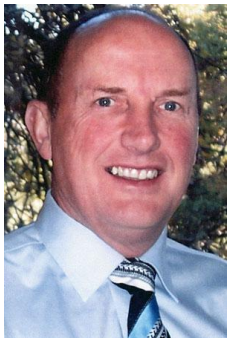
Challenging mathematics tasks: What they are and how to use them

Peter Sullivan

EPMC
Encouraging Persistence
Maintaining Challenge



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Overview

- A rationale for challenging students to think for themselves
- An illustrative sequence of tasks that ends with a challenge
- Another sequence that illustrate some constraints
- Advice for teachers

For students to learn, two sets of factors must align

- The first set of factors include that the:

–students have the requisite prior knowledge;

–curriculum is relevant to them;

–classroom tasks match their expectations;

–classroom tasks help them make connections

–pedagogies use their knowledge and experience;

–assessment regimes measure their learning.

The second set of factors relates to

- whether the students
  - –are motivated to learn
  - –see participation in schooling as creating opportunities
  - –are willing to persist
  - –connect effort and success

Where does the idea of "challenge" come from?

- Guidelines for school and system improvement (see, e.g., City, Elmore, Fiarman, & Teitel, 2009)
- The motivation literature (Middleton, 1995; 1999).
- Sets of teaching principles
  - Principles of Learning and Teaching, 2009
  - Productive Pedagogies, 2009

This connects to “mindsets”

- Dweck (2000) categorized students’ approaches in terms of whether they hold either *growth* mindset *or fixed* mindset

Students with *growth* mindset:

- Believe they can get smarter by trying hard
- Such students
  - tend to have a resilient response to failure;
  - remain focused on mastering skills and knowledge even when challenged;
  - do not see failure as an indictment on themselves; and
  - believe that effort leads to success.

Students with *fixed* mindset:

- Believe they are as smart as they will even get
- Such students
  - seek success but mainly on tasks with which they are familiar;
  - avoid or give up quickly on challenging tasks;
  - derive their perception of ability from their capacity to attract recognition.

Teachers can change mindsets

- This connects to
  - the things we affirm (effort, persistence, co-operation, learning from others, flexible thinking)
  - the way we affirm
    - You did not give up even though you were stuck
    - You tried something different
    - You tried to find more than one answer
  - the types of tasks we pose

Challenging tasks require students to

- plan their approach, especially sequencing more than one step;
- process multiple pieces of information, with an expectation that they make connections between those pieces, and see concepts in new ways;
- choose their own strategies, goals, and level of accessing the task;
- spend time on the task and record their thinking;
- explain their strategies and justify their thinking to the teacher and other students.

Consider this sequence of tasks

It can be adapted to a variety to levels

The end of the sequence might be this task.

A paddock in the shape of an L has an area of 1 hectare. What might be the perimeter of the paddock?

At what level would that task be "challenging"

The mathematical focus of the sequence

- Perimeter is the length around the outside
- We can measure length in cm and m
- Area is about the space covered on a surface
- We measure area in square cm and square m
- There are efficient methods for calculating area
- We can calculate the perimeter and area of composite shapes

A rationale for this sequence

- Area is used to measure both small and large shapes, and is used in everyday life and in other subjects
- Area forms the basis of volume and capacity calculations and many practical maximisation tasks
- Length, area and volume are all measures of size but they measure different attributes

To get everyone started we might pose …

- Draw some letters of the alphabet on squared paper using exactly 10 squares

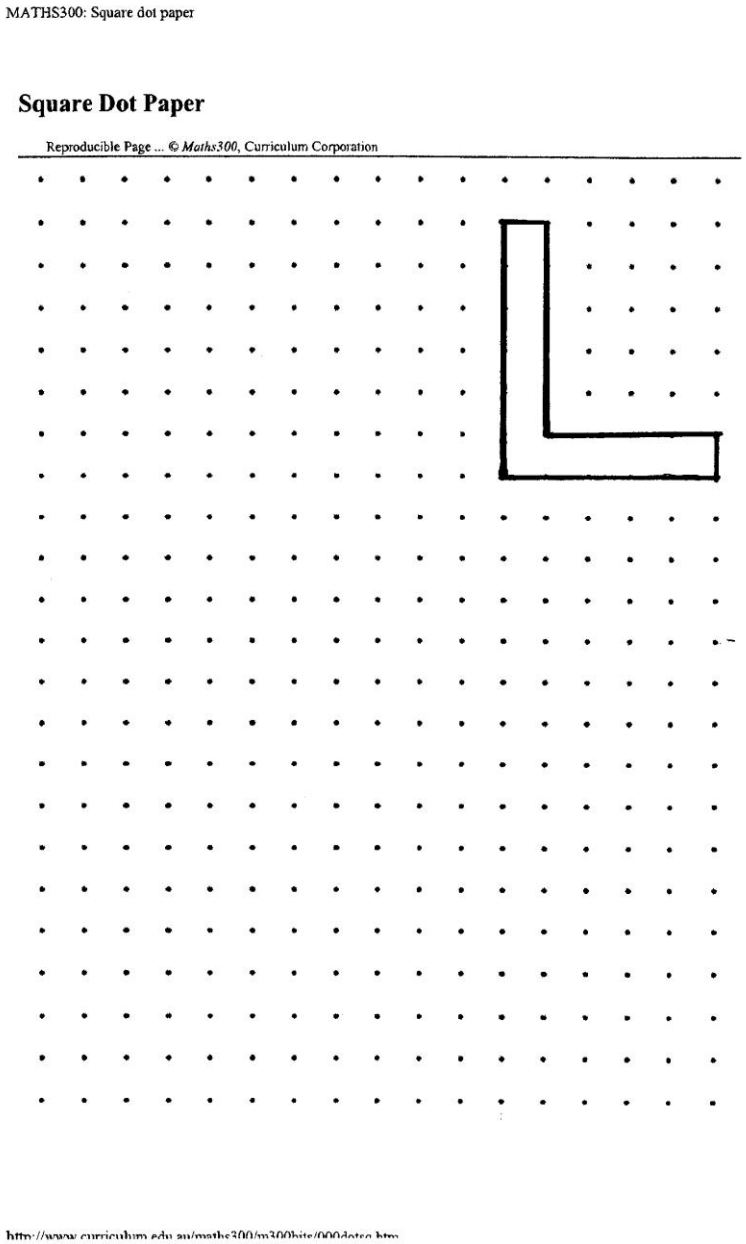
Anticipating some constraints

- Some students might not know what to do
  - Sometimes teachers respond by over explaining
- Some students might not know how to start
  - Sometimes teachers respond by making the task simpler (so removing challenge)
  - Sometimes teachers respond by giving some students different tasks (so exacerbating differences)

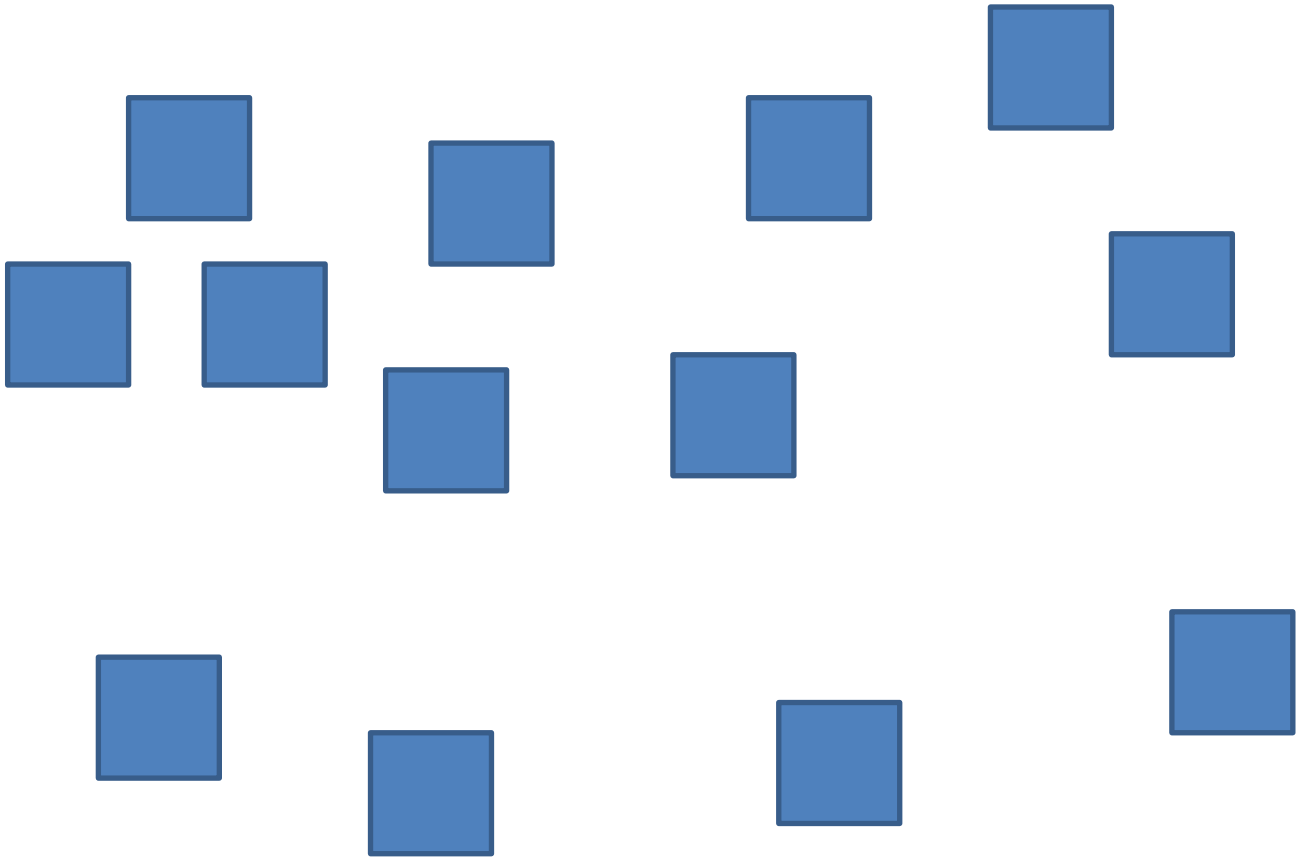
How might you assist students who still cannot get started?

If the students do not know what to do, we might offer them a model

But why did you not do that in the first place?



“Get some square counters …”



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“Make any shape you like out of 10 squares”

“Make any letter and count the squares”

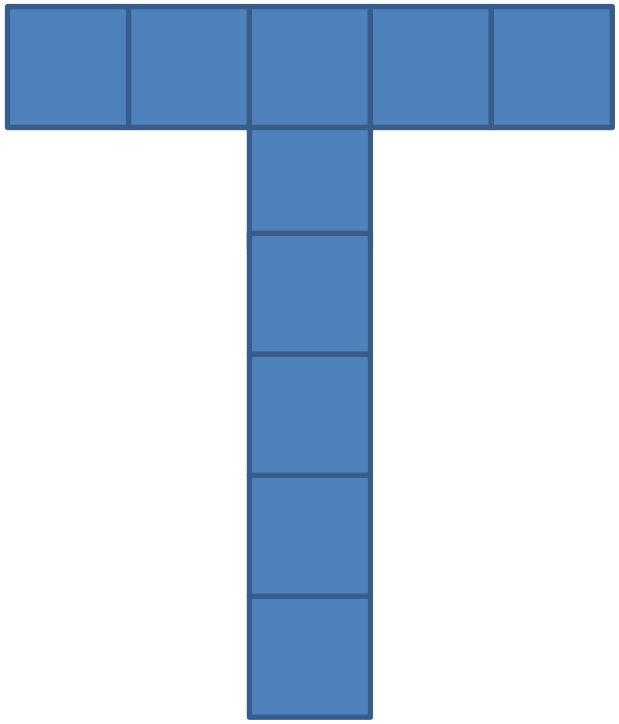
“Take a walk around …”

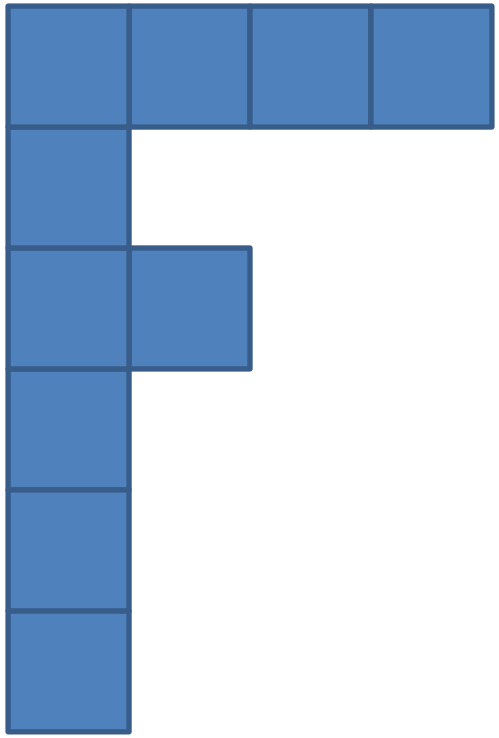
What are enabling prompts?

- Enabling prompts can involve slightly varying an aspect of the task demand, such as
  - the form of representation,
  - the size of the numbers, or
  - the number of steps,

  so that a student experiencing difficulty, if successful, can proceed with the original task.
- This approach can be contrasted with the more common requirement that such students
  - listen to additional explanations; or
  - pursue goals substantially different from the rest of the class.

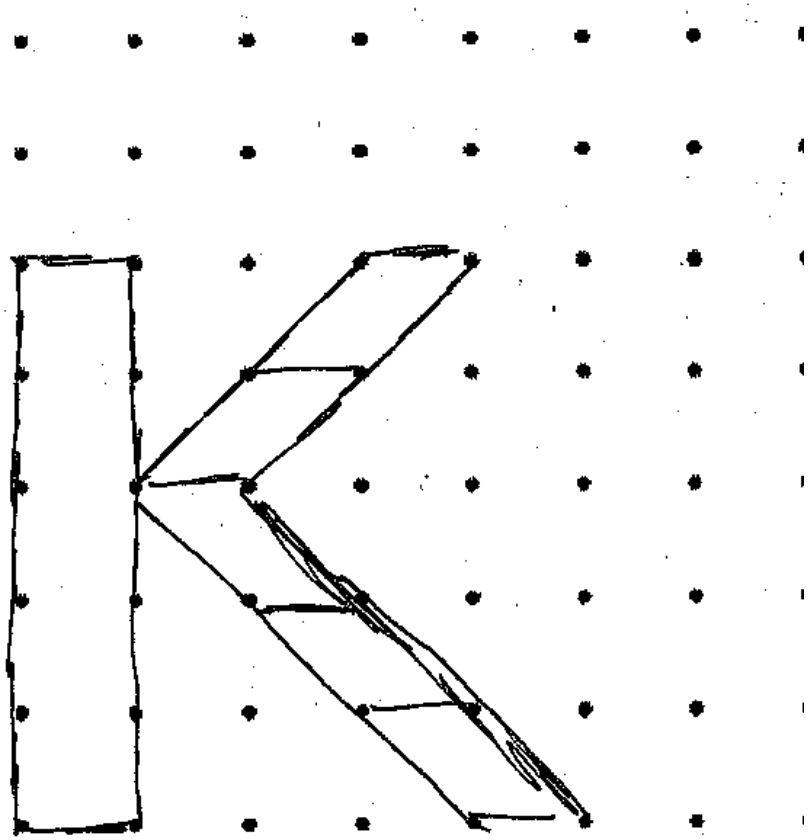
Each task in the sequence needs to be reviewed





The pedagogical dilemma: Do you

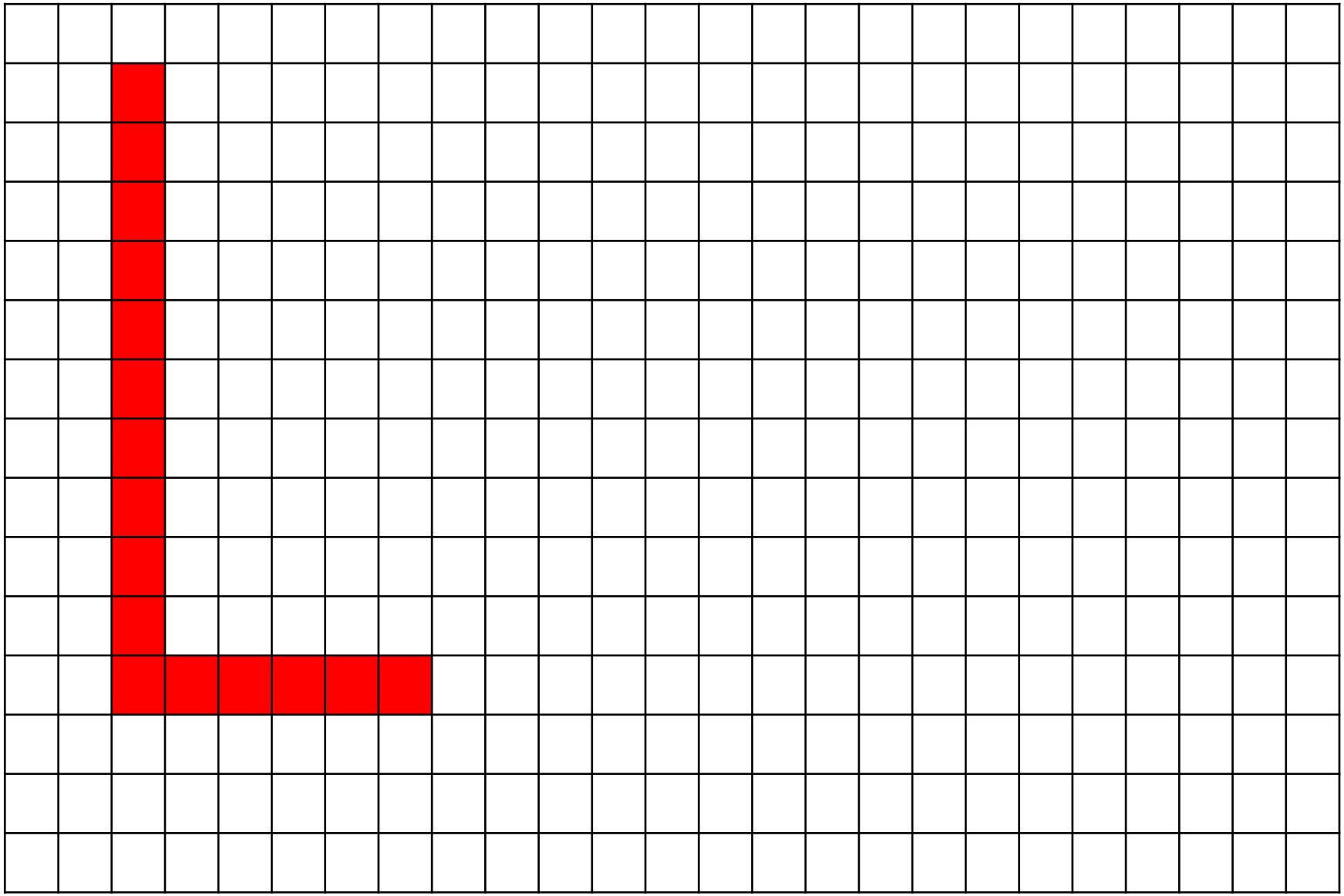
- Say “OK. That’s it. Let’s go onto the next task”
- Say “That is good work. See if you can write your name”
- Say “By the way, we are sending this work home to show your parents”
- Pretend you have not seen and wait to see if he can do the “k”

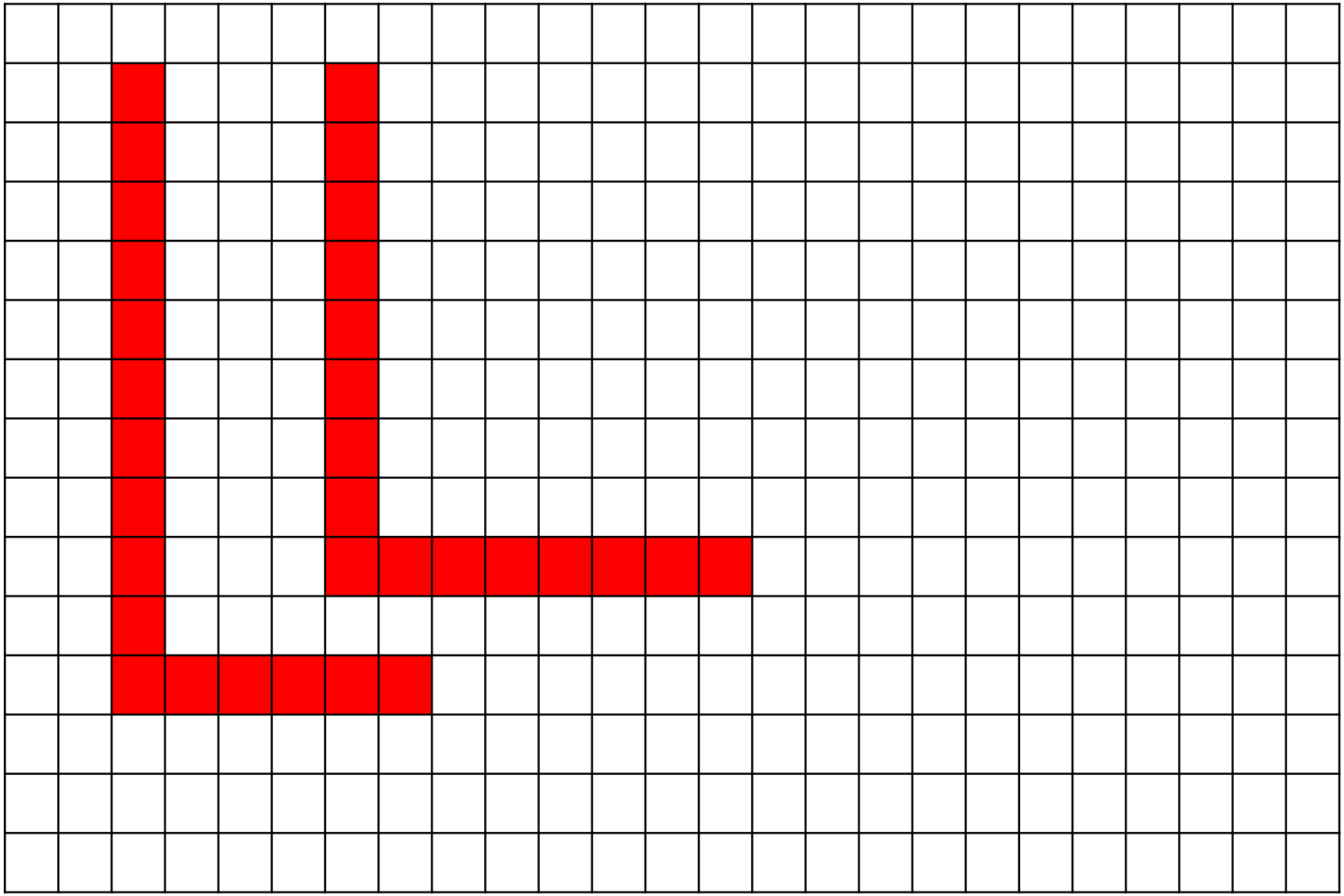


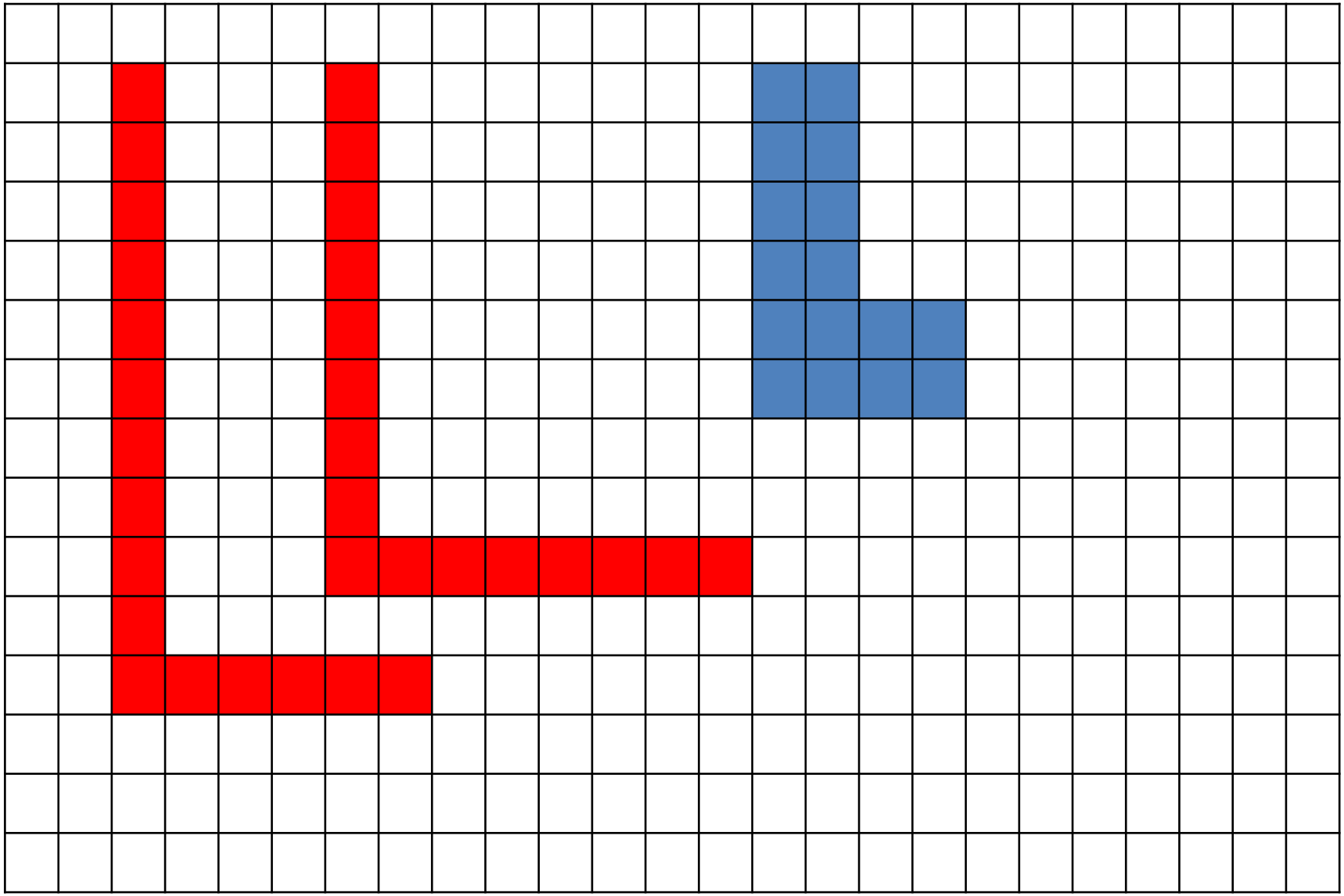
The sequence continues

On squared paper, draw an L shaped figure that covers 16 squares.

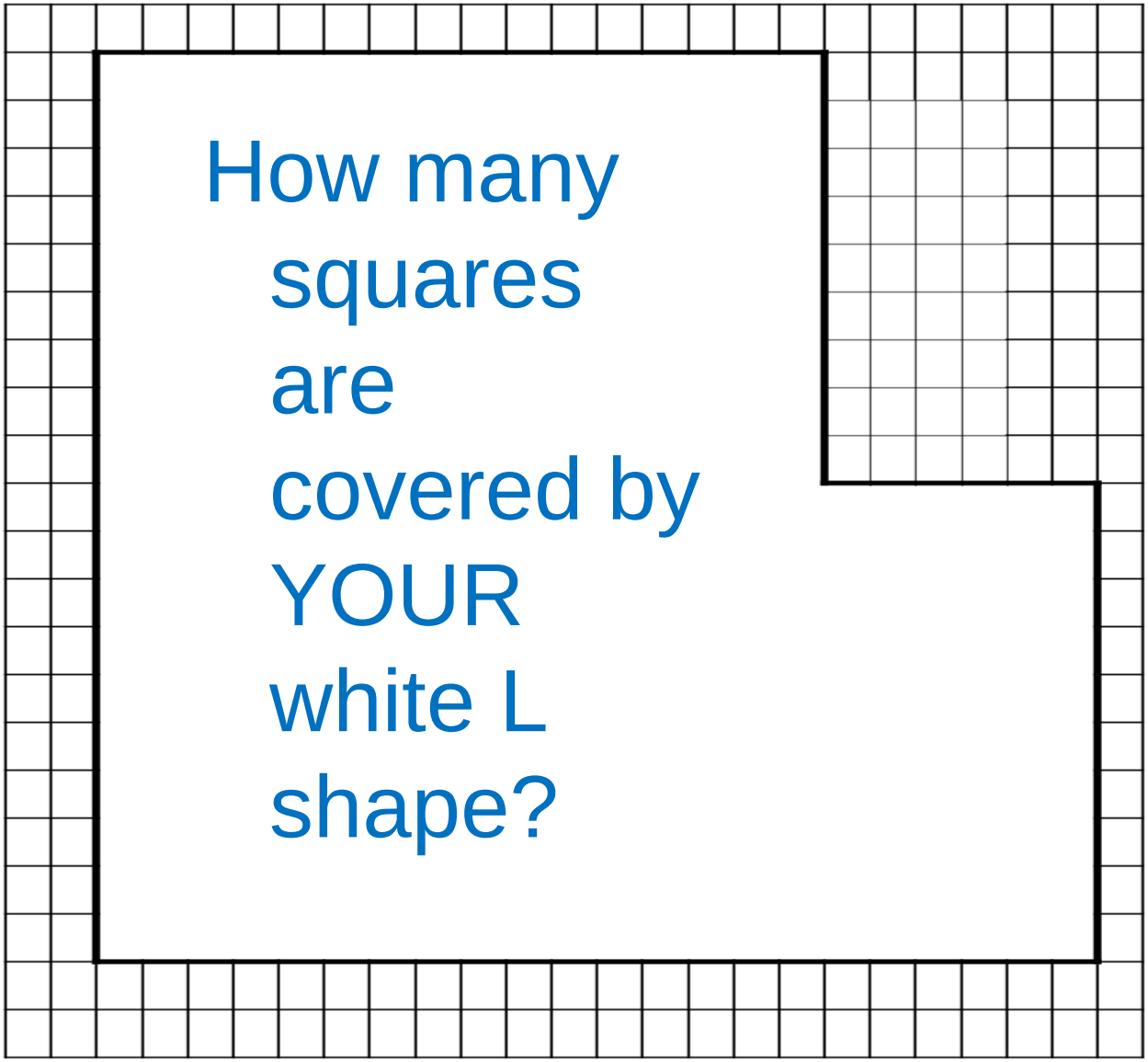
(Give more than one answer)

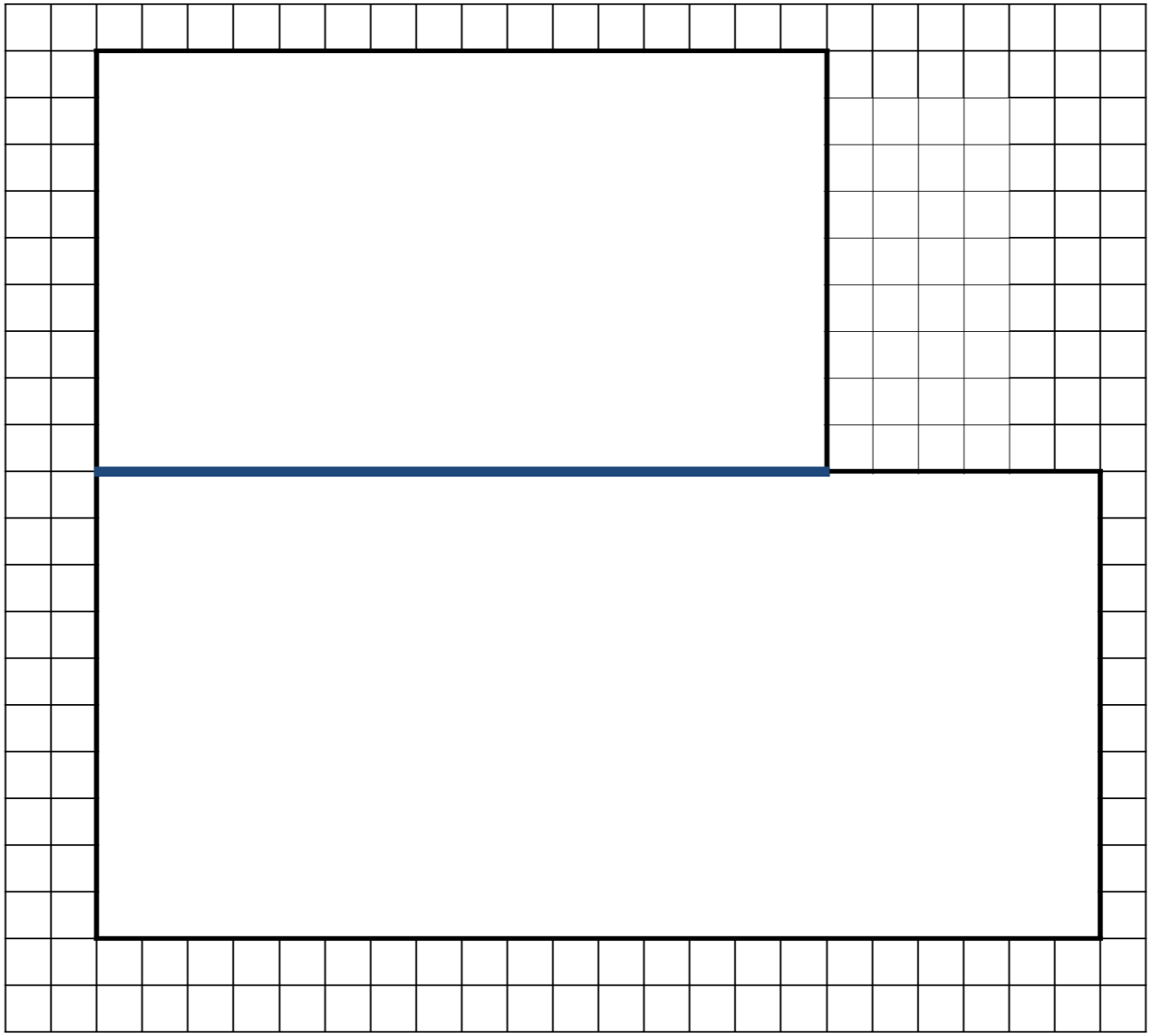


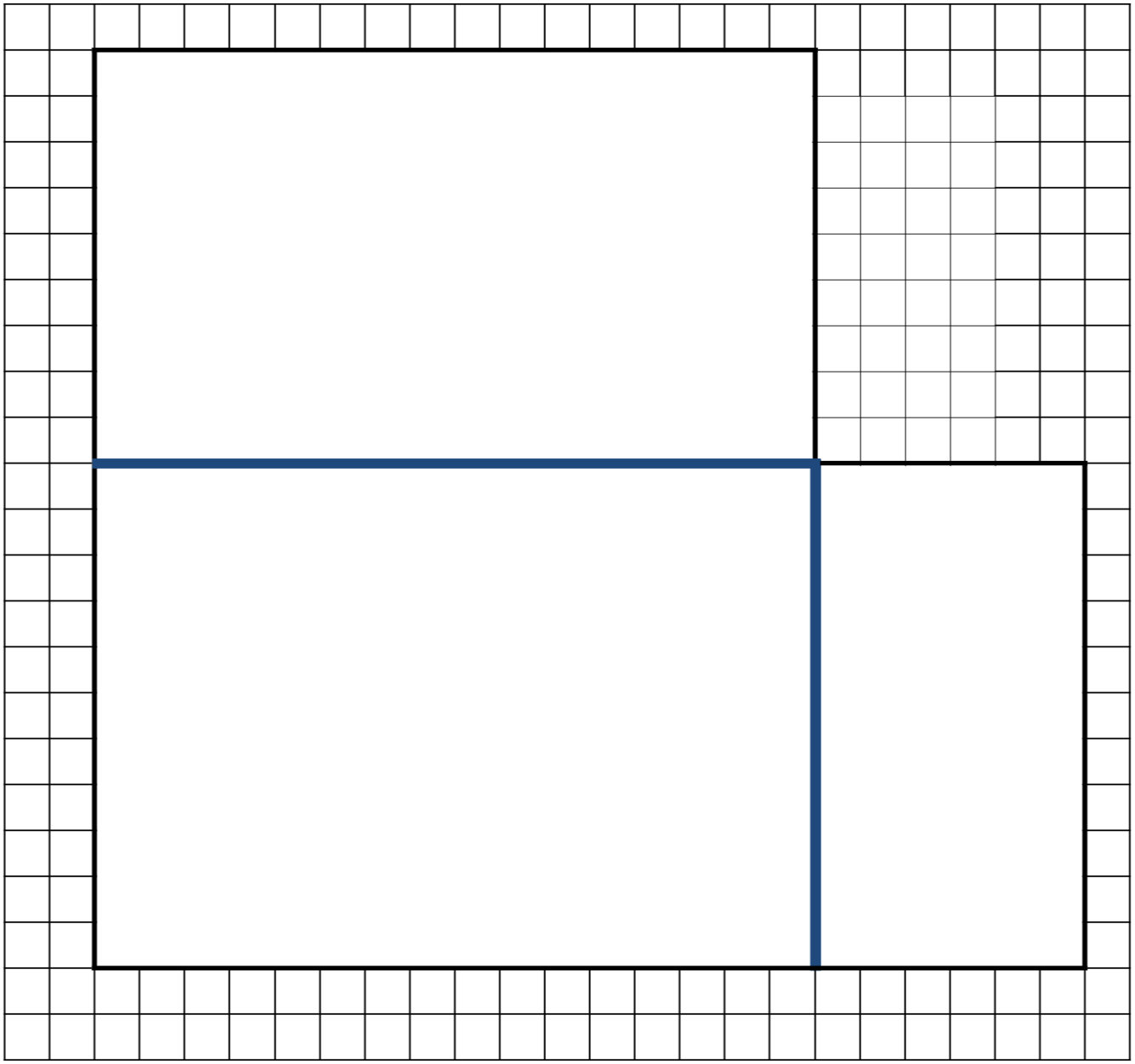


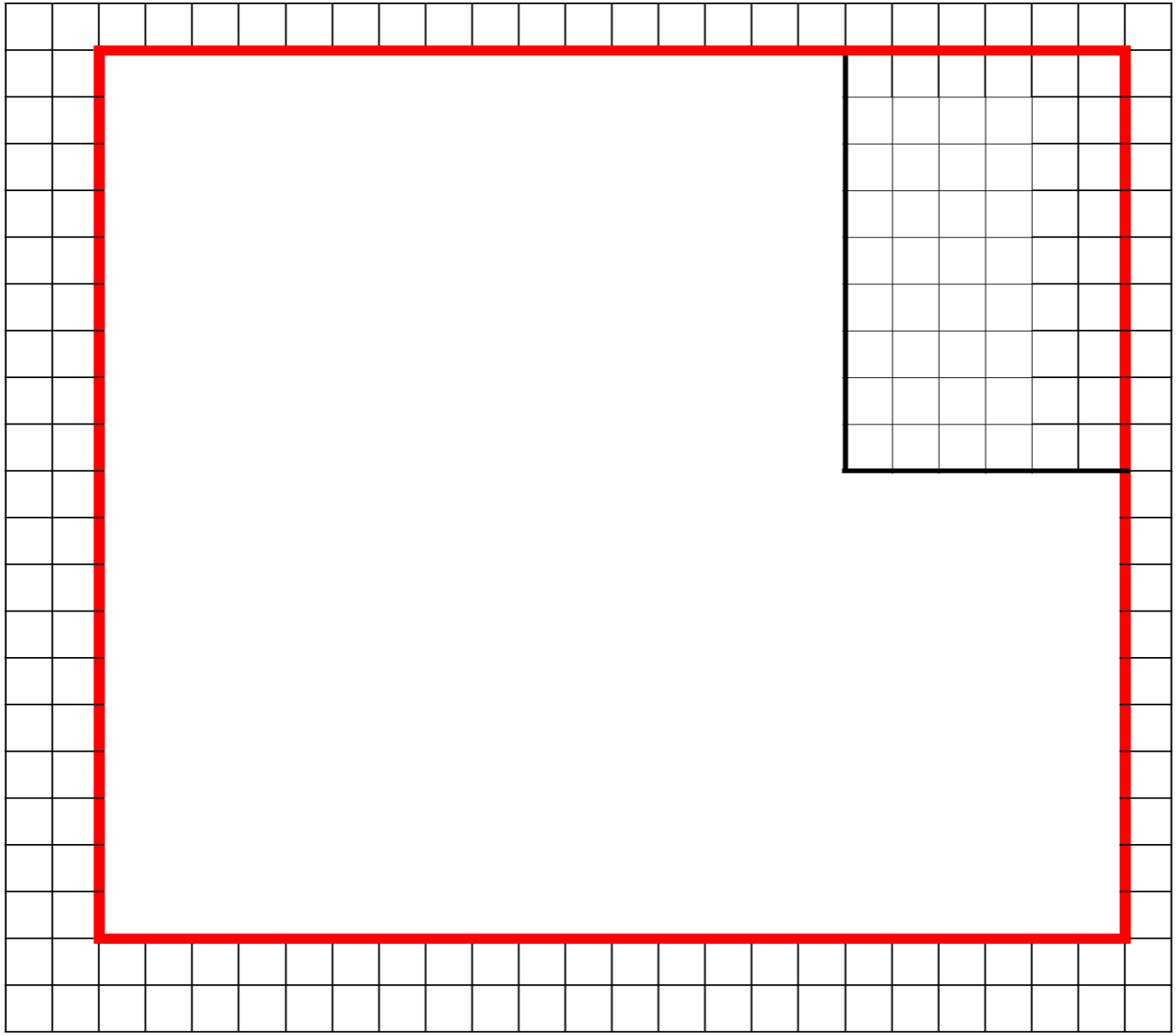


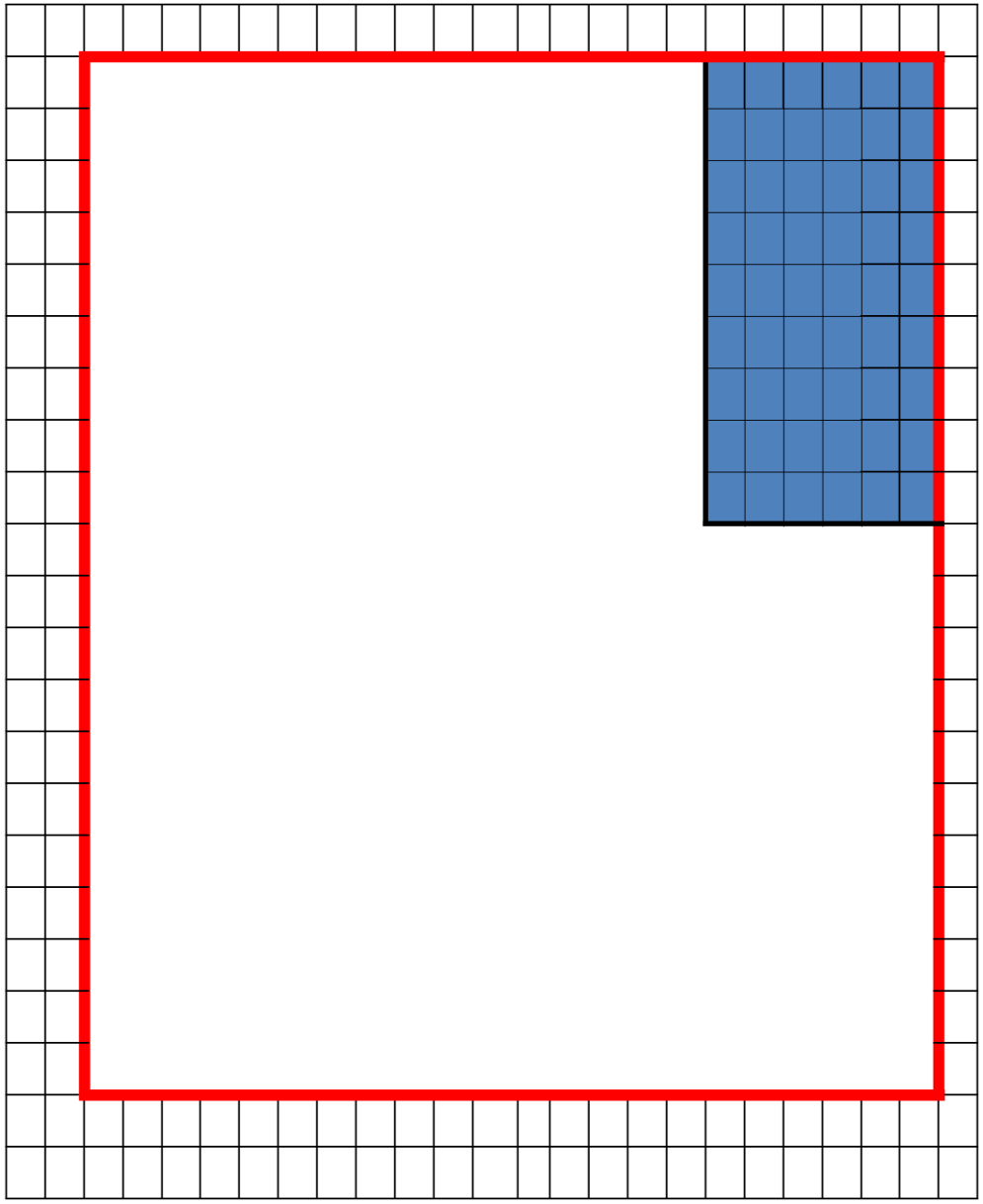
Of course, we need the students to get past counting squares

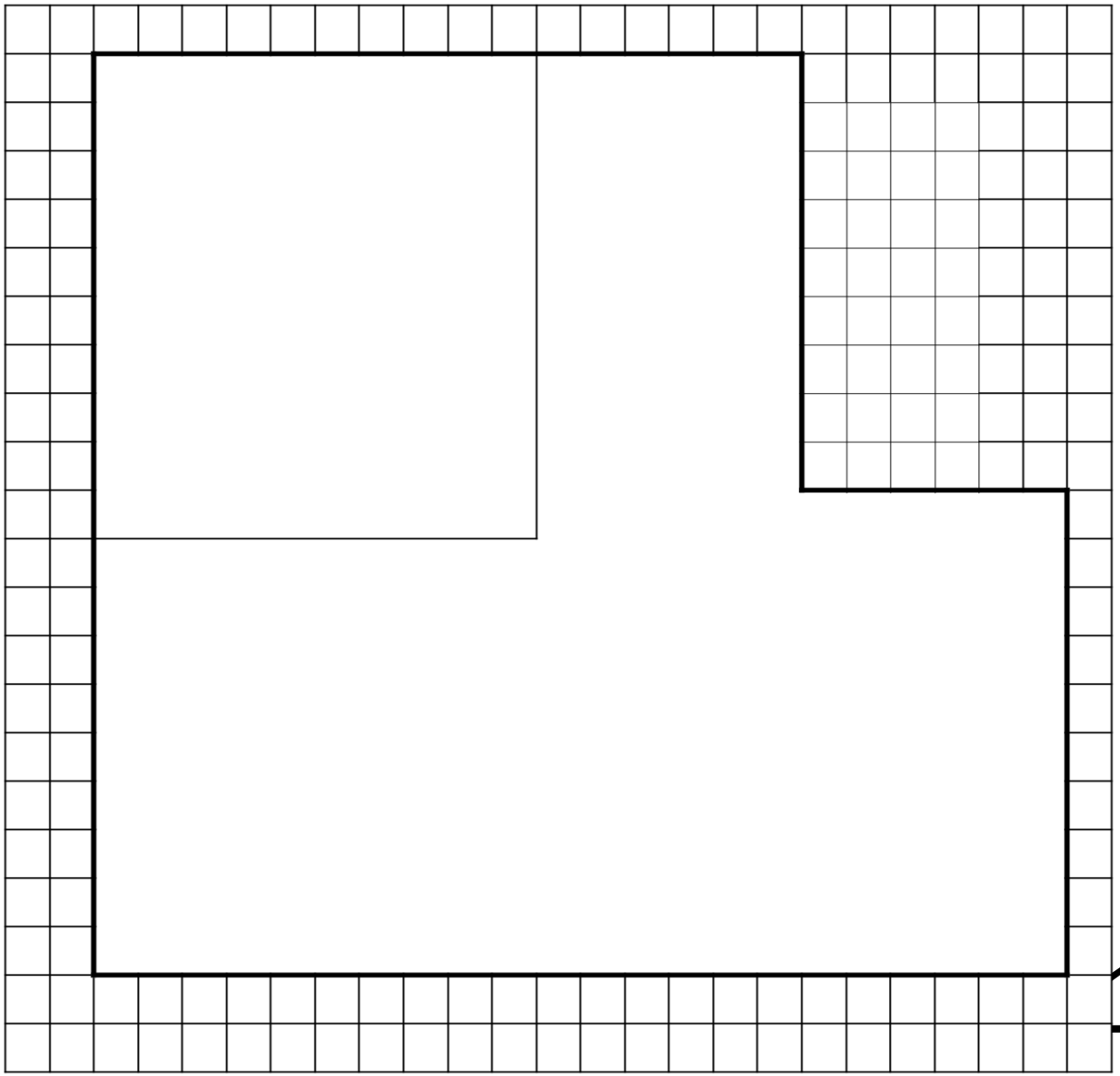

How many
squares
are
covered by
YOUR
white L
shape?











10 × 10

Choosing the right language

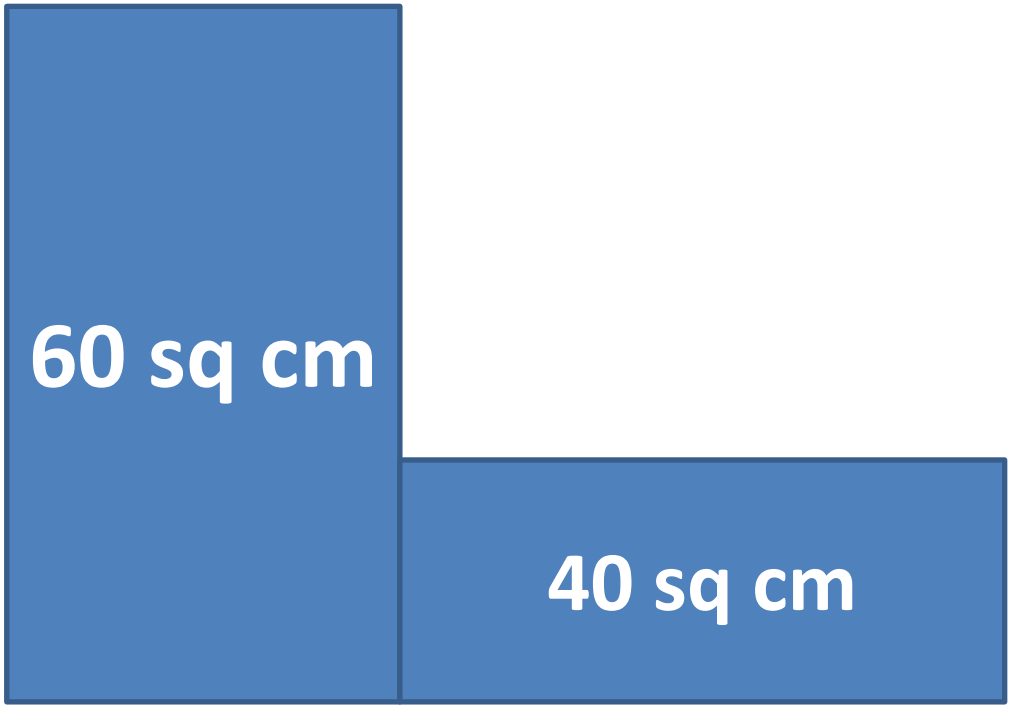
- A square on cm paper is called a square centimetre
- We sometimes write sq cm

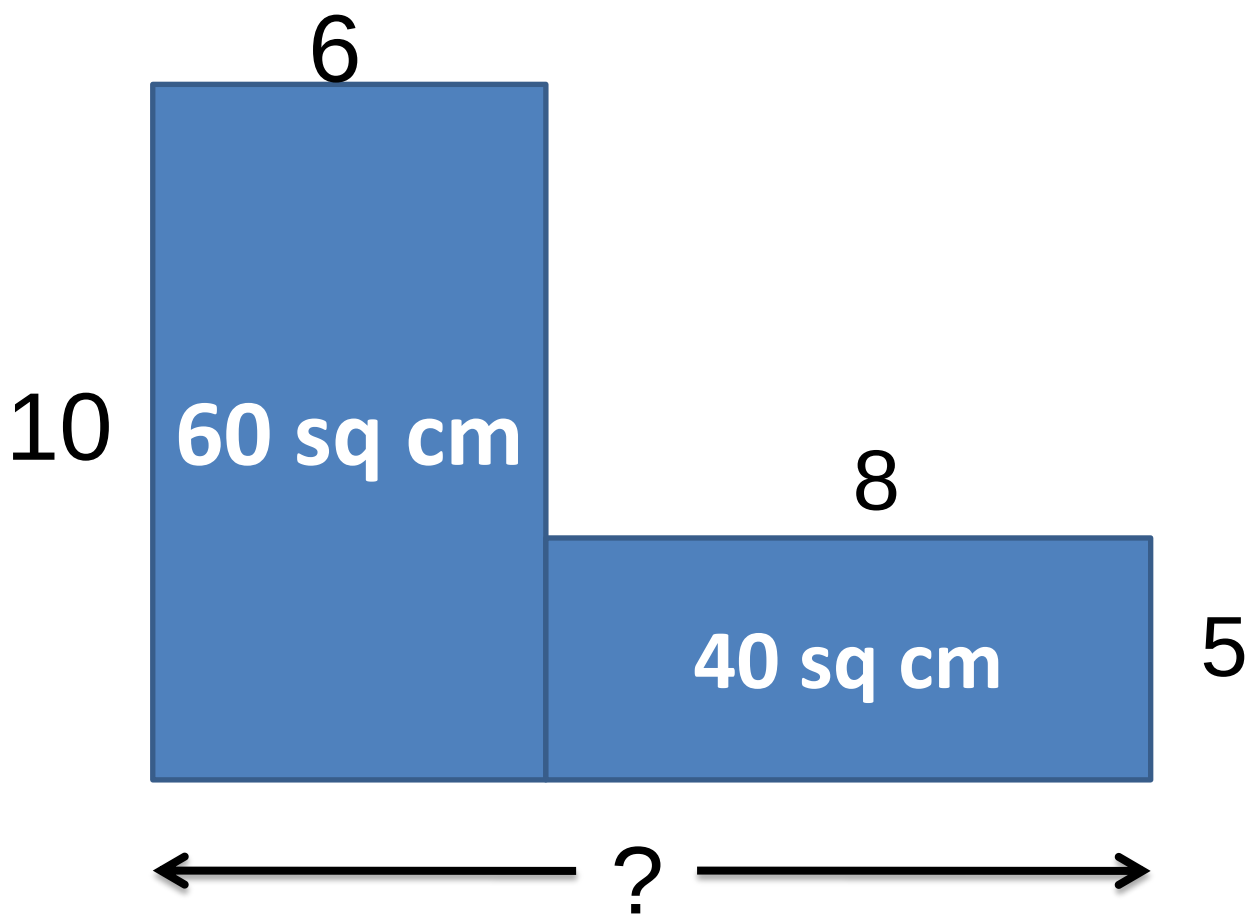
Next we might go on to …

Imagine that the letter L is drawn on a large piece of paper.

Its area is 100 sq cm.

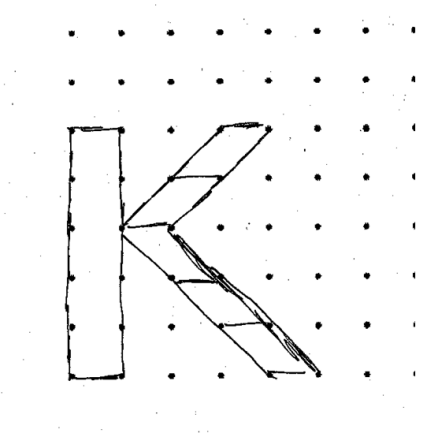
What might be its height and (the widest) width?

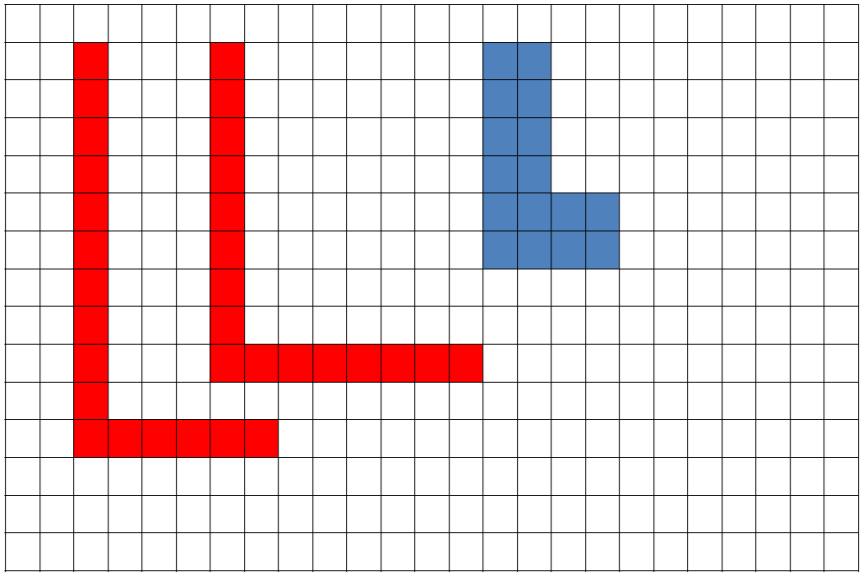

60 sq cm
40 sq cm

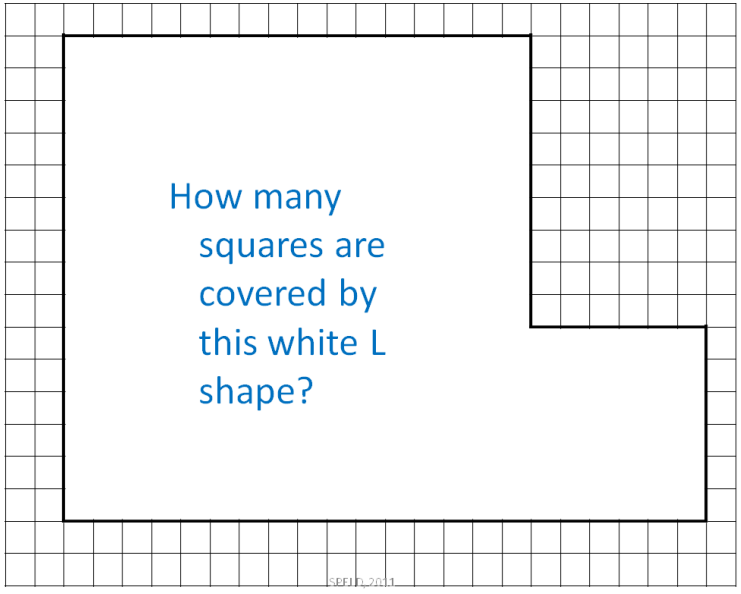

6
10
60 sq cm
8
40 sq cm
5
?

- A paddock in the shape of an L has an area of 1 hectare. What might be the perimeter of the paddock?

What was the nature of that sequence of experiences?







Imagine that the letter L is drawn on a large piece of paper.
Its area is 100 sq cm.
What might be its height and width?

- A paddock in the shape of an L has an area of 1 hectare. What might be the perimeter of the paddock?

Our experience observing teachers

- Seven observers watched the same lesson being taught by different teachers
- We wrote three types of observations
- The following is a synthesis of the discussion

It seems to encourage students if:

- the nature of the task is explained to the students (for example, if there are multiple possible solutions, this can be mentioned);
- the ways of working are explained to the students, including the type of thinking they are expected to do, what they should be recording, and what they should later report to the class;
- the classroom climate encourages risk taking, that even if the task is hard the students can do it, that errors are part of learning, and that students can learn even if they do not complete the task, and so on;
- the teacher communicates enthusiasm about the task, including encouraging the students to persist with it;

- students have a choice of ways that they can approach the task, and perhaps on the level of difficulty of the task itself;
- the class is organised to ensure that students have adequate time to work on the challenging task, including preparing some aspects such as group membership beforehand;
- some attempt is made to connect the task with the students' experience;
- the teacher moves around the class making regular contact with students, asking questions of the individuals or groups; and
- there is time allowed for lesson review so that students see the strategies of other students and any summaries from the teacher as learning opportunities.

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AER 59 reviews research into aspects of mathematics teaching, focusing on issues relevant to Australian mathematics teachers, to those who support them, and also to those who make policy decisions about mathematics teaching. It was motivated by and draws on the proceedings of the well-attended and highly successful ACER Research Conference *Teaching mathematics? Make it count: What research tells us about effective mathematics teaching and learning*, held in Melbourne in August 2010.

Section 2 describes the goals of teaching mathematics and argues that a practical orientation should be the focus of mathematics teaching in the compulsory years, and outlines the contribution numeracy-based perspectives can make to schooling. Section 3 uses assessment data to evaluate how well those goals are being met in Australia and introduces the challenge of seeking equity of opportunity in mathematics teaching and learning. Section 4 expands on the importance, to individuals and society, of achieving the mathematics goals; and Section 5 discusses six research-based principles of mathematics teaching. Section 6 argues for the importance of well-chosen mathematical tasks in supporting student learning, and models tasks and particular teaching strategies. Sections 7 and 8 analyse research which provides insights into a key issue facing Australian mathematics teachers, that of finding ways to address the needs of heterogeneous groups of students. Section 9 describes and recommends particular emphases and strategies for education programs for both prospective and practising teachers.

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