Diagnosis and Intervention: three dimensions to developing Numeracy in all children

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Mathematics is Multidimensional
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- **Intervention** *to develop and implement appropriate ways of thinking, generalising and applying mathematical ideas*
- ***Self confidence*** *to acquire and use mathematics as they develop conceptual understanding and fluent processes*

Numeracy

Children should have a robust sense of number:

- an understanding of place value
- meaning for the basic operations
- computational facility
- knowledge of how to apply this to problem solving

A thorough understanding of fractions includes

- being able to locate them on a number line
- represent and compare fractions, decimals and per cents
- estimate their size and carry out operations confidently and efficiently.

Final Report of the National Mathematics Advisory Board, 2008 pp 17 & 18

Assessment

Assessment is integral to teaching and learning

Well-focused assessment

- reveals how students think
- provides guidance to plan ongoing teaching
- furnishes information about how students are dealing with the mathematical tasks they are exposed to
- provides feedback on particular programs or learning activities:

Are they suited to the students and content in question?

Do they need to be modified to produce the expected learning?

Assessment

At the same time, assessment

- provides information to the student, teachers and others about the student's mathematical capabilities and potential
- highlights different outcomes across different groups of students, across different classes and schools

While it is important to know what students know, of even more importance is the way they know:

- is their knowledge simply memorised routines

or

- is there deep understanding based on well-understood concepts and fluent, meaningful processes applied in appropriate ways?

Diagnostic assessment

A need for Diagnostic assessment to

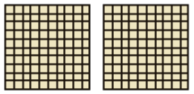
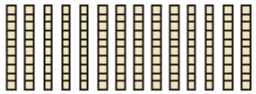
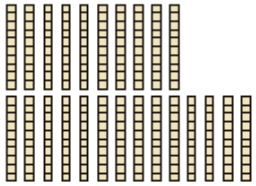
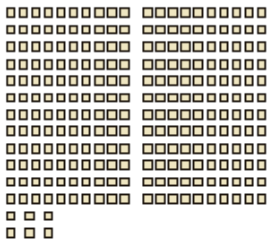
- reveal not only what a student knows but also *how they know*
- not what they do not know but reveal what they *need to know*
- reveal *gaps* in a student's mathematical knowledge

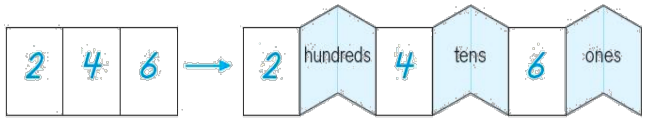
Critical ideas

- may not have become part of a student's way of thinking
- may not have been included in the steps used to build up a topic

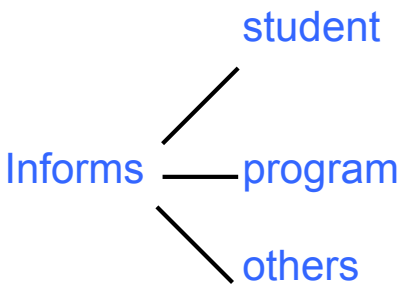
For example, the renaming needed for computation and other number processes may not be developed as an extension of place value or to provide a complete understanding of larger numbers

Renaming

	hundreds	tens	ones	
246				2 hundreds 4 tens 6 ones
renamed as				1 hundred 14 tens 6 ones
or				24 tens 6 ones
or				246 ones



Diagnostic assessment



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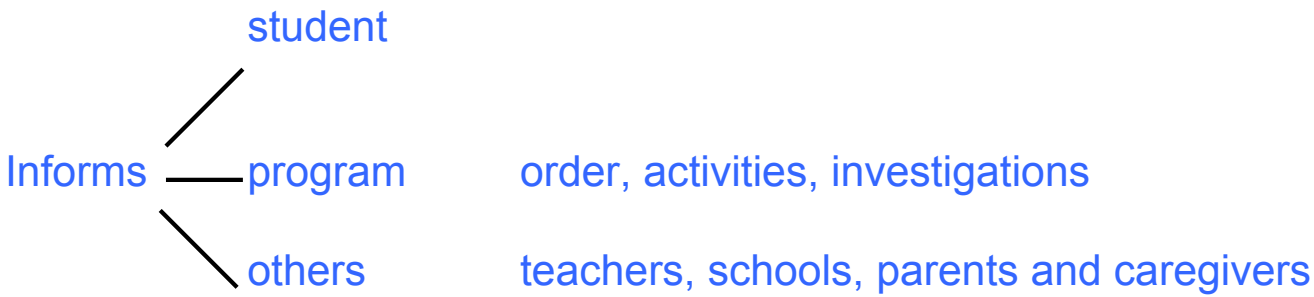

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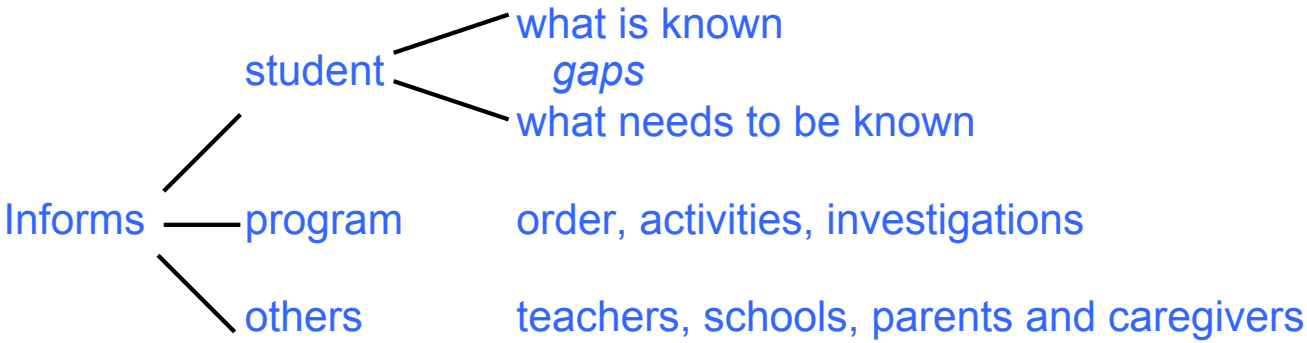

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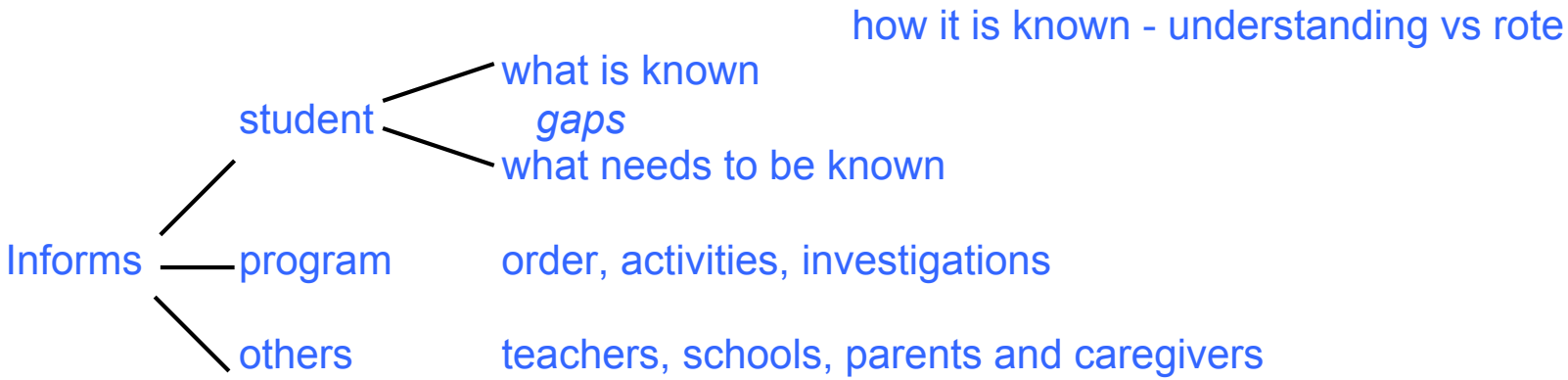


Diagnostic assessment

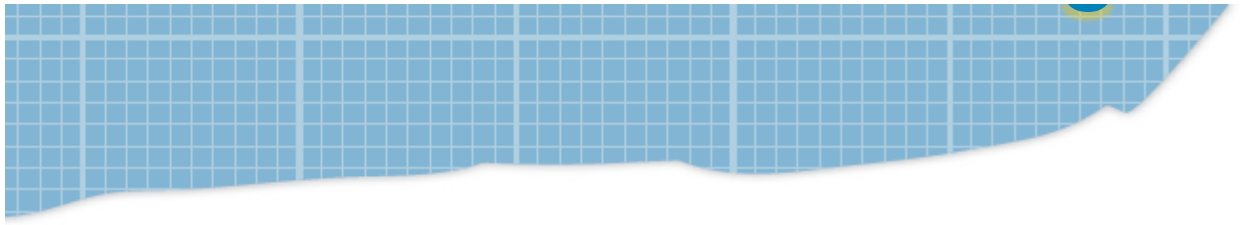
how it is known - understanding vs rote

Informs
- student
  - what is known
  - *gaps*
  - what needs to be known
- program — order, activities, investigations
- others — teachers, schools, parents and caregivers

Diagnostic tasks
- related to a mathematical framework
  conceptual not procedural

+

- insights into children's approaches
  inappropriate generalisations



$$\begin{array}{r} 6.7 \\ +\ 8.4 \\ \hline 14.11 \end{array}$$

$$\begin{array}{r} 0\ 5\ 0\ \ 1.2 \\ \hline 7\overline{)4\,{}^{3}5.2\,{}^{7}9} \end{array}$$

$$\begin{array}{r} \cancel{7}.\,{}^{0}\cancel{0}\,{}^{14}4 \\ -\ 3.\ 3\ \ 7 \\ \hline 4.\ 0\ \ 7 \end{array}$$

Difficulties with place value, renaming & zero

Diagnostic assessment

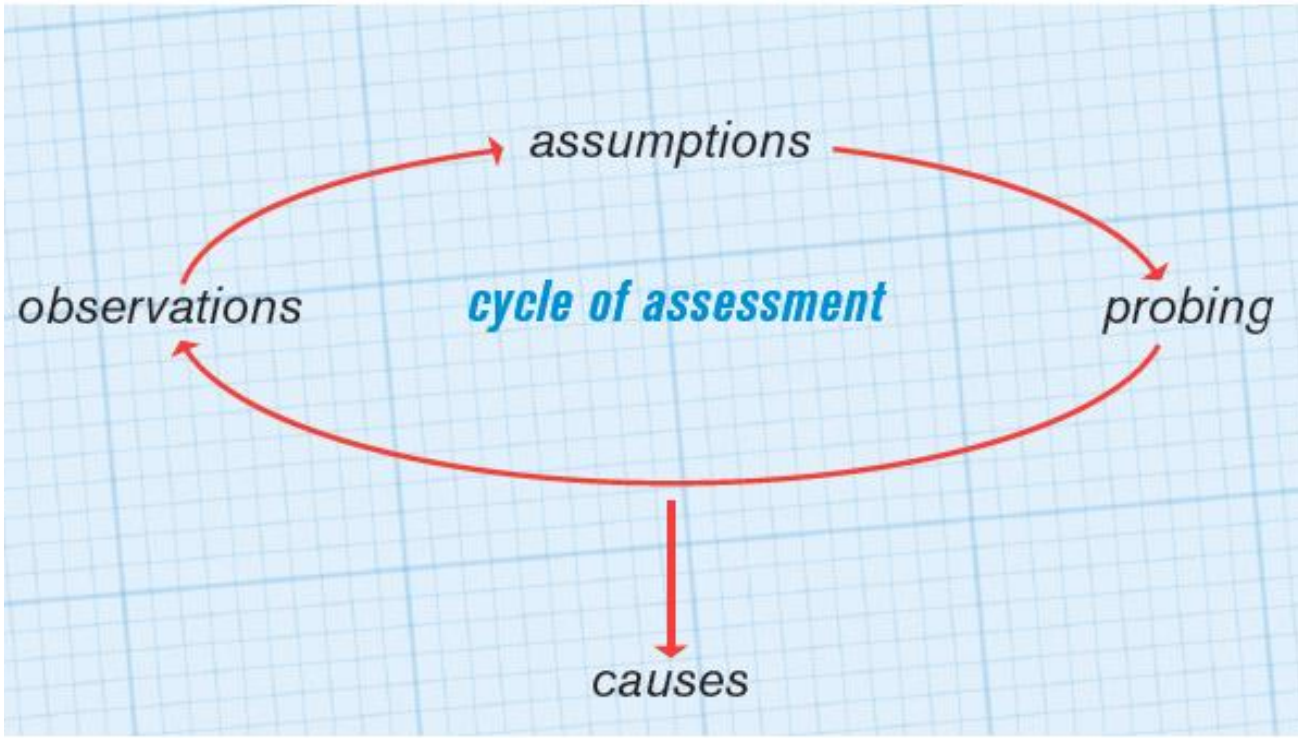
$$\begin{array}{r} {}^{2}5.6 \\ \times\ 7.4 \\ \hline 14.4 \end{array}$$

$$\begin{array}{r} {}^{2}5.6 \\ \times\ 7.4 \\ \hline 37.4 \end{array}$$

$$\begin{array}{r} {}^{42}5.6 \\ \times\ 7.4 \\ \hline 22.4 \\ 392.0 \\ \hline 414.4 \end{array}$$

Confusion with addition process

Diagnostic assessment



Diagnosis to Intervention

When misconceptions, difficulties and gaps in a student's knowledge have been identified, means to intervene in the learning can be planned and implemented in a manner appropriate to both the learner and the way in which concepts and processes are best established and consolidated

Simply showing a student what to do using a written procedure is rarely successful in replacing procedures that have led to errors.

At best they will try to copy and remember a teacher's approach but the link to what they do know may not be apparent in the purely recorded form

Diagnosis to Intervention

Construct new ways of thinking

- use materials to draw out the patterns on which ideas are developed
- link to a language that provides meaning
- move to symbolic expressions that express what is happening succinctly only when a learner has adopted the way of thinking as his or her own

Engaging practice activities, often in the form of games in which learners willingly participate, are an essential part of learning to bring a concept to the forefront of a learner's mind and enable a process to become fluent

Process of Intervention

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3. reveal inadequacies in thinking to a learner in order to build an appreciation of a need for change
4. implement means of constructing or re-constructing appropriate ways of thinking
5. provide practice that is focused and motivating to allow a way of thinking to become secure and provide a basis to
   - generalise to more complex problems and applications
   - the development of further mathematics

Building Numeracy

Numeracy Tests

BUILDING Numeracy

Moving from diagnosis to intervention

Extended background

6 numeracy class screening tests

Spread sheet for scoring and identifying needs

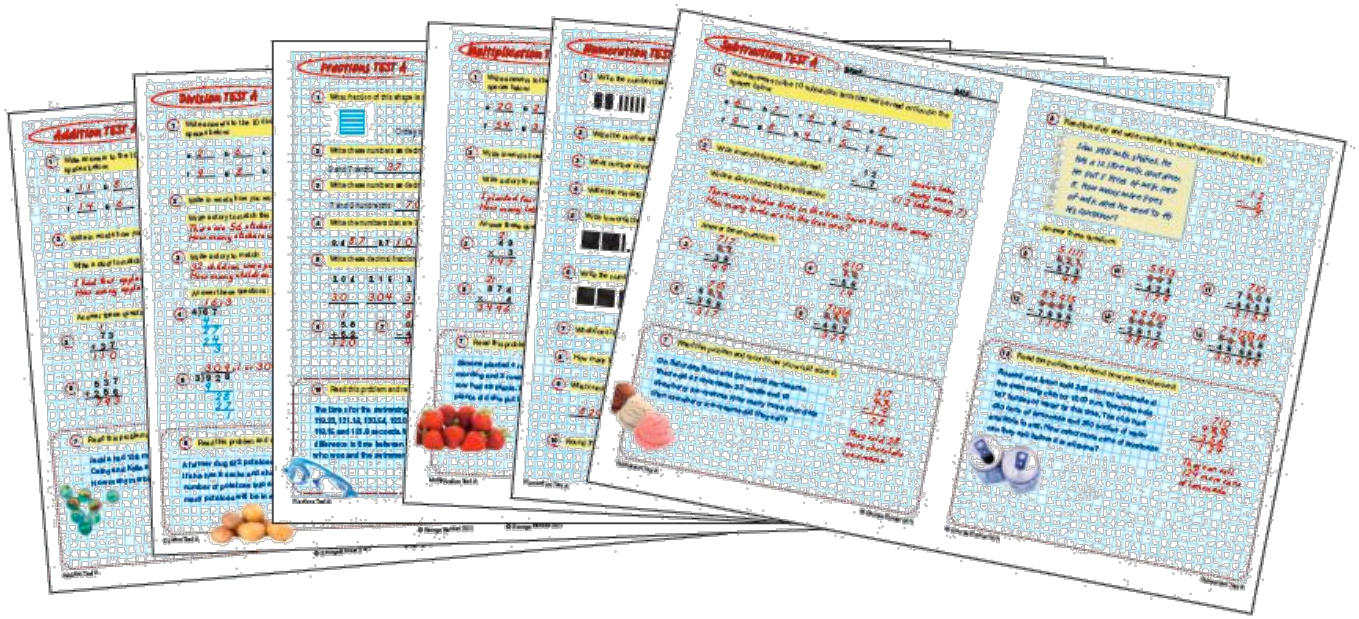
OXFORD

Building Numeracy

Numeracy Tests

Six different areas – 2 tests for each area

- *Numeration Tests A & B*
- *Addition Tests A & B*
- *Subtraction Tests A & B*
- *Multiplication Tests A & B*
- *Division Tests A & B*
- *Fractions Test A & B*



OXFORD

Fractions TEST A

Name: Date:

1 What fraction of this shape is shaded?

Circle your answer **2 eighths** *or* **2 tenths**

2 Write these numbers as decimal fractions:

3 and 7 tenths 3.7 8 tenths 0.8

3 Write these numbers as decimal fractions:

7 and 2 hundredths 7.02 16 hundredths 0.16

4 Write the numbers that are 3 tenths more:

8.4 8.7 9.7 10 6 6.3 0.8 1.1 2.9 3.2

5 Write these decimal fractions in order from least to greatest:

3.04 3.16 3.0 3.2 3.10

3.0 3.04 3.10 3.16 3.2

6
$$\begin{array}{r} {\scriptstyle 1} \\ 5.8 \\ +\,6.2 \\ \hline 12.0 \end{array}$$

7
$$\begin{array}{r} {\scriptstyle 8\,9\,12} \\ 9.0\,2 \\ -\,4.2\,3 \\ \hline 4.7\,9 \end{array}$$

8
$$\begin{array}{r} {\scriptstyle 4\,5} \\ 4.7 \\ \times\,6.8 \\ \hline 376 \\ 2820 \\ \hline 31.96 \end{array}$$

9
$$\begin{array}{r} 7.94 \\ 8\overline{)63.52} \end{array}$$

10 Read this problem and record how you would solve it:

The times for the swimming race were 119.23, 121.18, 120.54, 122.0, 119.7, 123.2, 119.15 and 121.8 seconds. What was the difference in time between the swimmer who won and the swimmer who came last?

$$\begin{array}{r} {\scriptstyle 1\,13\,1\,10} \\ 1\,\cancel{2}\,\cancel{3}.\,\cancel{2} \\ -\,1\,1\,9.1\,5 \\ \hline 4.0\,5 \end{array}$$

4.05 seconds

11 Franca has 1 dog, 2 cats and a fish tank with 6 tropical fish.

What fraction of her pets are cats? $\frac{2}{9}$

12 Write these numbers as common fractions:

4 ninths $\frac{4}{9}$ 3 and 5 eighths $3\frac{5}{8}$ 13 sixths $\frac{13}{6}$

13 Circle the fraction that is greatest: $\frac{1}{10}$ $\frac{1}{7}$ $\frac{1}{9}$ $\frac{1}{12}$ $\frac{1}{8}$

14 Write these common fractions as mixed numbers and improper fractions:

$3\frac{3}{4}$ *is the same as* $\frac{15}{4}$ $\frac{17}{6}$ *is the same as* $2\frac{5}{6}$

15 Which of these common fractions are equivalent fractions?

$\frac{2}{3}$ $\frac{4}{5}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{4}{9}$ $\frac{5}{9}$ $\frac{6}{9}$ $\frac{2}{3}$ $\frac{4}{6}$ $\frac{6}{9}$

16 Circle the fraction that is least: $\frac{1}{2}$ $\frac{5}{12}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{3}{8}$

Answer these questions:

17 $5\frac{2}{3} + 4\frac{5}{6}$

$$\begin{array}{r} {\scriptstyle 1} \\ 5\frac{4}{6} \\ +\,4\frac{5}{6} \\ \hline 10\frac{3}{6} \end{array}$$

18 $9\frac{5}{12} - 3\frac{3}{4}$

$$\begin{array}{r} {\scriptstyle 8\,\frac{17}{12}} \\ \cancel{9}\,\cancel{\frac{5}{12}} \\ -\,3\frac{9}{12} \\ \hline 5\frac{8}{12} \end{array}$$

19 $7 \times \frac{4}{5} = \frac{28}{5}$ or $5\frac{3}{5}$

20 Place these fractions on the number line below:

1.5 $\frac{2}{3}$ 0.7 $1\frac{3}{4}$ $\frac{2}{5}$ 0.25

0.25 $\frac{2}{5}$ $\frac{2}{3}$ 0.7 1.5 $1\frac{3}{4}$

0 1 2

Case study 1

Students were observed to have difficulties solving problems requiring subtraction with decimal fractions:

At the cycling competition, the times for the 200m sprint were 129.31, 131.15, 130.46, 132.0, 129.4, 133.1, 129.18 and 131.5 seconds.

What was the difference in time between the cyclist who came last and the cyclist who won the race?

Case study 1

At the cycling competition, the times for the 200m sprint were 129.31, 131.15, 130.46, 132.0, 129.4, 133.1, 129.18 and 131.5 seconds.

What was the difference in time between the cyclist who came last and the cyclist who won the race?

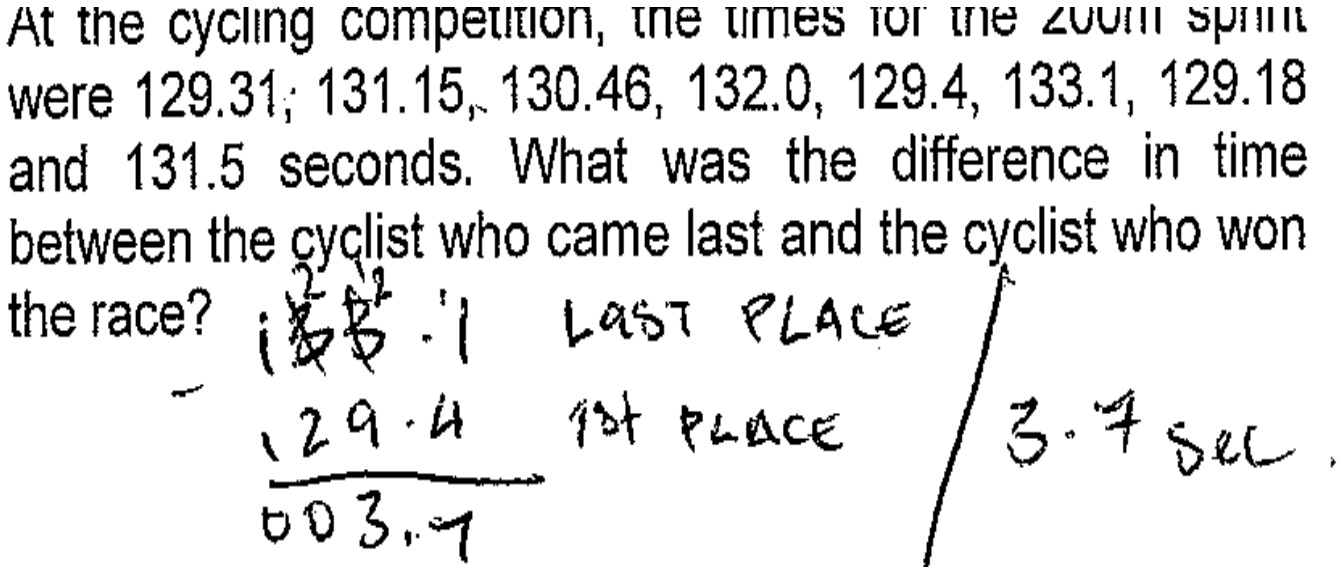
As well as subtraction to find the difference in time, this problem requires students to first identify the time taken by the winning cyclist and the time taken by the cyclist who came last.

$$\begin{array}{r} {}^{2}{}^{12}{}^{10} \\ 133.10 \\ 129.18 \\ \hline 003.92 \end{array}$$

Case study 1

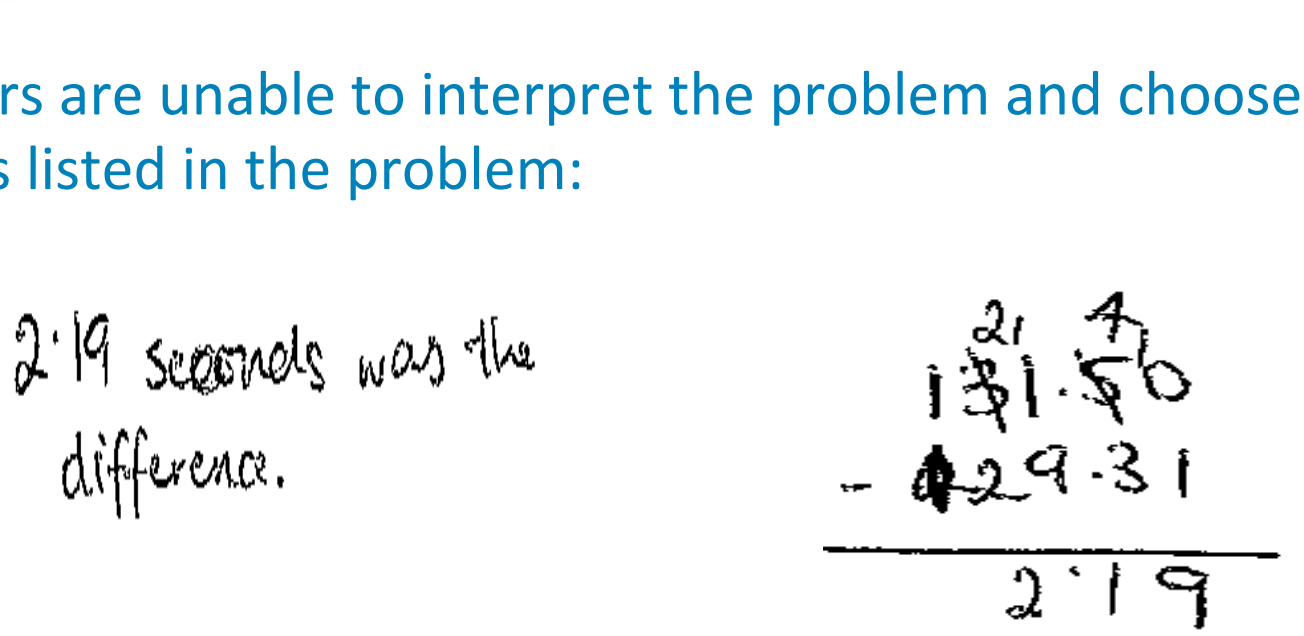
Students may be able to complete correctly the subtraction they set out to solve, but often are unable to determine which numbers to use

A common error is to see 129.4 seconds as the fastest time since 4 is less than 19.



Case study 1

Others are unable to interpret the problem and choose the first and last times listed in the problem:

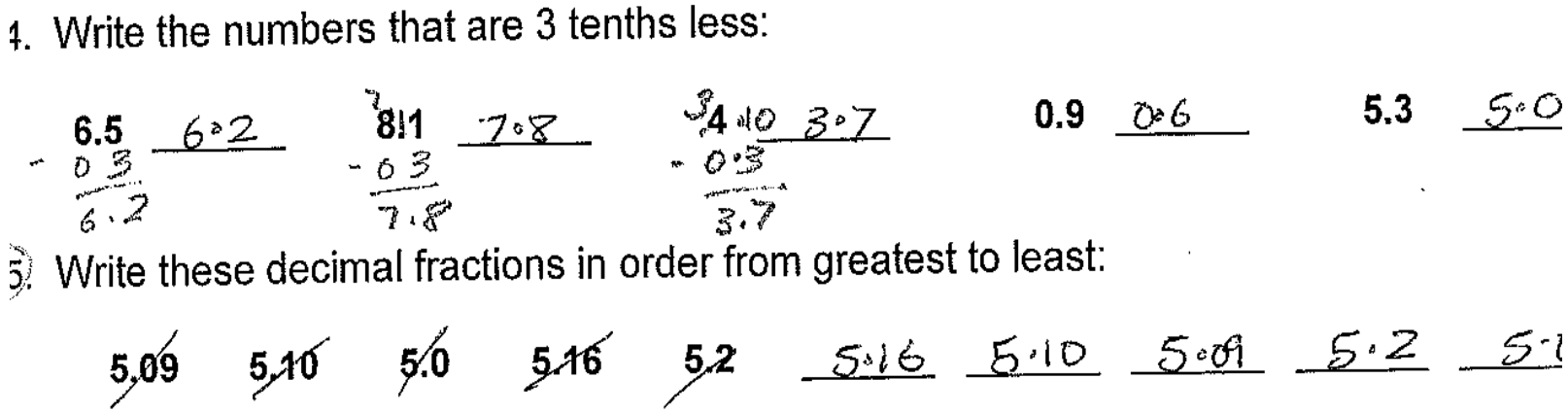


Some reverse the order of subtraction yet still obtain an “answer”:

Case study 1

Even when calculations are completed correctly, this does not indicate an understanding of decimal fraction computation, nor does it reveal where misunderstandings are sourced

The cause of their problem solving difficulties lies in an inability to order decimal fractions, in turn revealing that a lack of decimal place value needs to be addressed



Case study 1

This same student then continued:

$$\begin{array}{r} 6.7 \\ +\ 8.4 \\ \hline 15.1 \end{array}$$

$$\begin{array}{r} 5.6 \\ \times\ 7.4 \\ \hline 2224 \\ 3920 \\ \hline 41.44 \end{array}$$

7. $$\begin{array}{r} 7.014 \\ -\ 3.37 \\ \hline 3.647 \end{array}$$

9. $7 \overline{)45.29}$ = 6.47

42

29

28

1 (remainder)

49

At the cycling competition, the times for the 200m sprint were 129.31, 131.15, 130.46, 132.0, 129.4, 133.1, 129.18 and 131.5 seconds. What was the difference in time between the cyclist who came last and the cyclist who won the race?

winner: 129.4

last: 133.1

$$\begin{array}{r} 133.1 \\ -\ 129.4 \\ \hline 003.7 \end{array}$$

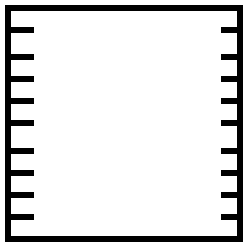
3.7 seconds.

Case study 1

In order to provide intervention on the underlying difficulties the diagnosis has revealed, the fraction concept and place value for decimal fractions need to be built up

This requires teaching to

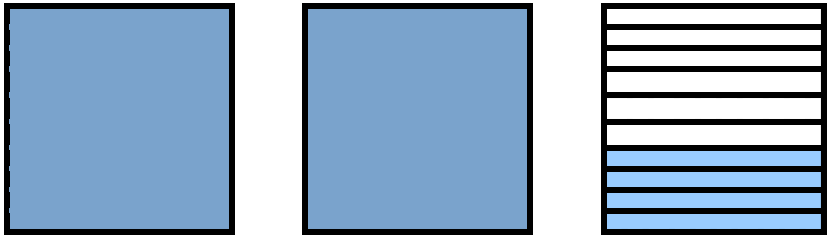
- revisit the model for fractions using rectangles to link model and names based on ordinal numbers
- use a square with the beginning lines and have the student connect the lines to see there are 10 equal parts – tenths



10 equal parts - tenths

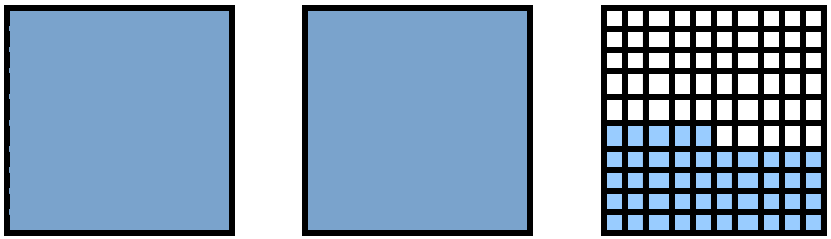
Case study 1

- name fractions with ones and tenths



2 ones and 4 tenths – 2 and 4 tenths

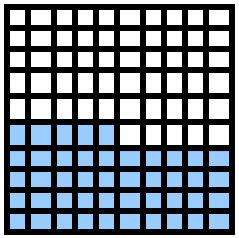
- name fractions with ones and hundredths



2 ones and 45 hundredths – 2 and 45 hundredths

Case study 1

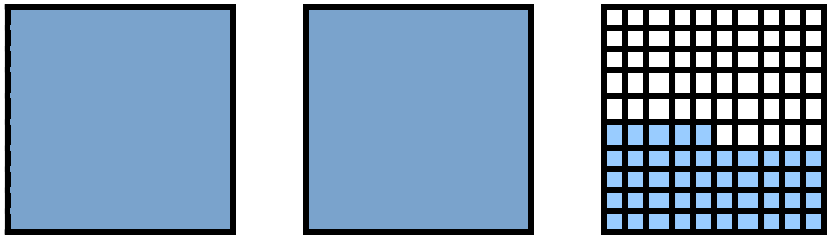
- rename hundredths as tenths and hundredths



1 tenth is 10 hundredths

45 hundredths is 4 tenths 5 hundredths

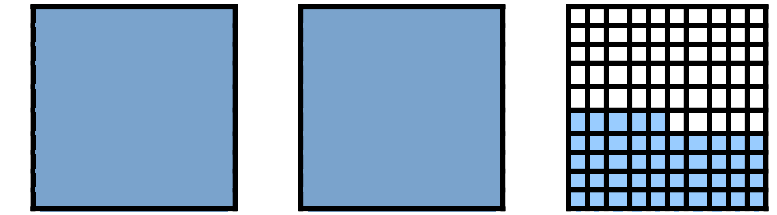
- name fractions with ones tenths and hundredths



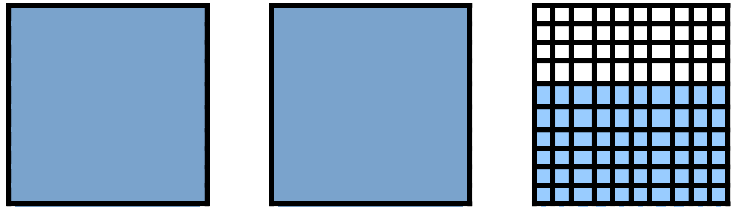
ones	tenths	hundredths	
2.	4	5	2.45

Case study 1

- compare fractions with ones tenths and hundredths



ones	tenths	hundredths	
2.	4	5	2.45



ones	tenths	hundredths	
2.	6	0	2.6

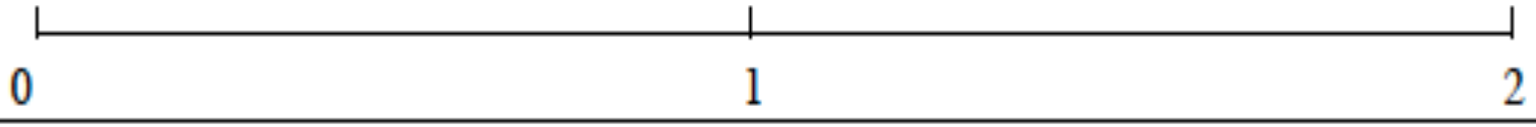
- elicit that 2.6 is greater than 2.45 because it has more tenths
- compare fractions using symbols only, drawing on place value

Case study 2

Students often have difficulties placing fractions on a number line

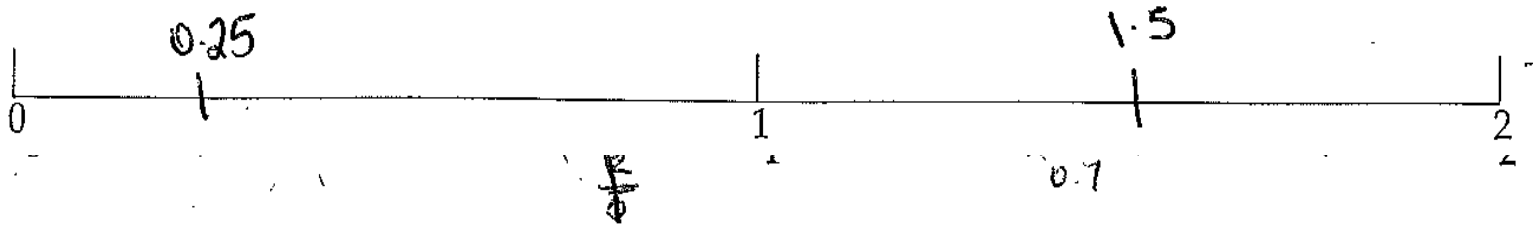
- an inability to see fractions as numbers among the whole numbers
- often viewed as two numbers arranged according to ‘decimal’ or ‘fraction’ rules
- a way of using whole numbers rather than extending their understanding of numbers

Place these fractions on the number line below: 1.5, $\frac{2}{3}$, 0.7, $1\frac{3}{4}$, $\frac{2}{5}$, 0.25



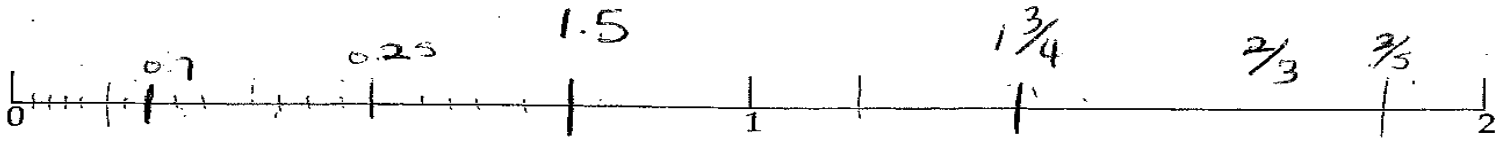
Case study 2

- Some would interpret the number line simply as a line and place some fractions where they believed they should be on the whole length and then place the others among them



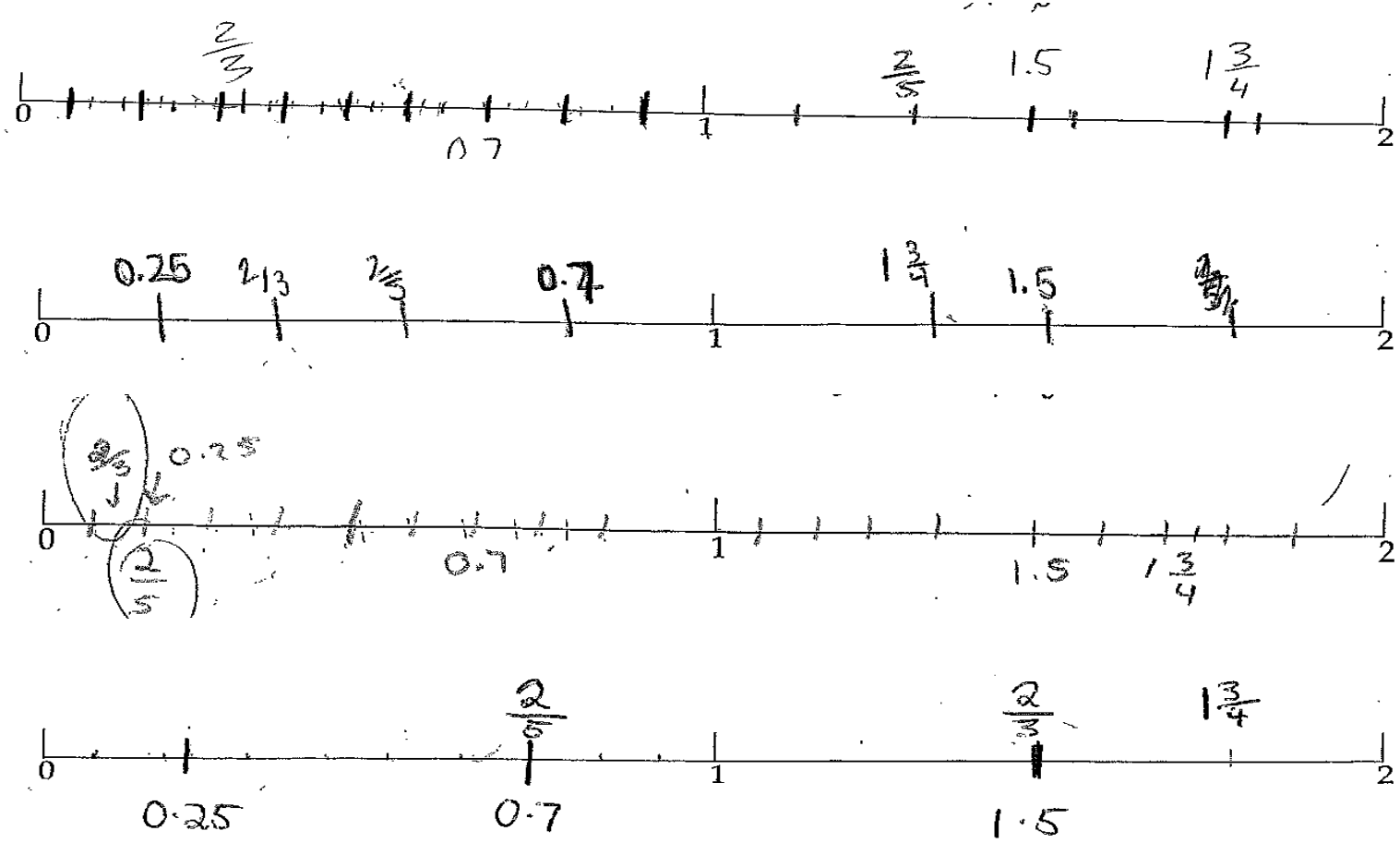
- Others could only place decimal fractions

- Fractions were often clustered around the whole numbers 0, 1, and 2



Case study 2

- When the line was divided to show parts, some fractions were placed correctly then others were simply put among them



Case study 2

Difficulties with number lines often stem from their use in the early years to count on and back

- this use focuses on the answers, using the points on the number line to count 1, 2, 3…
- emphasis on points on the line rather than the distances between numbers leads to difficulties with using a ruler to measure length as well as placing fractions along the number line
- impediments to the extension of the number line to provide meaning for negative as well as positive numbers

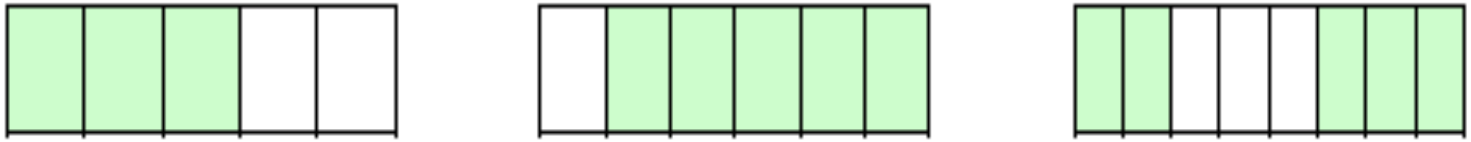
Case study 2

Intervention

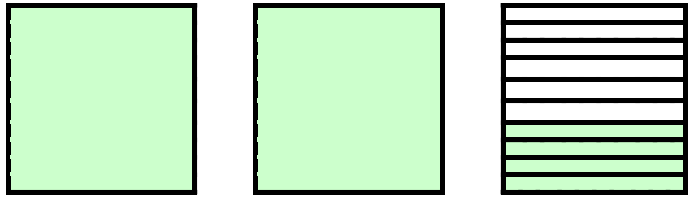
- A first approach to overcoming the underlying difficulties is to avoid premature uses of a number line in the first years of school
- Leaving number lines for fraction ideas and real numbers also provides a basis for understanding scale and other notions of proportional reasoning
- Build the fraction concept from the use of region models to show proper fractions and mixed numbers (whether decimal or common fractions)
- Gradually introduce representations on a number line

Case study 2

- Use rectangular models to establish the fraction concept as the number of parts out of the total number of equal parts

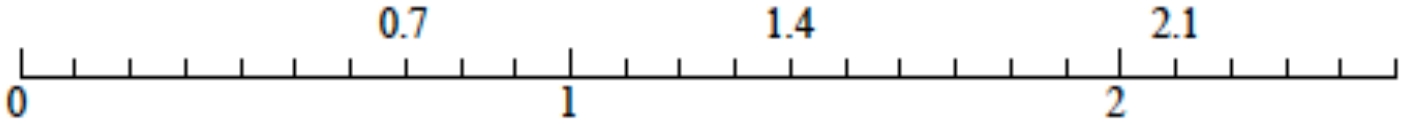


- Introduce meaning for decimal fractions as in Case Study 1



2 and 4 tenths – 2.4

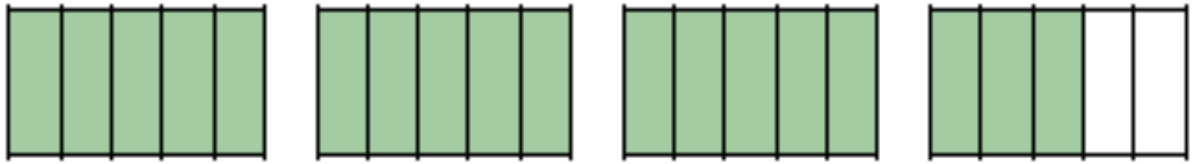
- Place decimal fractions onto a number line with 10 divisions between whole numbers



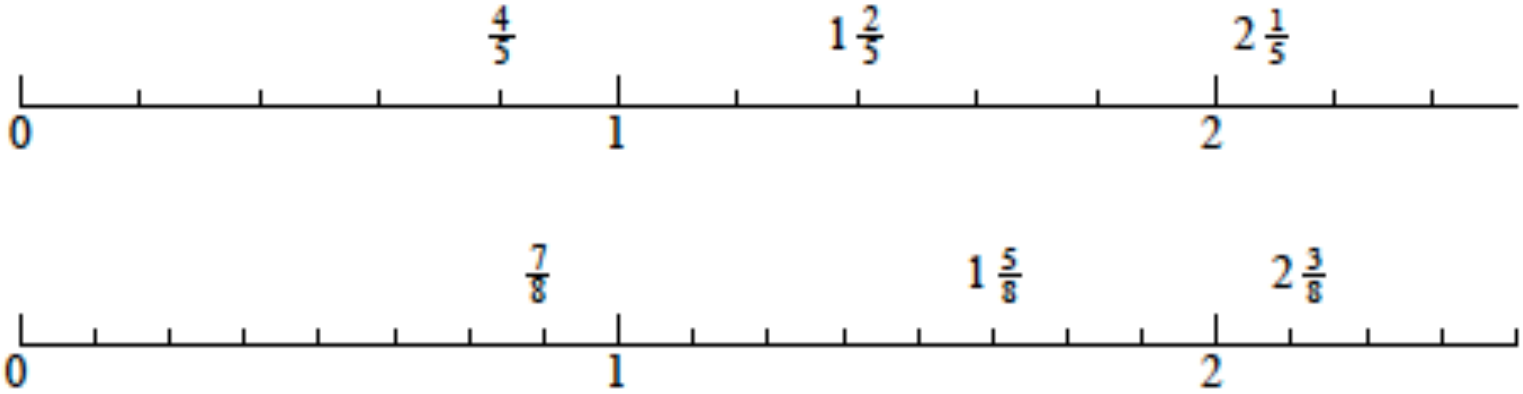
Case study 2

- Use rectangular models to introduce improper fractions and name them as mixed numbers

$3\frac{3}{5}$ or $\frac{18}{5}$

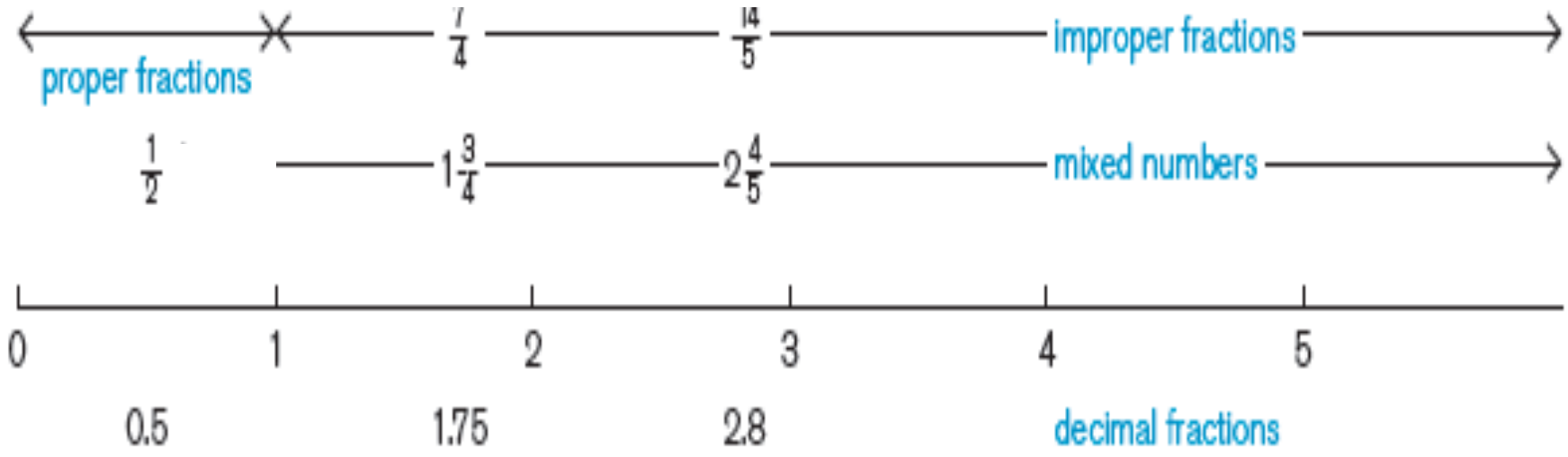


- Place different common fractions on a number line by drawing divisions to show the number of parts in each one



Case study 2

- Show how all fraction forms can be represented on a number line



- Have children determine the relative positions of the various fraction forms on a number line

Conclusion

Building numeracy in all students is a critical aspect of contemporary schooling. Understanding how concepts and processes are constructed and connected provides a basis for overcoming misconceptions and inappropriate ways of thinking that may have developed (Hiebert & Grouws, 2007, p.391).

Appropriate intervention programs can then be planned and implemented to build students' competence and confidence with fundamental mathematical ideas.

Conclusion

When all 3 dimensions – diagnosis of difficulties, intervention to build meaningful ways of thinking, and developing confidence in their mathematical ability – are present …

Students will be prepared to engage with further mathematical ideas and be inclined to use their knowledge of mathematics in the many everyday and work contexts where reasoning and sense making will be required