MATHEMATICS - OF PRIMARY IMPORTANCE. IS IT?

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What is called 'traditional mathematics education' has failed to educate. It may have taught, and students may have learnt, but it has failed to educate. Offering a diet of disconnected content parcels in a less-than-stimulating teaching environment is not educating. This paper suggests that teachers are having considerably more success at all levels by deliberately choosing best practice teaching craft strategies within a curriculum centred on learning to work like a mathematician - Working Mathematically.

Where Do I Stand?

I think it's time to stop teaching mathematics. In the past century or two the practice has hurt considerably more learners than it has helped. We all know someone, probably lots of someones, for whom that's true.

I think it's appalling that there is even one such someone.

I think mathematics content - the stuff you see listed in curriculum documents and outcome statements - offers very little in and of itself. The amount listed cannot be justified on the basis of 'you need it to pass an exam', or 'it will help you to get a job one day', or 'you might need this when you grow up'.

Worse, lists of content set learners up to fail. A content-driven view of mathematics learning ensures everyone succeeds to the first level of content they 'don't get' - and somehow, usually depending on their age when this happens, having not 'got that', they often decide they can't do maths.

Many of these learners later find themselves teaching mathematics with no vision of a mathematics classroom to inspire them other than the content-driven ones they survived.

Content-driven curriculum sets teachers up to fail too. It encourages us to think about teaching stuff, not people - whole human people with an integrity and a self-image formed by past experiences and within which they operate.

I believe these things ... and yet I have spent almost forty years teaching mathematics at all levels of primary and secondary school.

Learning to Work Like a Mathematician

In the last years of the sixties I did a degree in mathematics at Monash University. My academic record displays many credits, distinctions and high distinctions in mathematics units. At the end I felt I had not learnt much more than how to pass exams.

In 1969 I studied a Diploma course in Education, also at Monash, and through the wisdom and teaching experience of lecturers and tutors across the course, I began my teaching career believing that it *was not* my job to teach mathematics. It *was* my job to use my mathematics background to help learners feel better about themselves.

This is an objective I have failed to achieve in many of my classrooms over the years, but it is one from which I have never wavered.

Learning to become an EMIC (Exploring Mathematics in Classrooms) tutor in the eighties showed me I could achieve my objective more readily by building on success stories from other classrooms. By that time an anthology of such stories had begun to build through projects like MCTP (Mathematics Curriculum and Teaching Program) and RIME (Reality in Mathematics Education). Through the last fifteen years the sole purpose of my work has been to collect and retell such stories of success. Stories with the twin purposes of illustrating:

- features that encourage learning
- what it means to learn to work like a mathematician

Charles Lovitt invited me to assist him with the Mathematics Task Centre Project in 1992, that work continues to this day, and through that project we were able to detail what it means to be learning to work like a mathematician - Working Mathematically.

It was easy enough to do - we asked mathematicians. Their response was:

- *First give me an interesting problem.*

So, a Working Mathematically curriculum would begin with problems and our teaching craft would be called on to interest learners in them. That's very different to a content-driven curriculum.

But how do you approach one of these interesting problems? There is no promise of solution, there is only hope that there might be one, so what do you do? Mathematicians went on to describe their 'ways of knowing' as follows:

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Break the problem into smaller parts

- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- ...

If one way doesn't work I just start again another way.

This is the simple, yet subtle, description that has brought success to hundreds of classrooms from K to 12. It provides a higher order curriculum focus and a language for exploring it. Schools having most success use this language consistently across the school. And, perhaps because it makes sense, students don't ask 'Why do we have to do this?'

Email, Ian Pegram, Dandenong High School, 24th July 2007

I used Birth Month Paradox (HREF1) *with two Year 10 extension maths classes and one Year 11 VCAL class. The odd thing with the VCAL class is that I keep expecting them to say "What's the point of doing these types of questions?", but they never do.*

Examples of schools Working Mathematically and other information about learning to work like a mathematician can be found in the Working Mathematically link of the Mathematics Task Centre (HREF2).

So, a Working Mathematically curriculum is built around problems suitable for children at all year levels - real problems with unknown answers, not recipe exercises with expected answers. To help you consider my question *Mathematics - Of Primary Importance. Is it?* I invite you to consider two or three problems and the stories that surround them.

But first let me make it clear that the examples are drawn from my 15^+ years of professional development leadership which for the most part has been linked to Mathematics Task Centre, Maths300 and Calculating Changes. Clearly I derive income from these sources so some might see the chosen examples as promotion of these projects for my financial gain. That would be an incorrect assumption, but if the possibility concerns you then this the moment to either stop reading or jump to the conclusion of the article.

Sphinx

I am often asked about the Task Centre logo and its apparent connection to the flag of Indigenous Australians.

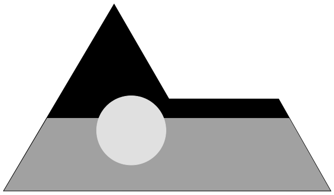


Figure 1. Task Centre Logo, normally coloured red, yellow and black.

The story is recorded in the Sphinx Album of the Mathematics Task Centre (HREF3) and the essence of it is retold here. The perimeter shape is called a Sphinx. It has an area of six equilateral triangles. Task 166, *Sphinx*, challenges students to use four of these shapes to make a Sphinx.

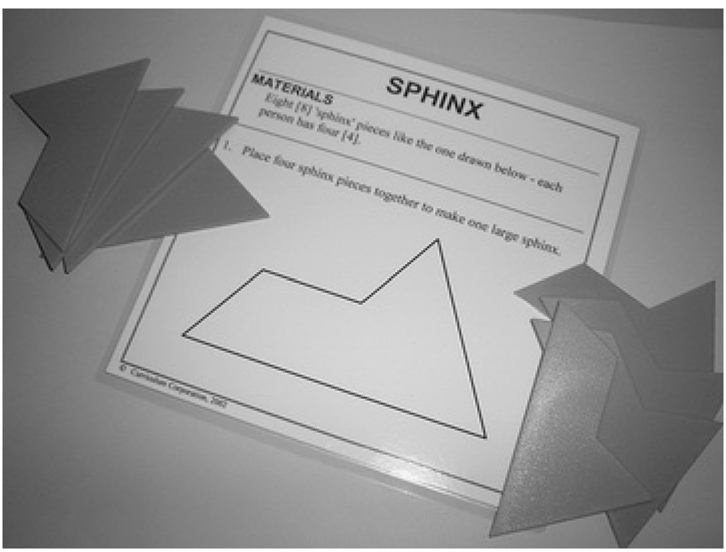


Figure 2: Task 166, Sphinx*, Mathematics Task Centre*

As it stands the task is a spatial challenge of moderate difficulty. In 1996 this was one of the tasks trialed at a student workshop day organised by a group of Brisbane teachers who were developing a task kit for urban Indigenous students. Michael and Tyler, two Year 5 Indigenous students from Norris Road School, accepted the challenge. They solved it quickly. However, all such tasks represent the tip of an investigation iceberg and they were challenged to dig deeper.

With help from one of the teachers they realised that if four Sphinxes make a Sphinx, then four of the new size would make the next Sphinx and making it would follow the template provided by the first solution. This was a bit of a mind experiment because they only had four wooden Sphinx pieces, some poster paper and markers. But they were able to perceive that if a single Sphinx is Size 1, the solution to the original puzzle could be called Size 2 and the one after that would be Size 4.

Then one of them asked: 'What happened to Size 3?'

The question flummoxed their teacher a little. To the best of his knowledge it had never been asked before and he said so. 'Can we see if we can make one?' That question started the equivalent of a never ending story which is detailed in the Sphinx Album.

The boys set to work with their tracing tools; but this was a regional workshop day for students and time ran out. The bus came to return them to school. 'Can we take this with us?' They could take their drawings, but not the wooden pieces. They belonged to the teachers' set. 'That's okay.'

Six months later they handed to the leader of the Brisbane project the solution of the Size 3 over-coloured with the colours of the Indigenous flag, claiming the solution for the Aboriginal people of Australia.

With the permission of their parents, the boy's work became the logo of the Mathematics Task Centre.

What is of primary importance here? The fact that 4 funny shapes fit together to form the same funny shape? The fact that the growth pattern leads to work in topics of patterns and powers, area and perimeter, prime and composite numbers, algebraic generalisation and symbolic representation and more.

Or how those boys felt about themselves?

Interestingly, until this solution was revealed, the project teachers were discussing removing Sphinx from the developing kit because they saw it as irrelevant to Indigenous students.

Poly Plug Values

Poly Plug is a preferred resource to support Calculating Changes (HREF4).

Figure 3: Poly Plug set

A grey scale picture doesn't do this resource justice and its colours are important to this story. The top board of the two shown is red with plugs 9mm thick. The bottom board is a sandwich of 9mm yellow with 9mm blue. Its plugs are 18mm thick, blue on one side and yellow on the other.

I will let Jacki Healey, Margate Primary School, tell this story as it developed in her class. It is recorded in Calculating Changes.

I was working with Place Value with my Grade 2 class and I wanted to do something to extend their understanding. I decided to use Poly Plugs. The first time I introduced the Poly Plugs I told them it would be interesting to use them to represent numbers.

How could they do this?

Initially I used low numbers, eg: 14, 17, etc. I discovered that given low numbers, most children would assign each Poly Plug a value of 1, ie: 1 to 1 correspondence.

The challenge lay in giving them larger numbers to represent, eg: 96, 43, 85, 74. Large numbers and odd numbers were harder and required more ingenuity to represent. At the end of the first 1 hour session, some children had considered giving, for example, a red plug the value of 1, a yellow 2, a blue 5 or 10.

Over the next 3 weeks I repeated the exercise, giving each group the opportunity to swap partners and share ideas. It was tremendously exciting to watch as the children's confidence and ingenuity increased each week.

They were also asked to record their ideas, so that they could share with the whole group. By the end of the month the children's ability to 'skip count' in 2s, 3s, 4s, 5s, 6s and 10s was amazing. Some children even assigned value to the empty places in the plug board. They also thoroughly enjoyed the whole process.

Jacki is using a teaching method here that has been devised by Calculating Changes teachers. It's called Threading (HREF5). The same rich task with a familiar structure is used for small amounts of time, often, over several weeks. The learner's intellectual energy is invested in the mathematics of the activity, rather than the structure of the activity, and developing learning can be easily tracked. Jacki's story continues:

One day I suggested that they had worked so hard they deserved a break. They could play on the brand new playground equipment, then come back to finish their Poly Plug Values. They unanimously refused and chose to complete the task first!

What is of primary importance here? The mathematics content or the classroom environment Jacki created and the way the children felt about themselves as learners?

Same or Different

Task 18, *Same or Different*, is easy to state and easy to start.

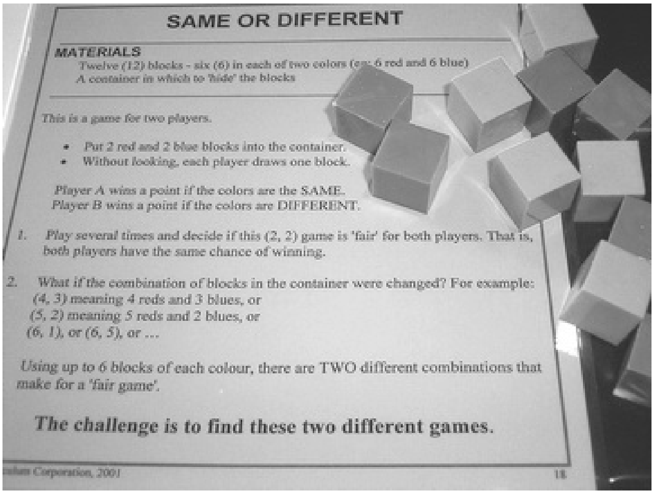


Figure 4. Task 18, Same or Different, Mathematics Task Centre

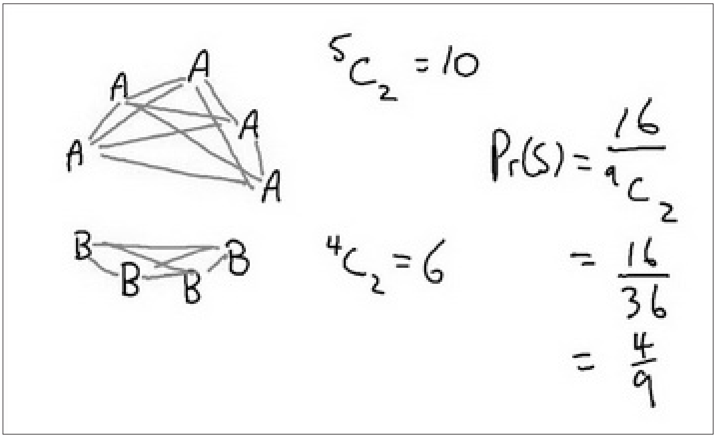
A bag contains a mixture of red and blue blocks. Two blocks are removed. What are the chances they are the same colour? Clearly the answer depends on the original combination of red and blue. So, is there a red to blue collection that gives a 50:50 chance of drawing the same colour - a fair game?

The investigation can be tackled in Year 3. It is only an experiment after all, but what are the features that captivate, fascinate and absorb students?

The investigation could also be tackled in Year 7, where, as the card suggests there are two fair games to be found using up to six blocks of each colour. Pushing further with the investigation these students might recognise the resulting fair game combinations (1, 3) and (3, 6) as special numbers - triangle numbers - and wonder if (6, 10) is also a fair game.

In senior secondary school, the same investigation could lead to exploring how theoretical probabilities lead to these triangle numbers and the discovery that this random event is actually governed by the discriminant of a quadratic equation (and a bit of rotational symmetry, but that's another story). And, as Damian Howison, MacKillop College, found, after years of learning to work like mathematicians students can make unexpected connections to earlier learning. Damian has recorded this story in the Task Cameo for *Same or Different* (HREF6).

I used the task in my Year 12 Methods class last week and we had a good time with it. In particular we used simple line diagrams to show the probabilities and this naturally lead to using the combinations function for calculating the probabilities (incidentally we derived the combinations function last year using 'Farmyard Friends', Task 129).



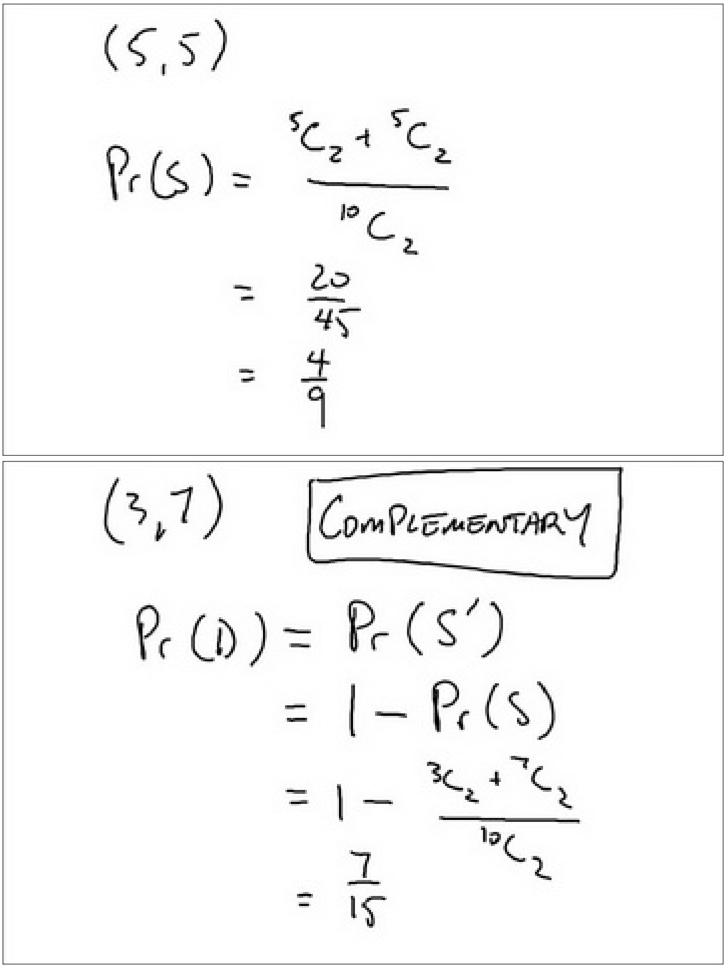


Figure 5. A series of whiteboard recordings from Damian's lesson sequence.

Using the combinations approach made it straightforward to search for fair games on Excel. The pattern 1, 3, 6, 10 came off the screen and one student recognised these numbers as $^2C_2$, $^3C_2$, $^4C_2$, $^5C_2$ ...

This suggested that one solution for a fair game was the general case of $(^NC_2, {}^{N+1}C_2)$.

And because of the combinations relationship with Pascal's Triangle, this sequence is also one of the diagonals of the triangle!

So as you can imagine, it was a wonderful little lesson of discovery - the best type where I the teacher learn just as much as the students.

What is of primary importance here? The undeniably extensive mathematics content totally relevant to an externally examinable curriculum, or...?

Back to Sphinx

Pam McGifford is a remarkable teacher. Her secondary school classrooms offer as much freedom to learn and responsibility for self-learning as any I have seen. In 1999 Pam learned of the Sphinx story in a professional development program built around making the Working Mathematically description above central to improving learning for Indigenous students in Tasmania. She introduced it to her Year 8 class at Cressy District High School and they explored it vigorously, largely without her assistance. When they presented her with their findings she asked them to make a record so she could show her colleagues at the next meeting of INISSS (Improving Numeracy for Indigenous Secondary School Students). After all, a mathematician has to publish results.

The students chose to make a video, which is now shown in many ‘Maths on the Move’ workshops (HREF7). Danielle Goss was one of the students in that class. In 2009, ten years later, she wrote:

Email, Danielle Goss, 20th March 2009

My name is Danielle Goss. I used to be a student at Cressy District High School where we studied the Sphinx problem in depth with our teacher Pam McGifford. As I’m sure you are aware, we recorded a video detailing our discoveries.

I am currently in the process of trying to track down a copy of this video (I don’t think I ever actually watched it!), and was wondering if you would be able to help me with this.

I have never met Dani, but as we emailed more I began to ask her some important questions. She responded:

Email, Danielle Goss, 4th May 2009

How has the Sphinx influenced my life to present? Well I will admit I have never been a mathematically minded person. Far from one in fact! Honestly, I couldn’t say how - if it has in anyway impacted my life over the last few years. However at the time it was a fantastic learning experience. I have always been a good student and achieved high marks but mathematics (apart from algebra - I still can’t understand why I can grasp that and not simple addition and multiplication!) was always a foreign language to me so I didn’t have any interest in that area. Studying the Sphinx problem sparked my interest, and got me actively involved in something I *could piece together and evaluate, rather than just work out on paper. Because I was interested by it all, I actually learnt a lot without realising it, and had a strong foundation to relate other problems to in the future. So in that sense, I suppose the Sphinx has helped me no end. I am still terrible at maths but I have some good practical knowledge arising from the Sphinx problem that I can draw upon if I need to.*

I think there was a question about the teachers in your last email? We had Pam McGifford for maths and John Bradbury for science, although I think he used to fill in for Pam from time to time and take our maths class. I'm not sure what to say about them, apart from how fantastic they are! They are both excellent teachers with that essential (well I think it is anyway) ability to relate to the students on a personal level whilst still maintaining the student-teacher order and respect. I only hope that when I complete my studies one day I can be half as good as them.

John was a participant in the same INISSS program as Pam and between them they nurtured their students from this small rural high school through Years 8 and 9. This class improved 20 percentile points on their state testing between Year 7 and Year 9 (HREF8).

Mathematics: Of Primary Importance?

So, is mathematics of primary importance? No.

- Why we teach is of primary importance.
- Learners are of primary importance.
- How we teach them is of primary importance.

The mathematics itself doesn't really matter. However, as has been shown by the Mathematical Association of Victoria with its syllabus documents *Working Mathematically in VELS: The Maths With Attitude approach* (HREF9), it is possible to produce curriculum documents from Years 3 to 10 centred on learning to work like a mathematician *and* address every demand of the current Victorian government document. Some would see it as a bonus that these syllabus documents are totally textbook free.

Using these documents as a model, other state associations are in a position to support their members in a similar way. Further, if a useable national curriculum document eventually develops, the Australian Association of Mathematics Teachers could accept a similar challenge nationally.

There is no reason for mathematics to be taught the way it always has been.

References

HREF1: **http://www.curriculum.edu.au/maths300/lesson_links/148lbirt.php3?ID=149**, Birth Month Paradox, Maths300, Curriculum Corporation. Ian Pegram's comment is recorded in the Classroom Contribution.

HREF2: **http://www.blackdouglas.com.au/taskcentre/work.htm**, Working Mathematically examples and support links, Mathematics Task Centre.

HREF3: **http://www.blackdouglas.com.au/taskcentre/sphinx.htm**, Sphinx Album, Mathematics Task Centre

HREF4: **http://www.blackdouglas.com.au/calchange**, Calculating Changes.

HREF5: **http://www.blackdouglas.com.au/calchange/cchbrief.htm#threading**, Description of Threading within the In Brief link of Calculating Changes.

HREF6: **http://www.blackdouglas.com.au/taskcentre/018same.htm**, Task Cameo for Same or Different, Mathematics Task Centre.

HREF7: **http://www.blackdouglas.com.au/taskcentre/monthem.htm**. Maths on the Move professional development information. Mathematics Task Centre.

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