USING PROGRESSION POINTS FOR DIAGNOSTIC (FORMATIVE) ASSESSMENT: CHANCE

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Find every Chance-related Progression Point, in Level-order, from Prep to Year 10. Translate each Progression Point into a pencil-and-paper task. This makes a developmentally or progressively graded worksheet-like "diagnostic profile" for assessing knowledge of, and skills with Chance. It is diagnostic because it starts with easy, early questions, and progressively gets harder and harder, as the concepts and skills in the Chance curriculum develop. Presenting this diagnostic profile to students at the beginning of a unit of work on Chance gives invaluable formative assessment information to guide your teaching. Using the same profile, or a parallel version at the end of the unit provides before-and-after summative assessment of the students' learning during the unit.

Chance in the Progression Points

Note from the outset, that despite the usual way that we speak jointly of "Chance and Data", or perhaps "Measurement, Chance and Data", there is no reason for connecting Chance in this way with other topics. Certainly chance-based events (such as rolling dice, flipping a coin, or observing passing road traffic) can be used to generate data that can then be analysed. Similarly chance-based actions (such as throwing a ball or picking leaves from a tree) can be used for measurement-based activities. But from a theoretical perspective, within the large body of knowledge we call Mathematics, the topic of Chance is a more or less self-contained sub-curriculum of its own — as self-contained, and/or as inherently connected, as any other topic that uses numbers, symbols, logical analysis, problem solving, and other relatable components of Mathematics.

The topic of Probability or Chance first arises in a young child's life when a parent, for example, responds to a child's request for a treat, perhaps, with a word such as "Probably", "Maybe" or "Perhaps". (Mathematically, an answer such as "No" or "Certainly" would also fit in the topic of Chance. But informally, and intuitively, we usually regard Chance as dealing with uncertainty, not the absolutes of NEVER or ALWAYS, or No or Yes.) Randomness, like daily routine, is a natural part of a young child's experience. Accidents happen. Milk is unintentionally spilt. Unexpected events occur. So, too, do routine predictable events, like breakfast in the morning (never at night), or Goldilocks eating Baby Bear's porridge (Baby Bear never eats Goldilocks's porridge), or "Play School" at 9 o'clock (or whenever it is scheduled within the child's experience).

It is hardly surprising, then, that as early as the first year of Primary school, Preparatory, or Level 1 in the Victorian Essential Learning Standards, and the VELS-based Progression Points, we find the beginning of the Chance curriculum.

From the "Dimension" of Measurement, chance and data:

At Level 1:

Students recognise and respond to unpredictability and variability in events, such as getting or not getting a certain number on the roll of a die in a game or the outcome of a coin toss.

1.5 (to be achieved by "average" or mainstream students half-way through the year):

— They use an expanded use of the language of chance such as ALWAYS and NEVER to describe the likelihood of events.

This builds progressively through later Progression Point Levels and year-levels.

At Level 2, students predict the outcome of chance events, such as the rolling of a die, using qualitative terms such as certain, likely, unlikely and impossible.

2.75 (to be achieved by "average" students by halfway through Year 2):

— They classify events according to likelihood using an expanded range of descriptors such as LIKELY through to UNLIKELY.

At Level 3:

Students compare the likelihood of everyday events (for example, the chances of rain and snow).

Students describe the fairness of events in qualitative terms.

Students plan and conduct chance experiments (for example, using colors on a spinner) and display the results of these experiments.

3.25

— Students use everyday language associated with chance, such as certain, impossible, fair, unfair and start to quantify likelihoods as fractions.

— Students can identify all possible outcomes from a one-step experiment and can identify successful outcomes based on simple descriptions.

— Students compare the likelihoods of outcomes, such as if a bag contains many red buttons and a few green buttons then the probability of selecting a green button from the bag is low.

This continues, with increasing detail, technical vocabulary, and mathematical formality, to middle Secondary, and beyond into Years 11 and 12, where the prescriptions of the Study Guides for mathematics subjects in the Victorian Certificate of Education (and maybe other post-Year 10 programs) replace VELS — and beyond that, into undergraduate studies, such as Probability 101.

At Level 5, students identify empirical probability as long-run relative frequency.

Students calculate theoretical probabilities by dividing the number of possible successful outcomes by the total number of possible outcomes.

Students use tree diagrams to investigate the probability of outcomes in simple multiple event trials.

5.25 (to be achieved halfway through Year 7)

— Students list event spaces (for combinations of up to three events) systematically, such as the possible outcomes when a four sided die is rolled three times.

— Students use tree diagrams to represent event spaces (for combinations up to three events each with equally likely outcomes), and use these to calculate probabilities such as showing the sum when two dice are thrown and then calculate the probability of getting seven.

— Students use a two-way table to calculate theoretical probabilities.

And, finally, within the Progression Points ...

6.5

— Students use tree diagrams to determine the probability of outcomes for sampling with or without replacement

— Students identify random variation and possible hidden variables in analysing association and possible causal relationship in bi-variate data.

6.75 (to be achieved halfway through Year 10)

— Students use conditional probability to distinguish between dependent and independent events.

Chance that is NOT in the Progression Points

Good as the Progression Points are, as a "map" of a topic, like a mini-curriculum within the whole of the P-10+ school mathematics curriculum, this says NOTHING about what can go wrong as students progress through the Chance curriculum. Interestingly, and notoriously, Chance is (probably) the sub-curriculum within mathematics that is most prone to serious MISCONCEPTIONS. This is probably because in all of mathematics, Chance is the topic that students naturally experience independently, and/or before they encounter formal instruction in the classroom. For the rest of mathematics, for most topics, students encounter the ideas for the first time when their teachers introduce them. Certainly students can misconstrue what their teachers present, in these cases. But the pre-instruction, everyday, intuitive pre-conceptions of Chance, because the student personally creates them, are far harder to eradicate when they are MIS-conceptions.

For example, probably through long hours rolling dice during Snakes and Ladders, or Monopoly ("I need a double-six to get out of Jail!"), many students get the (wrong!) idea that certain numbers on an ordinary (fair) dice are harder to roll than others. Or, perhaps worse, some students come to believe there is a skill in rolling a dice to get the number that is wanted. And, of course, notoriously, if a 6 hasn't come up for a while, then it is often believed to be more likely to appear next time.

These and other misconceptions, or "fallacies", MUST be confronted by the teacher: students must be helped through experience and deliberate, logical, abstract, theoretical analysis, to recognize the wrongness of these fallacies, and the rightness of the mathematics in Chance. For advice on this, consider Peard (1991), or Fischbein & Schnarch (1997), or a Google-search.

Another serious complication is that, like the possibility of misconceptions, some parts of the Chance curriculum seem counter-intuitive. Consider the Monty Hall problem (based on a popular TV Games Show in the USA), or the Blue-Green Cards Problem.

There are three cards in a bag. One card has both sides green, one card has both sides blue, and the third card has a green side and a blue side. You pull a card out, at random, and see that one side is blue. What is the probability that the other side is also blue? (Shaughnessy 1992)

We might also ask whether the Progression Points, and/or the related VELS statements, for the topic of Chance include the usual desired range of randomizing materials, such as dice (fair or loaded), spinners, packs of shuffled cards, urns or bags or jars of counters, road traffic, people in the street, tossed drawing pins, needles dropped on a ruled surface, ...; or include adequate mention of explanatory visual organizers or representations, such as Venn diagrams, tree diagrams, one-way, two-way, and three-way tables?

Where do the Levels come from?

A further point must be made about the way these Levels have been determined. We can fairly easily see that (logically) each Progression Point is correctly placed in sequence, THIS before THAT. What is far from clear is WHY, or maybe HOW, this should be THERE, and not HERE?

For example, how can we make sense of the first Chance Progression Point, about students recognizing ALWAYS and NEVER in terms of likelihood, if we do not, at the same time, consider MAYBE? But isn't that left until Level 2? Or do we have professional leeway to apply the words, "an expanded use of such language as ..." to include other relevant terms?

I think we do have this leeway. But I think this means that the placing of THIS Progression Point HERE is necessarily tentative. We will be better advised to let the students' responses to good classroom experiences guide us as to how far through the Chance curriculum (or any other subject, not just in mathematics, even!) we will let them go, or help them go!

Putting this another way, VELS and the Progression Points should be taken as indicative, not as bindingly prescriptive. Importantly, they were NOT based on normative research studies, only on professional rules of thumb, according to professional consultation, committee work, and professional consensus about classroom experience of "average" students and "average" teachers and "average" teaching. Hence, WHICH of the Progression Points or VELS statements apply to YOU and YOUR students is for YOU to determine, by open-ended diagnostic profile-based assessment.

Making Progression Points Practical for Chance

Taking any one of the Progression Points as a starting point, the basic question is: What will this mean in the classroom? Or, What will this look like if a student shows he or she can do this? We need to shift from OBJECTIVE to ACTIVITY.

Here is the first example, based on a blend of the first two Progression Points. (Obviously this question will be READ ALOUD to young students.)

Question 1.

Label each of the following events with A, N or P: A for Always, or N for Never, or P for Possibly.

- A rooster lays an egg.
- A triangle has three corners.
- A car is red.

- A dog is called Rover.
- A mother cat has babies that are ducklings.
- A circle has a corner.
- A winter day is snowy.
- A summer day is hot.
- A summer day is rainy.
- A tree leaf falls in autumn.

One of the next Progression Points is trickier when the Chance curriculum focuses on "likelihoods as fractions". If a student is to have a fair chance of responding to a question about this idea, we should not leave the student unsure what the question means. For this reason we introduce the question with a simple explanatory example.

Example:

Think about rolling an ordinary dice. How many ways can we get a number that is LESS than 3? We could get a 1, or a 2, which are both less than 3. Actually there are SIX possible different numbers we might get. We say that the CHANCE or PROBABILITY of an event, like getting a dice result less than 3, is the fraction we make from the number of ways the event can happen successfully, and the total possible outcomes of the event. In this case, the chance of a number less than 3 is 2 out of 6 possibilities, or 2/6 (which is the same as one-third, or 1/3).

Question 2.

Consider rolling an ordinary dice and getting a 2. What is the probability of this event?

Towards the end of the Progression Points (5.25) we meet the technique of using a table to analyse possible outcomes in an event.

Example:

Think about rolling two ordinary dice, and adding the numbers we get. This table shows all the possible outcomes.

		Outcome from rolling the first dice					
		1	2	3	4	5	6
Outcome from rolling the second dice	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Table 1: Possible outcomes from rolling two ordinary dice and adding the resulting numbers.

Using this table we can easily see, for example, that there are six different ways of rolling two dice and adding the results to get a total of 7.

Question X.

Use this table that shows the results of rolling two ordinary dice and adding the resulting numbers to find the probability that the two dice add to make 10.

Homework: Complete new questions for the remaining Progression Points ...

Conclusions, Perhaps ...

A diagnostic profile for a topic, such as Chance, begins with the early, easy questions we can ask as we start to introduce the topic. Then the questions become progressively harder, just as the ideas and skills of the topic, taught in a sequential, developing way, possibly across successive years (in the classic spiral curriculum of Jerome Bruner), become more conceptually demanding. The familiar Early Years Numeracy Interview is a classic example of a one-to-one diagnostic profile, except that it ranges across several mathematical topics, and stops about midway through Year 3. Hence, a diagnostic profile is a map of a learning trajectory for a focused topic (Mousley, Sullivan & Zevenbergen, 2004) — a pathway through the developing sequence of concepts and skills for that topic. It is also similar to the classic curriculum topic flow-charts developed by Richard Skemp (1971: for example p 310) and the learning hierarchies of Robert Gagne (1965).

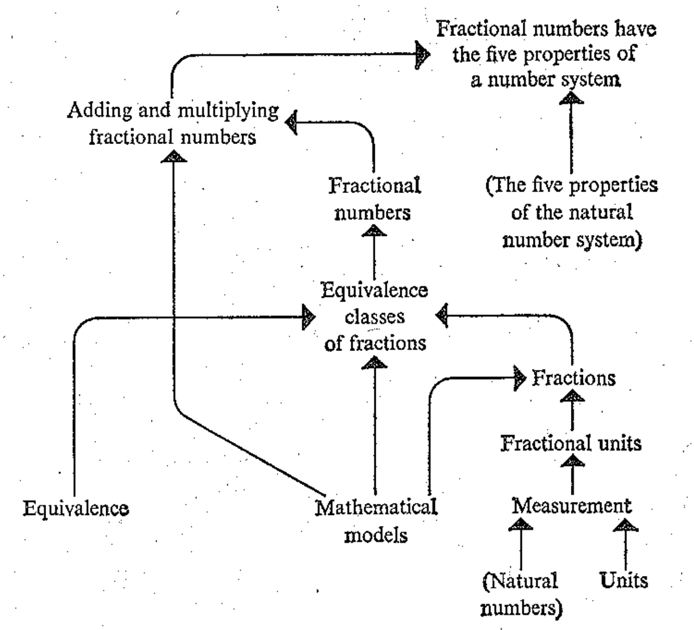

Figure 1: Richard Skemp's flowchart for part of the Number curriculum.

Homework: Make a Skemp-type flow-chart to map the Chance curriculum, including links with Set theory, and Fractions.

Note that this is NOT the first time I have outlined a Diagnostic Profile on Chance. The first such Profile was included in my across-year-levels topic profiling adaptation of the diagnostic year-level across-the-curriculum work of Howard Schleiger (Gough 1999). The second was based on the Curriculum & Standards Framework learning outcomes for Chance (Gough 2000). Despite any superficial changes in the Chance curriculum as outlined in VELS or the Progression Points, I also believe these earlier profiles are still effective.

In conclusion, any teacher who intends to find out what each student knows about a topic, at the beginning of a period of study or unit of work on that topic, and then build on what each student knows — don't we all? — needs to use some form of pre-"test" of entry-level knowledge. The results of this assessment are used formatively to guide instruction and select tailor-made learning-stimulus experiences for individualized (or small group) evidence-based teaching. If a pre-"test" is NOT used, then any post-"test" assessment can only describe what each student knows at the end of the study or unit of work, but can NOT itself ensure that the student did not already know this at the outset. In short:

— Learning = Post-"test" MINUS Pre-"test".

This paper has been an introductory, exploratory case-study of such an evidence-based approach to teaching, focusing on the topic of Chance, as outlined in the Progression Points. I would be glad to hear of anyone who works in this way, or uses any of my materials.

References and Further Reading

Fischbein, E., Schnarch, D. (1997), The Evolution with age of probabalistic, intuitively based misconceptions. *Journal for Research in Mathematics Education*, *28*(1), 96- 105.

Gagne, R. (1965). *The Conditions of Learning*, New York: Holt, Rinehart & Winston, 2nd edn 1970: 3rd edn 1977.

Gough, J. (1999). *Diagnostic mathematical profiles*. Geelong: Deakin University Press.

Gough, J. (2000). Making a Pre-'Test' / Post-'Test' Profile for Chance. In J. Wakefield (Ed.) (2000) *Mathematics: Shaping the Future* (pp. 106-117). Brunswick: Mathematical Association of Victoria.

Mousley, J., Sullivan, P., & Zevenbergen, R. (2004). Planning Alternative Learning Trajectories. MERGA [Mathematics Education Research Group of Australia — annual conference].

Peard, R. (1991). Misconceptions in probability. In J. O'Reilly & S. Wettenhall (Eds.). *Mathematics: Inclusive, Dynamic, Exciting, Active, Stimulating [IDEAS]* (pp. 496-499). Parkville: Mathematical Association of Victoria.

Shaughnessy, M. (1992). Research in Probability and Statistics: Reflections and Directions. In D.A. Grouws (ed.) *Handbook of Research on Mathematics Teaching and Learning.* Reston: National Council of Teachers of Mathematics [NCTM], p. 474.

Skemp, R. R. (1971). *The Psychology of Learning Mathematics*. Harmondsworth: Penguin, ands revised editions.

HREF1: Progression Points: **http://vels.vcaa.vic.edu.au/assessment/progresspoints.html**