USING PROBLEM SOLVING TO ENGAGE STUDENTS IN THEIR STUDY OF LINEAR EQUATIONS

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In 2008 Fairhills High School trialed a new semester long unit at Year 10 to improve students' skills in Algebra. Using the principles of the e5 instructional model (engage, explore, explain, elaborate & evaluate), the unit was designed to make extensive use of Problem Solving activities to engage students and to give purpose to the study of Algebra. The promising findings from this trial prompted the school to undertake a Teacher Professional Leave project investigating the integration of problem solving activities into the teaching of Year 8 Linear Equations and Graphs in 2009. This paper describes the model that was used in this project.

2009 Project Model

The model adopted to enhance the teaching of Year 8 Linear Equations and Graphs focused on the following three objectives.

Objective 1: Develop a Series Of Everyday Problem Solving Activities That Have Multiple Pathways To A Solution

Since student engagement was a major focus of the project, it was decided to establish a set of criteria as a starting point for the design of the problem solving tasks to be integrated into the Year 8 Linear Equations and Graphs topics. The criteria decided upon were:

(a) Each problem solving task should have practical links to the real world and provide a rationale for further study of linear equations and graphs
(b) Each task provides a meaningful learning experience for all students regardless of their level of competency in Maths.

While it was not hard to create tasks that met the first condition, it was found to be a lot more difficult designing activities that gave all students a practicable chance of meaningful learning and success. It became considerably easier to create activities when the second condition was unpacked to read:

> *Each task has to have a number of pathways to arrive at a prediction or proposal for the given scenario. The pathways available to students should typically have at least one elementary technique such as counting, making or trialing, and at least one higher order mathematical skill such as pattern analysis, algebra, use of formulae, application of theorems or the use of graphic calculators or computers.*

Because each task has more than one pathway to a solution, students at different learning levels can usually find an entry point that provides them with an achievable challenge. This in turn, makes the task engaging for all students. Also, since each student has their own learning experiences, they can then be drawn into the end of task reflection session, and after listening to other students' approaches to the problem, make their own decision as to the relevant merits of each mathematical method.

Objective 2: Develop And Promote A Common *Problem Solving Toolbox*

A 2008 audit of Fairhills High School's Year 7-10 Mathematics curriculum showed that while there were problem solving activities embedded into the courses for each year level, they tended to be one-off activities, with no common thread that systematically developed students' skills and confidence with new and challenging problems. Consequently it was decided that not only should more activities be developed but there also needed to be an agreed pedagogy and set of strategies that should be presented with each task.

As the project progressed the pedagogy evolved into the lesson structure described in the following section and the list of strategies (which we called the *problem solving toolbox*) became the following set of 20 starting points divided into 6 categories.

1. *Use your intuition*
   a. Guess & check
   b. Use number sentences
2. *Simulate the problem*

a. Draw a diagram
b. Make a model
c. Act out the problem
d. List all possibilities

3. *Put your information in a table or on a graph*

a. Use a table to help you count or list possibilities
b. Use a table to help you to set out your working (calculations)
c. Put your data onto a graph and draw a line of best fit

4. *Find/Use a pattern*

a. Find a pattern and apply it
b. Work backwards
c. Solve a simpler related pattern

5. *Use Algebra*

a. Plot a graph from a rule
b. Solve a linear equation
c. Solve two simultaneous equations
d. Solve a quadratic equation

6. *Use Excel*

a. Create a table with a formula and then use guess and check
b. Create a table to continue counting patterns
c. Create a table and draw a graph with a trendline
d. Create a table, draw a graph and then find a rule

This toolbox was presented with each of the problem solving tasks that were designed with the criteria described in Objective 1. Before starting any problem students were usually asked to make a prediction and then to choose/discuss a strategy before solving the task. No statement was to be made regarding the relative merits of any particular approach apart from the fact that the best approach is the one that makes the most sense to the student involved. Students were also encouraged to change strategies if their perspective of the problem changed, and in some cases combine two strategies if they felt it necessary.

Objective 3: Develop A Common Lesson Structure For Running Problem Solving Activities

To achieve the most effective outcome from a well designed problem solving activity, it is essential to deliver it in a way that engages all students and incorporates a reflection activity. The lesson structure that was generally found to work well consisted of the following three stages.

Stage 1: Introduce The Problem

Teachers should *take time to set the scene* for any problem solving activity and to explain it to the students in at least two ways to cover more than one preferred student learning style. Next, it is essential to reinforce the *universal method* for solving any new problem:

1 Understand the problem
2 Decide on a plan (pick a strategy)
3 Solve the problem
4 Check your answer

Going through these steps gives the teacher the opportunity to ask the class again for any questions about the problem and then to begin a discussion about possible plans/strategies for undertaking the task (using the Problem Solving Toolbox discussed previously). From this point students are ready to nominate their preferred strategy and to start solving the problem.

Stage 2: Solving The Problem

To keep students engaged and help them to maintain their focus on the task objectives, most of the problem solving activities created had a problem solving working stage of between 30 to 80 minutes. Three points that were continually stressed to students during this time were:

1 Record all your working either under a heading in your workbook, on a worksheet or on a blank piece of paper. Let your ideas flow onto your working page, but try to make it clear what you are doing so that someone else can easily follow the processes that you used to reach your prediction or answer. Dot points are recommended for strategies that require multiple non algebraic steps to a solution and short descriptive phrases are important when algebra is being used.
2 Discussion is an important avenue for learning. Feel free to discuss your ideas with your fellow students
3 Feel free to change your strategies if you get a different perspective of the problem and in some cases you may find it useful to combine two strategies.

Since most tasks are designed with multiple pathways to a solution most students should be able to make a reasonably prompt start during the problem solving stage. There may however be a couple of students who need the problem re-explained to them and they may need prompting with choosing their strategy or following it through. When assisting these or any other students during the problem solving stage try and use questioning techniques to enhance students' thinking and continually reinforce the three points above.

Lesson Stage 3: Reflect On The Problem

The student reflection stage of any activity is usually a time of significant learning and can often be an excellent opportunity for establishing a rationale for the study of new skills. Consequently, it is important to set aside a reasonable amount of time for students to discuss what they found and how they found it. This reflection can be tackled in a number of different ways:

1 A class discussion - where students present their findings and discuss the relative merits of the strategy that they used.

2 Summary sentences - where students write a quick summary of their work by adding endings to about 5 or 6 sentence starters. These summary sentences can often be a good prompt to the class discussion mentioned in the previous point.

3 A formal written report – where students write a report on the set task that will be reviewed by the teacher or their peers.

Sample Task

Storyline

Ken has been asked to build a 23 metre long pole fence with the design shown below:

Each section of fence is made from 1m long poles and the example above was made from 18 poles.

Problems to Solve

1 How many 1m poles does Ken need to build the 23m fence?
2 How much would the fence cost the client if they were charged $20 per pole and a further $10 to set each vertical pole and $5 to attach each horizontal pole?
3 What length of fence would $4000 purchase?
4 Ken has to produce quotes for eight prospective clients who are considering post and rail fences for their properties. Because each of the fences are to be built to different lengths, Ken has asked you to recommend a method for quickly working out the cost of each of the fences. What method would you recommend?

Possible Strategies/Pathways

Students can be directed towards the following strategies for problem 1:

- Use your intuition with number sentences
- Simulate the problem with a diagram or a model (with matches)
- Use a table to help you count the number of poles after each metre of fence
- Find/Use a pattern

After completing problem 1 students should refer to the toolbox for the remaining questions.

Outcomes from a trial of this problem

In a typical trial of this activity, problem 1 was set as the primary task and problems 2 to 4 were presented as "have a go" exercises. In most trials, approximately 30 minutes was allotted as problem solving time giving all students the opportunity to complete problem 1. In review time, students who tried different strategies were called upon to describe how they solved problem 1 and then, after seeing all the presentations, they were asked what strategy they would use for a fence that was 100 metres long. At this point, most students usually said that using a rule would be the best alternative, and since most of the rules were in a verbal form, it was a good opportunity to show how a rule can be formalized into an algebraic expression.

Overall, this problem proved to be a very useful starter to the linear equations topic because it gave real life credence to further study into equations by demonstrating how formulae can save us time and effort.