quad 11 + 7 = 1 + 17$$

$$15 + 3 = 30 - 12$$

$$17 + 1$$

$$16 + 2$$

$$10 + 0 =$$

Student B

$$6 + 6 + 6 = 14 + 16 - 2$$

Student C

$$17 + 1 = 1 + 17$$

$$19 - 1 = 100 - 82$$

$$6 \times 3 = 3 \times 6$$

$$9 \times 2 = 2 \times 9$$

$$9 + 9 = 200 - 182$$

$$18 \times 1 = 1 \times 18$$

$$300 - 282 = 400 - 382$$

$$500 - 482 = 600 - 582$$

$$\begin{array}{r} 6 + 6 + 6 = 14 + 5 - 2 \\ 18 \qquad 18 - 6 \end{array}$$

Effective Teachers of Numeracy

Principles to Action (NCTM, 2015)

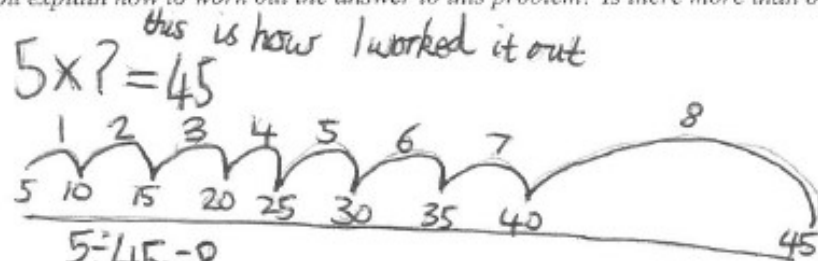
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

A team of 5 players scores a total of 45 points in a game of basketball.

Each player scores the same number of points.

How many points does each player score?

Can you explain how to work out the answer to this problem? Is there more than one way?



$$5 \div 45 = 8$$

$$5 \times 8 = 45$$

The Answer is eight

Can you write your story using only **mathematical symbols** and **numbers** (a number sentence)? You might be able to do this more than one-way? (Creativity)

$$5 \times ? = 45$$

$$5 \div 45 = 8$$

$$5 \times 8 = 45$$

A team of 5 players scores a total of 45 points in a game of basketball.

Each player scores the same number of points.

How many points does each player score?

Can you explain how to work out the answer to this problem? Is there more than one way?

$$40 + 40 = 80 + 5 = 85$$

85

$$80 \div 5 = 8 + 1 = 9$$

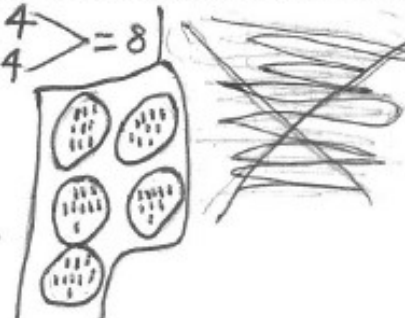
Can you write your story using only **mathematical symbols** and **numbers** (a number sentence)? You might be able to do this more than one-way? (Creativity)

$$40 \div 5 = 4 \rightarrow = 8$$

$$40 \div 5 = 4$$

$$5 \div 1 = 2$$

$$8 + 1 = 9$$



IMPACTS ON TEACHING

**Collective Teacher Efficacy- 1.57
(John Hattie- revised 2017)**

When a teacher team is empowered to believe that working together makes a difference in student learning- they do!





1. Setting Goals

Overview

Lessons have clear learning intentions with goals that clarify what success looks like.

Lesson goals always explain what students need to understand, and what they must be able to do. This helps the teacher to plan learning activities, and helps students understand what is required.

Key elements

- Based on assessed student needs
- Goals are presented clearly so students know what they are intended to learn
- Can focus on surface and/or deep learning
- Challenges students relative to their current mastery of the topic
- Links to explicit assessment criteria



2. Structuring Lessons

Overview

A lesson structure maps teaching and learning that occurs in class.

Sound lesson structures reinforce routines, scaffold learning via specific steps/activities. They optimise time on task and classroom climate by using smooth transitions. Planned sequencing of teaching and learning activities stimulates and maintains engagement by linking lesson and unit learning.

Key elements

- Clear expectations
- Sequencing and linking learning
- Clear instructions
- Clear transitions
- Scaffolding
- Questioning/feedback
- Formative assessment
- Exit cards



3. Explicit Teaching

Overview

When teachers adopt explicit teaching practices they clearly show students what to do and how to do it.

The teacher decides on learning intentions and success criteria, makes them transparent to students, and demonstrates them by modelling. The teacher checks for understanding, and at the end of each lesson revisits what was covered and ties it all together (Hattie, 2009).

Key elements

- Shared learning intentions
- Relevant content and activities
- New content is explicitly introduced and explored
- Teacher models application of knowledge and skills
- Worked examples support independent practice
- Practice and feedback loops address misunderstandings



4. Worked Examples

Overview

A worked example demonstrates the steps required to complete a task or solve a problem.

By scaffolding the learning, worked examples support skill acquisition and reduce a learner's cognitive load.

The teacher presents a worked example and explains each step. Later, students can use worked examples during independent practice, and to review and embed new knowledge.

Key elements

- Teacher clarifies the learning objective, then demonstrates what students need to do to acquire new knowledge and master new skills
- Teacher presents steps required to arrive at the solution so students' cognitive load is reduced and they can focus on the process
- Students practice independently using the worked example as a model



5. Collaborative Learning

Overview

Collaborative learning occurs when students work in small groups and everyone participates in a learning task.

There are many collaborative learning approaches. Each uses varying forms of organisation and tasks.

Collaborative learning is supported by designing meaningful tasks. It involves students actively participating in negotiating roles, responsibilities and outcomes.

Key elements

- Students work together to apply previously acquired knowledge
- Students cooperatively solve problems using previously acquired knowledge and skills
- Students work in groups that foster peer learning
- Groups of students compete against each other



6. Multiple Exposures

Overview

Multiple exposures provide students with multiple opportunities to encounter, engage with, and elaborate on new knowledge and skills.

Research demonstrates deep learning develops over time via multiple, spaced interactions with new knowledge and concepts. This may require spacing practice over several days, and using different activities to vary the interactions learners have with new knowledge.

Key elements

- Students have time to practice what they have learnt
- Timely feedback provides opportunities for immediate correction and improvement



7. Questioning

Overview

Questioning is a powerful tool and effective teachers regularly use it for a range of purposes. It engages students, stimulates interest and curiosity in the learning, and makes links to students' lives.

Questioning opens up opportunities for students to discuss, argue, and express opinions and alternative points of view.

Effective questioning yields immediate feedback on student understanding, supports informal and formative assessment, and captures feedback on effectiveness of teaching strategies.

Key elements

- Plan questions in advance for probing, extending, revising and reflecting
- Teachers use open questions
- Questions used as an immediate source of feedback to track progress/understanding
- Cold call and strategic sampling are commonly used questioning strategies



8. Feedback

Overview

Feedback informs a student and/or teacher about the student's performance relative to learning goals.

Feedback redirects or refocuses teacher and student actions so the student can align effort and activity with a clear outcome that leads to achieving a learning goal.

Teachers and peers can provide formal or informal feedback. It can be oral, written, formative or summative. Whatever its form, it comprises specific advice a student can use to improve performance.

Key elements

- Precise, timely, specific, accurate and actionable
- Questioning and assessment is feedback on teaching practice
- Use student voice to enable student feedback about teaching



9. Metacognitive Strategies

Overview

Metacognitive strategies teach students to think about their own thinking.

When students become aware of the learning process, they gain control over their learning.

Metacognition extends to self-regulation, or managing one's own motivation toward learning. Metacognitive activities can include planning how to approach learning tasks, evaluating progress, and monitoring comprehension.

Key elements

- Teaching problem solving
- Teaching study skills
- Promotes self-questioning
- Classroom discussion is an essential feature
- Uses concept mapping



10. Differentiated teaching

Overview

Differentiated teaching are methods teachers use to extend the knowledge and skills of every student in every class, regardless of their starting point.

The objective is to lift the performance of all students, including those who are falling behind and those ahead of year level expectations.

To ensure all students master objectives, effective teachers plan lessons that incorporate adjustments for content, process, and product.

Key elements

- High quality, evidence based group instruction
- Regular supplemental instruction
- Individualised interventions

Learning Log – Algebra

Year _____ Level _____ Task _____ Date _____

This Learning Progression describes how a student becomes increasingly able to identify a pattern as something that is a discernible regularity in a group of numbers or shapes. Figuring out how a pattern works brings predictability and allows the making of generalisations. As students become increasingly able to connect patterns with the structure of numbers, they create a foundation for algebraic thinking (that is, thinking about generalised quantities).

<p>Identifying and creating patterns (Make It)</p> <p>The student:</p> <ul style="list-style-type: none"> identifies standard patterns (dice or domino) without counting individual items. creates repeating patterns with numbers and shapes (circle, square, circle, square or 1,2,3 1,2,3 1,2,3). 	<p>Continues repeating patterns (Draw It)</p> <p>The student:</p> <ul style="list-style-type: none"> identifies the pattern unit within the repeating pattern (continues the pattern) finds the missing element in a pattern involving shapes or objects. 	<p>Continuing number patterns (List or Table)</p> <p>The student:</p> <ul style="list-style-type: none"> continues patterns where the difference between each term is the same number (2, 4, 6, 8, 10 ...) describes rules for continuing patterns where the difference between each term is the same number (to find the next number in the pattern 3, 6, 9, 12 ... you add 3) sequences numbers to identify a pattern or rule. 	<p>Generalising patterns - finds a rule (Writes it as a Number Sentence)</p> <p>The student:</p> <ul style="list-style-type: none"> identifies a rule in numerical patterns and records it in words or as a numerical expression (2, 4, 6, 8, 10 ... is $n + 2$, or 2, 6, 18, 54 ... is $3n$) predicts a higher term of a pattern using the pattern's rule. 	<p>Algebraic expressions- reverses the rule (Graph It)</p> <p>The student:</p> <ul style="list-style-type: none"> Can reverse the rule (can see the relationship between the numbers in a table either way) and record it. Represent rules from patterns in a number of ways e.g. $2n+6=y$ can also be written as $3(n+2)=y$
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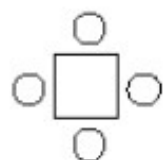
Details of progression provide nuanced and detailed descriptions of student learning – what students can say, do, make or write. Examples of student learning in each step are not hierarchical.

Squares and Circles

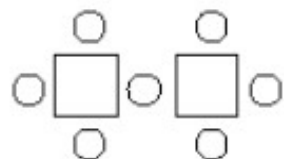
This problem gives you the chance to:

- find and use a pattern
-

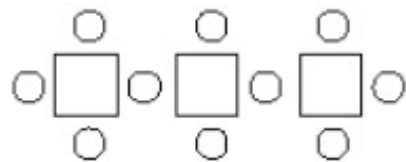
Jack makes patterns using squares and circles.



1 square
4 circles



2 squares
7 circles



3 squares
10 circles

1. Draw a diagram to show Jack's pattern using 4 squares.

Jack makes a table to show the number of circles he needs to make patterns using different numbers of squares.

Number of squares	1	2	3	4	5
Number of circles	4	7	10		

2. How many circles does Jack need to make a pattern using 4 squares?
Write your answer in the table above.

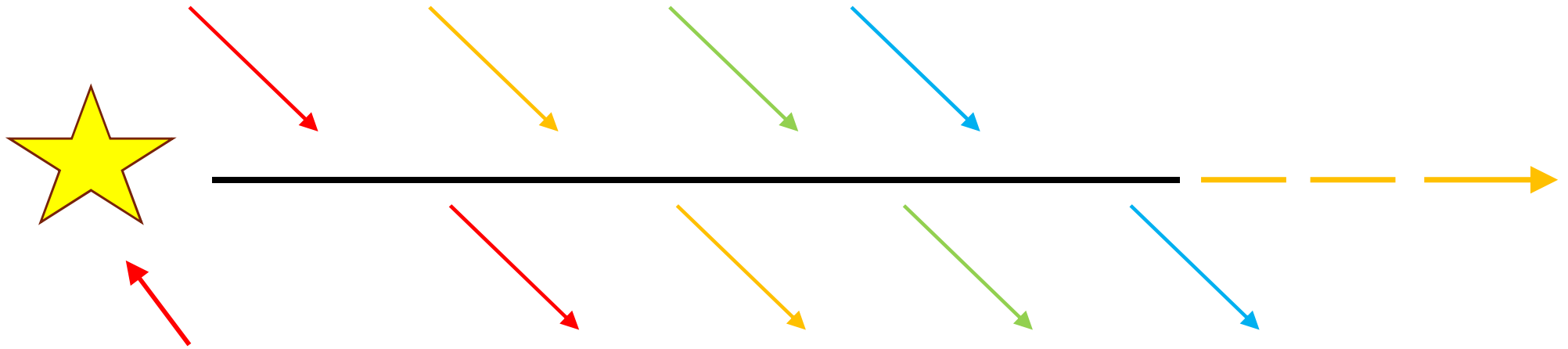
3. How many circles does Jack need to make a pattern using 5 squares?
Write your answer in the table
Explain how you figured it out.

4. Jack makes a pattern using 10 squares.
How many circles does he use? _____
Show how you figured it out.

5. How many squares does Jack need to make a pattern that uses 40 circles?

Explain how you figured it out.

Multiple Entry & Exit Points

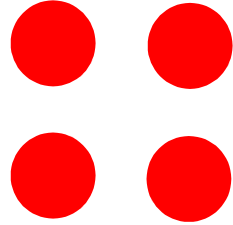


Starting Point: EVERYONE must be able to access the task.

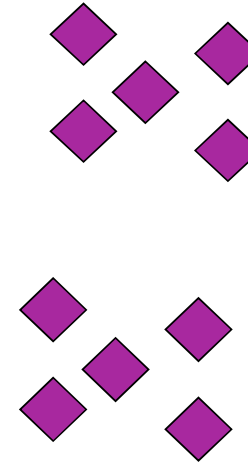
All students will enter the task at different levels. The goal is for them to exit at a point further along the line.

NO TASK IS EVER FINISHED!

Reference: Charles Lovitt



Collections, counters, numberlines.....



Make

Materials

Conceptual Understanding

Name

Language

read, say, write and say it another way

Record

Symbols

recognise, read it and write it in a number sentence

2 Groups of 5 is 10

four

$2 \times 5 = 10$

4

Reference: Di Siemon

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