Meet the Examiners

Mathematical Methods 2017

Paper 1

A Production by Rod Watson
&
Mary Papp

rodwatson1@hotmail.com
mary.a.papp@hotmail.com

mav.vic.edu.au
Mathematical Methods MA082

Written Examination Paper 1 2017.

General comments.

* The number of candidates who sat for the 2017 examination was 16 355. Completion of past papers (available on the VCAA website) will enhance examination technique. The advice given in previous Examination Reports continues to contain valuable information. There were fewer candidates who scored 0 or 40.
* Reports from Chief Assessor is be available on VCAA web site.

* It was pleasing to note that many students wrote responses that clearly demonstrated correct **methodology with legibly set out and logically sequenced reasoning**. Adherence to **correct mathematical notation** also showed an improvement.

* Students seemed most confident and competent in handling the questions that tested probability and differentiation techniques, especially the product and quotient rules.

* Correct deployment of the **chain rule**, however, was not as well handled.

* Students who persisted with the later parts of questions, even if they did not fare as well in the earlier part of the question were often rewarded and were able to gain marks. This was evident in Q.5c, Q.7c and Q.8c.

* It is disappointing to note that while many students could make solid in-roads to a solution, their efforts were **marred by poor arithmetic skills or incorrect algebraic manipulation**. This was evident in Q.4, Q.5a, Q.5c, Q.8a, and Q.9a – all of which involved operating fractions.
* 2018 students are advised to practise **simplifying algebraic expressions that involve fractions, negatives, square roots and inequalities**. Correct **placement of brackets** is crucial to success in Mathematics in situations with or without CAS.

* Students who re-read a question to ensure that their answer did **address what was specified by the question** fared better than those that

did not. This included instructions such as **‘show that’** (Q.3a and Q.9b) and **‘hence’** (Q.2b and Q.6b) and these instructions should not be ignored. Questions 3a and 9b required a step by step demonstration of how one side of the given equation became the other side of the equation. Q.2b (integration by recognition) and Q.6b (solving a cubic equation involving trigonometric expressions) required utilisation of an answer or given statement of fact that appeared in the previous part of the question.

*   Questions 2 and 6 proved to be the most challenging problems for students. Practice with these types of problems is recommended.

§   Important note: markers will mark precisely what is provided by the candidate. They can make no assumptions about what the candidate “might have meant to write”.

*   **Problem areas**

- Probabilities outside [0, 1]
- Solution of non-routine equations
- **Very poor** basic arithmetic including manipulation of fractions §§
- Some seemed to have a problem bringing all necessary equipment to a Mathematics examination – pen**s**, pencil**s**, eraser, straight edge.
- Those candidates who were “short & sweet” generally fared better.
- Omission of *dx* was still sighted.

§§ Common examples include $5 \times 6 = 36$, $8 \times 5 = 45$, $3^3 = 9$, $100^2 = 1000$, $9 \times 4 = 37$, $\frac{36}{3} = 13$ as well as the inability to cancel $\frac{30000}{16}$.

Question 1

a. **Let $f: (-2, \infty) \to R,\ f(x) = \dfrac{x}{x+2}$.**

Differentiate f with respect to x.

$$f'(x) = \frac{(x+2)-x}{(x+2)^2}$$

Needs to be simplified

$$= \frac{2}{(x+2)^2}$$

STOP!

Marks	0	1	2	Mean
%	12	19	69	1·6

Points to note:

* This question was well handled with those choosing to use the quotient rule being more successful than those using the product rule (all sorts of trouble with the division).

* Some responses remind one of Schubert's Symphony Number 8 – unfinished! Specifically, those who did not simplify $x + 2 - x$ must have parents who tidied their rooms for them!!!

* There were problems with those who used the quotient rule by being too smart by "cancelling" x+2 in the numerator with **an** x+2 in the denominator.

A common misconception!!!!!!!

$$f'(x) = \frac{\cancel{(x+2)} - x}{(x+2)^{\cancel{2}}}$$

* Others ***unnecessarily*** expanded $(x+2)^2$ and did so incorrectly.

* Strangely, some who performed poorly here performed competently on later questions.

b. **Let $g(x) = (2 - x^3)^3$.**
Evaluate $g'(1)$.

$$g'(x) = 3(2 - x^3)^2 \times (-3x^2)$$
$$= -9x^2(2 - x^3)^2$$
$$g'(1) = -9 \times (2 - 1)^2$$
$$= -9$$

Marks	0	1	2	Mean
%	14	21	65	1·5

Points to note.

* This type of double-barrelled question has appeared in every Paper 1 from 2008, except in 2015. Again, the substitution of 1 for x was missed (as similarly in previous years).

* Sometimes the $\frac{du}{dx}$ term $(-3x^2)$ was AWL.

* Some encountered problems with negatives.

* Those who chose to expand the brackets often came to grief.

Question 2

Let $y = x\log_e(3x)$

a. Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \log_e(3x) + x \times \frac{1}{3x} \times 3 \quad \left[\frac{d(3x)}{dx} = 3\right]$$
$$= \log_e(3x) + 1$$

Marks	0	1	2	Mean
%	12	31	57	1·5

Points to note.

* Most knew to use the product rule. But there were problems with differentiating $\log_e(3x)$ with answers such as $\log_e(3x) + 3$ and $\log_e(3x) + \frac{1}{3}$ emerging.

* Some left $\frac{x}{x}$ unsimplified or simplified it to 0.

b. **Hence, calculate $\int_1^2 (\log_e(3x) + 1)\,dx$. Express your answer in the form $\log_e(a)$ where a is a positive integer.**

$$\int_1^2 (\log_e(3x) + 1)\,dx = [x\log_e(3x)]_1^2$$
$$= 2\log_e(6) - \log_e(3)$$
$$= \log_e\left(\frac{36}{3}\right)$$
$$= \log_e(12) \text{ or } \ln(12)$$

Marks	0	1	2	Mean
%	36	19	45	1·1

Points to note.

* This question was not well handled. Students generally were not able to form an integral from their previous answer, often ignoring the "hence" instruction by attempting to integrate the given expression from scratch.

* Some very poor application of log laws and/or log notation was observed.

* Calculation mistakes involving multiplication tables were rife.

* Poor notation such as $\boxed{\mathbf{log}e^{(12)}}$ was seen but not as much as has occurred in previous years. Encourage recalcitrant students to use **ln(*x*).** Such students seem to lack basic observational skills.

Question 3

Let $f: [-3, 0] \to R, f(x) = (x+2)^2 (x-1)$.

a. Show that $(x+2)^2 (x-1) = x^3 + 3x^2 - 4$.

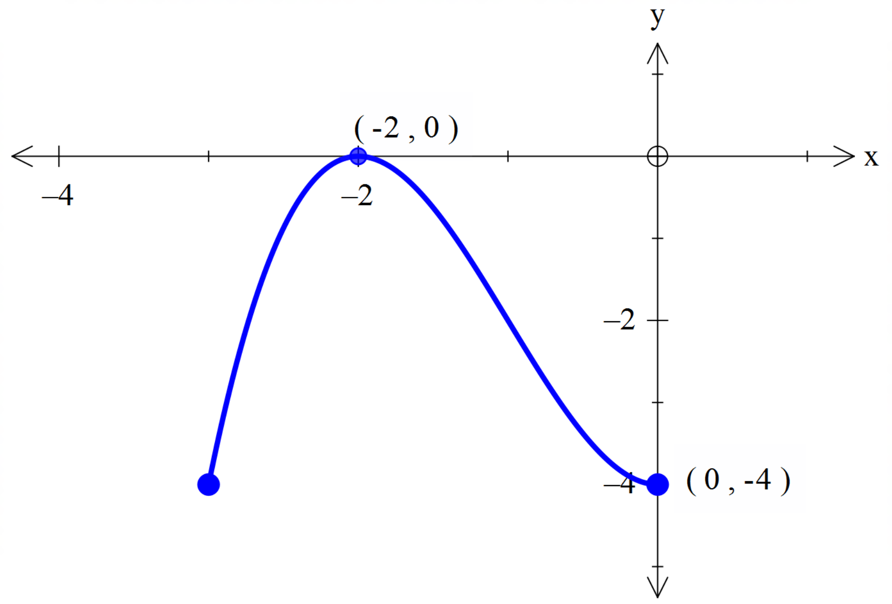
$$
\begin{aligned}
\text{LHS} &= (x+2)^2 (x-1) \\
&= (x^2 + 4x + 4)(x-1) \quad \text{[Need to see.]} \\
&= x^3 + 4x^2 + 4x - x^2 - 4x - 4 \\
&= x^3 + 3x^2 - 4 \\
&= \text{RHS, as required.}
\end{aligned}
$$

Marks	0	1	Mean
%	21	79	0·8

Points to note.

* Whilst LHS and RHS could be omitted from an acceptable solution, they are included as a reminder to 2018 candidates to "play it safely".
* Lines 2, 3 and 4 are essential in order to "show" correctly.
* Essential brackets were omitted. It should be noted in that $x^2 + 4x + 4(x-1)$ is not equivalent to $x^3 + 4x^2 + 4x - x^2 - 4x - 4$
* In some cases, the expansion was wrong but the candidate was able to "cook" the answer. Jamie Oliver would not be amused!
* Surely Year 12 students could expand $(x+2)^2$ directly, here and in Q.1a.
* Many "showed" by poor setting out, assuming to be true what was to be proven.

b. **Sketch the graph of *f* on the axes below. Label the axis intercepts [sic!] and any stationary points with their coordinates.**



Marks	0	1	2	3	Mean
%	11	10	43	36	2·0

Points to note.

* Some very good graphs were seen even by weaker candidates.
* Common mistakes included using **R** as the domain.
* Some graphs took on the shape of an inverted parabola rather than a cubic, due to the failure of recognising a stationary point at (0, -4) whilst other graphs could have been a segment of a quartic or quintic graph.
* A correct graph had to include closed circles at the end-points.

Question 4

In a large population of fish, the proportion of angel fish is $\frac{1}{4}$.

Let $\hat{P}$ be the random variable that represents the sample proportion of angel fish for samples of size n drawn from the population. Find the smallest integer value of n such that the standard deviation of $\hat{P}$ is less than or equal to $\frac{1}{100}$.

Common errors:

* n not included under the radical

* transposition for n while handling the inequality

* basic arithmetic

$$p = \frac{1}{4}$$

$$\sqrt{\frac{\frac{1}{4} \times \frac{3}{4}}{n}} \le \frac{1}{100}$$

$$\sqrt{\frac{3}{16n}} \le \frac{1}{100}$$

$$\frac{3}{16n} \le \frac{1}{10\,000}$$

$$\frac{16n}{3} \ge 10\,000$$ **[Note changed direction of inequality.]**

$$n \ge \frac{30\,000}{16}$$

$$n \ge 1875$$

¥12 *<u>should be able to</u>* cancel 2s.

Smallest integer value of n is 1875.

Marks	0	1	2	Mean
%	28	41	31	1·0

Points to note.

* Most identified the correct formula.
* Disappointingly many were unable to correctly transpose the inequality to solve for n or correctly manipulate the arithmetic involving rational numbers.
* Some students had poor notational work in that they did not extend the square root sign to include 'n' on the denominator. Some even had the n under the radical in one line and outside in the next as though the n was moving around on a tourist visa!

* A significant number started correctly but had $\frac{1}{100}$ on the LHS and pronumerals on RHS. Then, in proceeding, $\leq$ or $\geq$ were treated as an $=$ sign and not reversed when the reciprocal was taken of both sides or when sides were swapped. That is, $\geq$ and $\leq$ were deemed by some to be interchangeable.

* Some found cancellation of $\frac{30000}{16}$ too hard or got to $\frac{7500}{4}$ but could not complete the simplification. Knowledge of basic multiplication tables???

Question 5.
For Jac to log onto a computer successfully, Jac must type the correct password. Unfortunately, Jac has forgotten the password. If Jac types the wrong password, Jac can make another attempt. The probability of success on any attempt is $\frac{2}{5}$. Assume that the result of each attempt is independent of the result of any other attempt. A maximum of three attempts can be mad.

a. What is the probability that Jac does not log on to the computer successfully?

Let X be the number of attempts.

$$\Pr(X = 0) = \left(\tfrac{3}{5}\right)^3 \text{ or } \frac{27}{125} \qquad \textbf{[Either answer.]}$$

Marks	0	1	Mean
%	24	76	0.8

Point to note.

* Students clearly identified what was required but many could not gain the mark due to poor arithmetical evaluation. E.g., $3^3 = 9$.

b. Calculate the probability that Jac logs onto the computer successfully. Express your answer in the form $\frac{a}{b}$, where a and b are positive integers.

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) = 1 - \frac{27}{125}$$
$$= \frac{98}{125}$$

Marks	0	1	Mean
%	34	66	0.7

Point to note.

* In the main, students recognised that the required event was the complement of the event in part (a). Others took the longer and time consuming route utilising a tree diagram to identify all possibilities for Jac to log on successfully.

c. Calculate the probability that Jac logs on successfully on the second or on the third attempt. Express your answer in the form $\frac{c}{d}$, where c and d are positive integers.

Pr(log on 2^nd or 3^rd attempt) = Pr(N,S) + Pr(N,N,S)

$$= \frac{3}{5} \times \frac{2}{5} + \left(\frac{3}{5}\right)^2 \times \frac{2}{5}$$

$$= \frac{6}{25} + \frac{18}{125}$$

$$= \frac{48}{125}$$

Marks	0	1	2	Mean
%	26	24	50	1.2

Point to note.

* A tree diagram would also provide a neat solution.
* This question was well attempted. Most pleasing was that many students who struggled with previous parts of the question, generally made use of a tree diagram to find the two required cases.
* Common errors included, use of conditional probability, use of binomial theorem, or not realising that once Jac logged in, there was no need to keep attempting (three cases).
* A small number of students recognised that Pr(success on second or third attempt) = Pr(success) – Pr(success on the first attempt) = $\frac{98}{125} - \frac{2}{5}$.

Question 6.

Let $(\tan(\theta) - 1)(\sin(\theta) - \sqrt{3}\cos(\theta))(\sin(\theta) + \sqrt{3}\cos(\theta)) = 0$

a. Stated all possible values of $\tan(\theta)$.

$$\tan(\theta) = 1, \pm\sqrt{3}\ .$$

Marks	0	1	Mean
%	75	25	0.3

A SHOCKER !!!!!

Point to note.

* This was very badly done and not attempted by all. Most did not see that $\tan(\theta)$ was all that was needed and that values of θ were required in Q.6b and so over-engaged the problem. This question is a good illustration of the necessity of reading questions carefully both before **&** **after** writing the solution.

b. Hence, find all possible solutions for $(\tan(\theta) - 1)(\sin^2(\theta) - 3\cos^2(\theta)) = 0$, where $0 \leq \theta \leq \pi$.

$\tan(\theta) = 1$, whence $\theta = \frac{\pi}{4}$ in Q_1 only OR

$\tan(\theta) = \sqrt{3}$, whence $\theta = \frac{\pi}{3}$ in Q_1 OR

$\tan(\theta) = -\sqrt{3}$, whence $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ in Q_2.

Hence $\theta = \frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}$.

Marks	0	1	2	Mean
%	41	34	25	0.8

Points to note.

* The "hence" instruction was often not acknowledged. Nevertheless, some managed to fumble their way to obtaining the three answers.

* Some could obtain $\frac{\pi}{4}$ but could not obtain the other two.

* It is worrying that others who managed to obtain $\tan(\theta) = 1$ and $\sqrt{3}$ gave the third value as $\frac{1}{\sqrt{3}}$ instead of $-\sqrt{3}$.

* Giving a solution in the form such as $\tan(\theta) = \frac{\pi}{4}$ is not accepted.

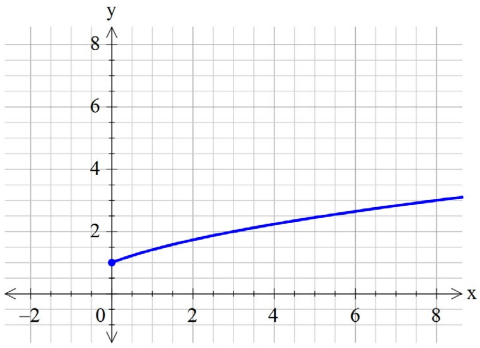
Restricted domain

Question 7.

Let f: $[0, \infty) \to R, f(x) = \sqrt{x+1}$.

a. State the range of f.

Range = $[1, \infty)$



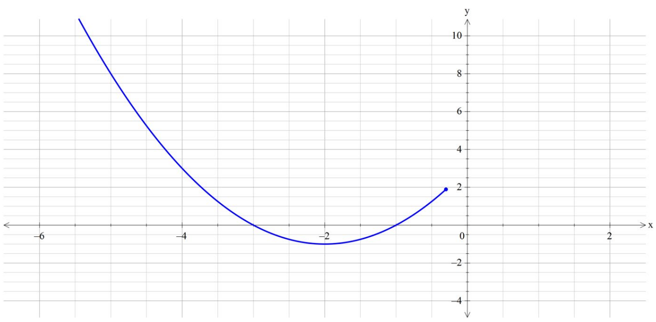
Marks	0	1	Mean
%	35	65	0.6

Points to note.

* Reasonably well done. √1 **was** required to be simplified to 1.
* There was some incorrect use of [and (. Markers may not interpret what the candidate meant; answers must be **explicit.**
* Reminder: $\sqrt{a}$ refers to the positive square root of a.

b. Let g: $(-\infty, c] \to R$, $g(x) = x^2 + 4x + 3$, where $c < 0$.

i. Find the largest possible value of c such that the range of g is a subset of the domain of f.



ran$g$ ⊆ dom$f$ means that $g(x) \geq 0$

Domain of g can be $(-\infty, c]$

Whence $c_{MAX} = -3$

Marks	0	1	2	Mean
%	53	18	29	0.8

Points to note.

* Students who successfully solved this question used the equation or a graph or both.

* Those who let $g(x) = 0$ usually succeeded. Occasionally $c = -1$ was chosen.

ii. For the value of c found in part b.i., state the range of $f(g(x))$.

Range = $[1, \infty)$

Marks	0	1	Mean
%	80	20	0.2

Point to note.

* This question was poorly handled. Many students tried to find the range of $g(x) \subseteq f(x)$ but did not realise that it is the domain of $g(x)$ is the required one. Students gave the answer $[3, \infty) \subseteq [0, \infty)$ and did not know what would be the next step from here. Teachers are urged to practise this style of question with their students

c. Let h: $R \to R$, $h(x) = x^3 + 3$.

State the range of $f(h(x))$.

dom[$f(h(x))$] = R

$\therefore$ ran[$f(h(x))$] = $[2, \infty)$

Marks	0	1	Mean
%	70	30	0.3

Point to note.

* Most students could identify the composite function but struggled with determining its range.

Question 8.

For events A and B from a sample space, $\Pr(A|B) = \frac{1}{5}$ and $\Pr(B|A) = \frac{1}{4}$. Let $Pr(A \cap B) = p$.

a. Find $Pr(A)$ in terms of p.

$$Pr(A \cap B) = Pr(A) \times Pr(B|A)$$

$$p = Pr(A) \times \frac{1}{4}$$

$$\therefore\ Pr(A) = 4p$$

Marks	0	1	Mean
%	27	73	0.7

Points to note.

* This question was generally well handled. The most common mistakes included **solving** for Pr(B), (failure to answer the specific question).

* Another problem was incorrectly transposing $\frac{p}{Pr(A)} = \frac{1}{4}$.

* Sometimes an answer was presented in the form $p = \frac{\Pr(A)}{4}$.

b. Find $Pr(A' \cap B')$.

First prepare a table of values. Black entries completed from information so far.

$\mathscr{E}$	A	A'	
B	p	$4p$	$5p$
B'	$3p$	$1-8p$	$1-5p$
	$4p$	$1-4p$	1

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

$$Pr(B) = \frac{Pr(A \cap B)}{Pr(A|B)}$$

$$= \frac{p}{\frac{1}{5}}$$

$$= 5p$$

NB This enables the entries in $\boxed{\text{RED}}$ to be completed.

Or $Pr(A \cup B) = 4p + 5p - p = 8p$.
And $(A \cup B)' = A' \cap B'$
Hence $Pr(A' \cap B') = 1 - 8p$

Marks	0	1	2	Mean
%	55	8	37	0.8

Points to note.

* Successful students usually used a Karnaugh map or a Venn diagram to arrive at their answer.
* There were a lot of misconceptions of the connection between conditional probabilities and $Pr(A' \cap B')$. Many students assumed that events A and B are independent and incorrectly used $Pr(A' \cap B') = Pr(A') \times Pr(B')$.

c. Given that $Pr(A \cup B) \le \frac{1}{5}$, state the largest possible interval for p.

$$Pr(A \cup B) \le \frac{1}{5}$$

$$4p + 5p - p \le \frac{1}{5}$$

$$8p \le \frac{1}{5}$$

$$p \leq \frac{1}{40}$$

BUT $p \neq 0$ since the values of the conditional probabilities would be indeterminate.

$$\therefore\ 0 < p \leq \frac{1}{40}.$$

$$\text{Or } p \in (0, \frac{1}{40}]$$

Marks	0	1	2	Mean
%	38	52	10	0.7

Points to note.

* This question was well attempted. Most students identified that $\Pr(A \cup B) = 8p$, even if not from part b. But only a few students identified the correct interval because students did not consider that in this case $p \neq 0$.

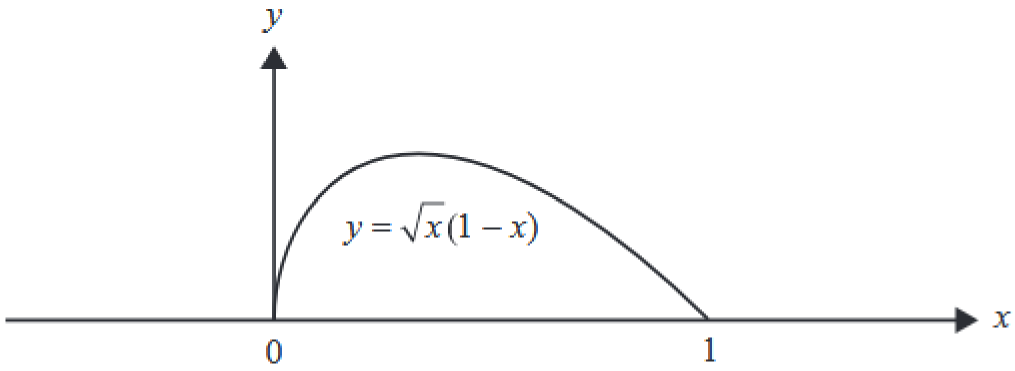
* Common incorrect answers included $p = \frac{1}{40}$ or $p \leq \frac{1}{40}$ (allowing negative probabilities) and $0 \leq p \leq \frac{1}{40}$.

* Some who obtained $8p \leq \frac{1}{5}$ went on to write $p \leq \frac{8}{5}$. This lack of accuracy with primary school fractions is worrying.

* Sometimes "interval" interpreted as referring to a single value.

Question 9.

The graph of f: $[0, 1] \to R$, $f(x) = \sqrt{x}(1 - x)$ is shown below.



a. **Calculate the area between the graph of f and the x-axis.**

$$\text{Area} = \int_0^1 (x^{1/2} - x^{3/2})\,dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right]_0^1$$

$$= \frac{2}{3} - \frac{2}{5}$$

$$= \frac{10}{15} - \frac{6}{15}$$

$$= \frac{4}{15}$$

Marks	0	1	2	Mean
%	44	9	47	1.0

Points to note.

* Concise solutions were produced by many and correct solutions were produced by candidates who had not fared well earlier in the paper. Fraction manipulation was handled surprisingly well.

* It is highly disturbing that so many candidates used a fictitious "product rule for antidifferentiation":

$$\int f(x)g(x)\,dx = \int f(x)dx \times \int g(x)dx.$$

* More disturbingly, x was sometimes taken from the integral as a "common factor".

b. For x in the interval (0, 1), show that the gradient of the graph of f is $\dfrac{1-3x}{2\sqrt{x}}$.

$$f'(x) = \frac{1}{2}x^{-1/2} - \frac{3}{2}x^{1/2}$$

$$= \frac{1}{2\sqrt{x}} - \frac{3\sqrt{x}}{2}$$

$$= \frac{1 - 3\sqrt{x}\times\sqrt{x}}{2\sqrt{x}}$$ **[Use LCD or rationalise the numerator.]**

$$= \frac{1-3x}{2\sqrt{x}}$$ **as required.**

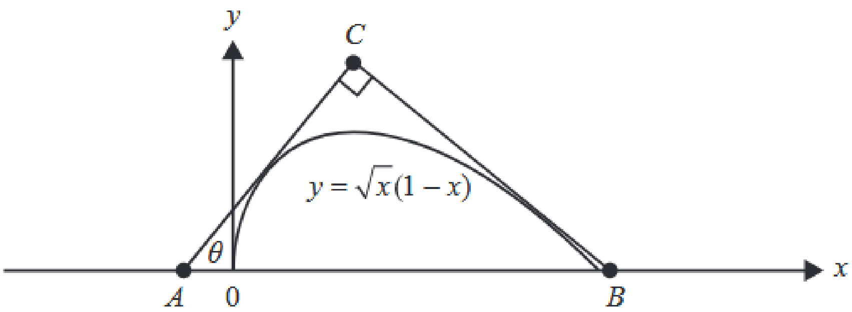
*** Note that $x \in (0, 1)$ since the gradient is not defined at the end-points.**

Marks	0	1	Mean
%	65	35	0.4

Points to note.

* "Show" questions require including all of the steps to demonstrate exactly what was done, but many students often omitted steps. A common pattern was to go straight from the first line of differentiation, immediately to the final line, with no indication of making denominators equal. The "show" had to include a clear use of LCD or rationalising for the numerator of one of the terms.

The edges of the right-angled triangle ABC are the line segments AC and BC, which are tangent to the graph of *f*, and the line segment AB, which is part of the horizontal axis, as shown below. Let *θ* be the angle that AC makes with the positive direction of the horizontal axis, where 45° ≤ θ < 90°.



c. **Find the equation of the line through B and C in the form $y = mx + c$, for $\theta = 45°$.**

Gradient of AC = 1 and so gradient of BC = -1.

To find the value of *c*, we need to find the point of intersection of BC with the curve.

$$\frac{1 - 3x}{2\sqrt{x}} = -1$$

$$1 - 3x = -2\sqrt{x}$$

$$3x - 2\sqrt{x} - 1 = 0$$

In solving this equation, do not square both sides

Let $\sqrt{x} = a$ [Not necessary, but helpful.]

$$3a^2 - 2a - 1 = 0$$

$$(3a + 1)(a - 1) = 0$$

$$\therefore \quad a = -\frac{1}{3} \text{ or } 1$$

$$\therefore \quad \sqrt{x} = -\frac{1}{3} \text{ or } 1$$

BUT $\sqrt{x}$ cannot be negative by definition and so $x = 1$ only. Thence $y = 0$.

BC meets the curve at (1, 0).

$y = mx + c$ **becomes** $0 = -1 \times 1 + c$

$c = 1$

$y = 1 - x$ **as required.** **(A)**

Marks	0	1	2	Mean
%	77	6	17	0.4

Points to note.

* Though a poorly attempted question, many students were able to recognise $m = -1$ and gave the correct equation of tangent BC, but were not able to fully determine B as (1,0) due to insufficient working, or assumed it to be so.

* Students experienced difficulty solving $\frac{1 - 3x}{2\sqrt{x}} = -1$. The best performances were where students used a pronumeral such as in 'Let $a = \sqrt{x}$' as above.

* **Poor notation:** quite a few replaced $\sqrt{x}$ by x instead of, say a, leading to $x = 1$ or $-\frac{1}{3}$.

* It is worrying to say the least that some candidates thought that the gradient of BC was 1. Others assumed incorrectly that from the diagram, the graph passed through B rather than establish it.

* Others used the algebraic expression from Q.9b as the gradient in a linear equation!!!

d. Find the coordinates of c **when** $\theta = 45°$**.**

AC will touch the curve when the curve has a gradient of 1.

$$\frac{1 - 3x}{2\sqrt{x}} = 1$$

$$1 - 3x = 2\sqrt{x}$$

$$3x + 2\sqrt{x} - 1 = 0$$

Let $\sqrt{x} = b$

$$3b^2 + 2b - 1 = 0$$

$$(3b - 1)(b + 1) = 0$$

$$\therefore \quad b = \frac{1}{3} \text{ or } -1$$

$$\therefore \quad \sqrt{x} = \frac{1}{3} \text{ or } -1 \text{ which is impossible}$$

Whence $x = \frac{1}{9}$.

$$f\left(\frac{1}{9}\right) = \sqrt{\frac{1}{9}}\left(1 - \frac{1}{9}\right)$$

$$= \frac{1}{3} \times \frac{8}{9} = \frac{8}{27}$$

Equation of AC is $y - \frac{8}{27} = 1\left(x - \frac{1}{9}\right)$

$$y = x + \frac{8}{27} - \frac{1}{9}$$

$$y = x + \frac{5}{27} \qquad \text{(B)}$$

Solving (A) and (B), $x + \frac{5}{27} = 1 - x$

$$2x = 1 - \frac{5}{27} = \frac{22}{27}$$

$$x = \frac{11}{27}$$

From (A), $\quad y = -\frac{11}{27} + 1$

$$= \frac{16}{27}$$

Co-ordinates of C when $\theta = 45°$ are $\left(\frac{11}{27}, \frac{16}{27}\right)$.

Marks	0	1	2	3	4	Mean
%	75	7	5	5	8	0.7

Points to note.

* Many students did not attempt this question; there were, however, a large number of successful responses. These students knew to equate the equation from **Q.9c.** with their equation in Q.**9d.** and solve these equations simultaneously.

* There were a number of errors calculating the co-ordinates accurately.

* Many who used +1 in **Q.9c.,** then used -1 in **Q.9d**.

* An elegant geometric solution involved mid-points and the recognition that ΔABC was an isosceles triangle; this was rarely seen.

* Finally, some candidates "interchanged" parts **c** and **d**.

"Here endeth the first lesson of the 2017 paper!"