AMSI
AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

The Australian Curriculum: Mathematics

- Open Access Online Resources for Teachers

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Who are we?

The Australian Mathematical Sciences Institute

AMSI was established in November, 2002

We aim to improve the teaching of mathematics at primary and secondary level by joining with teachers, mathematics teacher associations and government agencies to develop a strategy to address issues such as teacher shortfalls and under-qualified teachers.

AMSI has 31 members including university mathematics departments, the Australian Mathematics Trust, the Australian Bureau of Statistics and CSIRO.

Who are we?

The developers of ICE-EM Mathematics

AMSI's education division, the International Centre of Excellence for Education in Mathematics (ICE-EM), has been undertaking wide ranging education programs at primary, secondary and tertiary levels since 2004.

The ICE-EM Mathematics Program provides books, professional development and teacher resource materials.

The Australian Curriculum versions of the books are published by Cambridge University Press.

Who are we?

The developers of ICE-EM Mathematics

Written by teachers together with mathematicians.

Smooth transition from upper primary to secondary - Years 5 to 10.

Mathematically accurate.

Supports development of teacher content knowledge.

Supplementary material.

TIMES Project

Minister Gillard's announcement ~ 13 October, 2009

Australian Government

"Adding $2 million to maths education"
The Minister for Education, Julia Gillard, today announced funding of $2 million to help improve mathematics education in Australian schools and universities.
The Government firmly believes that all Australian students need to be proficient in the basics like numeracy to be able to fully participate in the world of work and further study.
The Australian Government is funding the Improving Mathematics Education in Schools Project which is run by the Australian Mathematical Sciences Institute (AMSI) in collaboration with business, industry and teacher professional associations.
The project supports the teaching of mathematics in targeted low socio-economic schools and raises the awareness of maths-related career opportunities.(cont.)

TIMES Project

Minister Gillard's announcement ~ 13 October, 2009

"Adding $2 million to maths education"
Support for initiatives that have been shown to work, such as the Improving Mathematics Education in Schools Project, builds the capacity of teachers and schools to improve the educational outcomes of their students.

Part of the funding will be used to extend the successful BlueScopeSteel/AMSI Illawarra Outreach Program model which links schools, local business and industry with other schools in low socio-economic regions.

The project will also develop innovative school mathematics resources to support the implementation of the new national mathematics curriculum being delivered by the Rudd Government...(continues...)

TIMES Project

Careers materials

Careers materials have been developed to raise awareness among students, parents, school career advisers and others about the mathematics requirements for different careers.

An initial research and review process established key directions with a broad range of employer groups and key stakeholders involved in the process.

Packs of TIMES careers Materials were sent to every school in the country in 2010.

TIMES Project

CSIRO collaboration

Mathematics and Statistics by Email
CSIRO, ABS and AMSI jointly develop and operate a regular enewsletter, Maths by Email (MbE). MbE is based on the highly successful Science by Email model operated by CSIRO. It is aimed at primary and secondary school teachers, students of age 10+ and families.
www.csiro.au/mathsbyemail

Mathematicians in Schools
AMSI participates in a joint collaboration to support the Mathematicians in Schools program run by CSIRO. http://www.mathematiciansinschools.edu.au/

TIMES Project

Careers materials

maths: make your career count

We have produced careers materials that demonstrate how a range of people use mathematics in their work:

- 12 Posters
- 10 three-minute videos
- A 16 page brochure containing 20 profiles

TIMES Project

CSIRO collaboration

TIMES Project

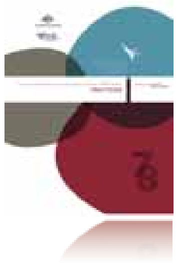
CSIRO collaboration

• To become one of more than 13 500 subscribers

www.csiro.au/mathsbyemail

TIMES Project

The Modules



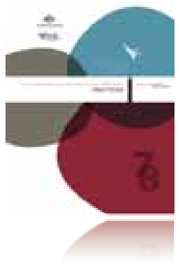
The use of the modules has been a key element of the AMSI's Outreach work.

In each school the way in which they are used is different.

The one common factor is that the modules focus teacher attention on the importance of enhancing their understanding of and confidence with mathematics content.

TIMES Project

The Modules



There are 68 modules. The material has been developed for years F-10.

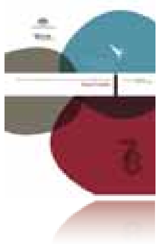
They are organised through the strands of the Australian Curriculum. These are:

Number and algebra
Measurement and geometry
Statistics and probability

Go to www.amsi.org.au/teachermodules

TIMES Project

The Modules



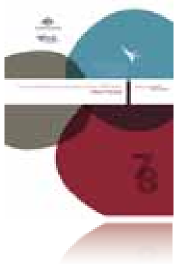
The Modules are teaching and learning resources for teachers - supporting the implementation of the national mathematics curriculum.

Modules have been written to support teachers in their teaching of mathematics. Teachers in Outreach clusters have worked with us to develop the materials. These will provide teachers with a valuable mathematical resource.

The Modules are available through Scootle and FUSE or via www.amsi.org.au/teachermodules

TIMES Project

The Modules



Each module has the following components:

Assumed knowledge
Motivation
Content
Links forward
History

Each module contains a small number of exercises for the teacher throughout.

TIMES Project

AMSI Modules
MAV 2011
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Who are we?

The TIMES team

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The modules are organised according to the content strands of the Australian Curriculum

- Number and Algebra
- Measurement and Geometry
- Probability and Statistics

The organisation of the modules:

- Assumed knowledge
- Motivation
- Content
- Links forward
- History

Proficiency strands

Adapted from the recommendations in *Adding it up* (Kilpatrick, Swafford, & Findell, 2001).

Understanding, which includes building robust knowledge of adaptable and transferable mathematical concepts, the making of connections between related concepts, the confidence to use the familiar to develop new ideas, and the 'why' as well as the 'how' of mathematics.

Proficiency strands

Fluency, which includes skill in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily.

Proficiency strands

Problem solving, which includes the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively.

Reasoning, which includes the capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying, and generalising.

Negative integers are not a trivial idea

'God made the natural numbers; all else is the work of man'
Kronecker

From the module:
The Integers

- *A debt subtracted from zero is a fortune.*
- *A fortune subtracted from zero is a debt.*
- *The product of zero multiplied by a debt or a fortune is zero.*
- *The product of zero multiplied by zero is zero.*
- *The product or quotient of two fortunes is a fortune.*
- *The product or quotient of two debts is a fortune.*
- *The product or quotient of a debt and a fortune is a debt.*
- *The product or quotient of a fortune and a debt is a debt.*

1 The Integers

From the module:
The Integers

There is no evidence of the Greeks mentioning negative numbers. For example, the Greek mathematician Diophantus (275 AD) described the equation $4x + 20 = 4$ as impossible.

Brahmagupta, an Indian Mathematician, wrote important works on mathematics and astronomy including a work called Brahmasphutasiddhanta (The Opening of the Universe) in the year 628. This book is believed to mark the first appearance of the rules for negative numbers in the way we know today.

He gives the following rules for positive and negative numbers in terms of fortunes (positive numbers) and debts (negative numbers).

From the module:
The Integers

After Brahmagupta other Hindu writers used negative numbers and all Hindu mathematicians used the rules discussed by Brahmagupta from about 850AD.

In Europe, Fibonacci (1225) interpreted a negative solution of a linear equation in a financial problem as a loss instead of a gain but little else was done in medieval times in Europe.The first of the 16th century writers to consider negative numbers was Cardano. In his book Ars Magna (1545) he found negative solutions of equations and gave a clear statement of the rules we have discussed in this chapter.

Why are the modules relevant for the proficiency strands Reasoning and Understanding?

Lets look at multiplication of the integers

For example, for $a = -2$, $(-2) \times (-1) = -(-2) = 2$. This allows us to multiply any negative integers together.
Two ways of doing this are shown here:

$$\begin{aligned}(-3) \times (-5) &= (-1 \times 3) \times (-5)\\ &= -1 \times (3 \times (-5))\\ &= -1 \times (-15)\\ &= 15\end{aligned}$$

$$\begin{aligned}(-3) \times (-5) &= (3 \times (-1)) \times (-5)\\ &= 3 \times (-1 \times (-5))\\ &= 3 \times 5\\ &= 15\end{aligned}$$

The careful reader will notice that we assumed both the commutative and associative rules for multiplication.

EXERCISE 3

The distributive law states $a \times (b + c) = a \times b + a \times c$. We want the distributive law to hold for the integers. By considering the fact that $-3(5 + (-5)) = 0$ show that $-3 \times (-5) = 15$

MULTIPLICATION

The product $5 \times (-3)$ means 5 lots of -3 added together.
That is, $5 \times (-3) = (-3) + (-3) + (-3) + (-3) + (-3) = -15$.

We want multiplication of integers to satisfy the commutative law
$a \times b = b \times a$.
Just as $8 \times 6 = 6 \times 8$ we will take $(-3) \times 5$ to be the same as $5 \times (-3)$.

The question remains of what multiplying two negative integers together means.
We have seen that $-(-a) = a$ for any integer a. If a is a positive integer $a \times (-1) = -a$.

We would like this to be true for all integers. That is, we agree that $a \times (-1) = -a = -1 \times a$ for all integers.

$3 \times 5 = 15$	$3 \times (-5) = -15$
$2 \times 5 = 10$	$2 \times (-5) = -10$
$1 \times 5 = 5$	$1 \times (-5) = -5$
$0 \times 5 = 0$	$0 \times (-5) = 0$
$-1 \times 5 = -5$	$-1 \times (-5) = 5$
$-2 \times 5 = -10$	$-2 \times (-5) = 10$
$-3 \times 5 = -15$	$-3 \times (-5) = 15$

Can we prove that the product of two negatives is a positive?
Not easily and we shouldn't pretend that we can. All of the above arguments are consistency arguments.
Yes, we can prove it but we first need to construct the negative integers.
Should we undertake the construction with year 7 or 8 students? Probably not.

From the Appendix of the module, The Integers.

In this module we developed the integers intuitively in a way that would be appropriate for classroom use. Our approach was basically geometric, using the number line to move forwards and backwards and to motivate 'opposites', addition and subtraction. We wanted the integers to satisfy the usual rules of arithmetic such as the commutative, associative and distributive laws of addition and multiplication, and this insistence led us to the rules for multiplication and division for integers. A careful look at how we derived, for example, the rule that the product of two negative numbers is positive, will reveal that we tacitly used the rules mentioned above when dealing with integers.

ORDERED PAIRS

The starting point is to take the set of ordered pairs (a, b) of whole numbers. Intuitively, we will think of this ordered pair as representing the integer $a - b$.

Thus (7, 4) will be thought of as representing the number $7 - 4 = 3$, while (4, 7) will represent the number $4 - 7 = -3$.

You will immediately realize then, that (5, 3) and (8, 6) both represent the number 2 and so we will say that two ordered pairs (a, b) and (c, d) are equivalent if $a + d = c + b$

Note that if $a + d = c + b$ then $a - b = c - d$, which is what we want.

ADDITION

We now define the addition of two ordered pairs as follows:

$$(a, b) + (c, d) = (a + c, b + d).$$

Hence, for example, $(1, 3) + (5, 2) = (6, 5)$.

This will correspond to $-2 + 3 = 1$.

Multiplication

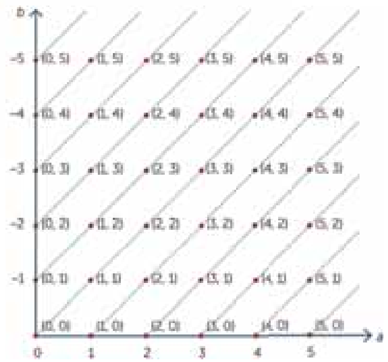
The multiplication rule is a little trickier. To see what the correct definition should be, we recall that (a, b) and (c, d) can be thought of as $a - b$ and $c - d$, so expanding the product $(a - b)(c - d)$, we obtain $ac + bd - ad - bc = ac + bd - (ad + bc)$.

Hence we define multiplication of the ordered pairs by

$$(a, b).(c, d)= (ac + bd, ad + bc).$$

Thus, for example, $(1, 3).(5, 2) = (5 + 6, 2 + 15) = (11, 17)$.

This corresponds to $-2 \times 3 = -6$.



A little geometry from the module: Introduction to plane geometry

Geometric reasoning Year 7 substrand

- Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (ACMMG166)
- Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (ACMMG163)
- Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning (ACMMG164)

CONVERSE STATEMENTS

Many statements in mathematics have a converse, in which the implication goes in the opposite direction.
For example, the statement
'Every even number ends in 0, 2, 4, 6 or 8.'
has converse
'Every number that ends in 0, 2, 4, 6 or 8 is even.'
This particular statement and its converse are both true, but this is by no means always the case.

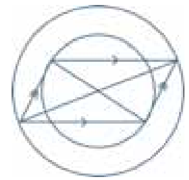
Statement:
If the lines are parallel, then the alternate angles are equal.
Converse:
If the alternate angles are equal, then the lines are parallel.

Statement:
If the lines are parallel, then the co-interior angles are supplementary.

Converse:
If the co-interior angles are supplementary, then the lines are parallel.

From the module: Parallelograms and Rectangles

As a consequence of this property, the intersection of the diagonals is the centre of two concentric circles, one through each pair of opposite vertices.



For example, the following two statements are converses of each other:

'Every multiple of 4 is an even number.'

'Every even number is a multiple of 4.'

and here, the first statement is true, but the second is false

From the module: Parallelograms and Rectangles

Theorem
The diagonals of a parallelogram bisect each other.

The converse:

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram:

This converse gives a very simple construction of a parallelogram.

Draw two intersecting lines, then draw two circles with different radii centred on their intersection. Join the points where alternate circles cut the lines. This is a parallelogram because the diagonals bisect each other.