

MIND YOUR LANGUAGE: SPEAKING IN AND ABOUT THE MATHEMATICS CLASSROOM

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This keynote presentation has two distinct sections. First, research is reported into the role of spoken mathematics in mathematics classrooms in six different countries. The focus is frequency of public and private talk by students and the extent to which this talk makes use of the language of mathematics. Some classrooms around the world deliberately aim to develop student use of spoken mathematics and some do not. How is this done and what difference does this appear to make for student learning and student ability to ‘speak mathematics’? Second, mathematics teachers in different countries (particularly non-English speaking countries) employ different professional vocabularies to describe what they do and what occurs in their classrooms. Many of the terms, such as ‘mise en commun’ (French), ‘Kikan-Shido’ (Japanese) and ‘jianping’ (Chinese), routinely used in the local language, have no direct equivalent in English. Mathematics teachers in these countries routinely refer to classroom activities for which we have no name. If you can name an activity then you can both recognise it and seek to improve it. Without the name it is difficult to do either. Students or Teachers: How important is it to be able to talk about what we do?

Talking Mathematics in the Classroom

How important is it that students have the opportunity to talk mathematics in class? The recent review by Walshaw and Anthony (2008) makes the statement: “What these researchers have demonstrated is that effective instructional practices demand students’ mathematical talk” (p. 523). This is a very strong statement in support of spoken mathematics by students in the classroom. The promotion of spoken mathematics by students is a strategic instructional activity by the mathematics teacher. A particular focus of the current research by my colleagues and myself is the role of spoken mathematics in both instruction and learning.

The instructional value of the spoken rehearsal of mathematical terms and phrases central to a lesson’s content could be justified by reference to several theories of learning. Interpretation of this spoken rehearsal as incremental initiation into mathematics as a discursive practice could be justified by reference to Walkerdine (1988), Lave and Wenger (1991), or Bauersfeld (1994). The instructional techniques employed by the teacher in facilitating this progression could be seen as “scaffolding” (Bruner, 1983) and/or as “acculturation via guided participation” (Cobb, 1994). Speaking mathematics can be seen as facilitating learning or as a valuable skill in itself. It might also be seen as irrelevant.

Our research into spoken mathematics has two essential aspects: classroom practice and its consequences. The key questions are: “What is the occurrence of spoken mathematics in the different classrooms studied and, in particular, what is the frequency of the students’ use of technical mathematical terms in their public and private speech?” and “What use do these same students make of the language of mathematics in describing their experience of the mathematics classroom and what evidence does this provide regarding the learning arising from their classroom experiences?” This paper examines the use of spoken mathematics in classrooms internationally, in public discussion and in private student-student classroom conversations. It also examines students’ use of mathematical terms to describe the mathematics classroom as they experienced it.

Research and theorising regarding the role of language in mathematics classrooms is culturally-situated to a remarkable extent. The review by Walshaw and Anthony (2008) omits any reference to research into classrooms situated in Asian countries. Given the success of school systems in countries such as Japan, Korea and Singapore in international tests of mathematics achievement, this omission is remarkable. In the analyses summarised in this paper, I suggest that the instructional practices of the teachers in the various classrooms are based on pedagogies that assign spoken mathematics a very different function in the learning process.

It is important for Australian educators to give serious consideration to these alternative pedagogies and to give thought to the different assumptions underlying the practices in classrooms overseas. This sort of self-critical reflection need not lead to the adoption of overseas approaches, but could lead to the expansion of the Australian mathematics teacher's instructional tool kit and, perhaps, to the refinement of current classroom practice.

Studying Spoken Mathematics in the Classroom

In this paper, I summarise research conducted by the research team at the International Centre for Classroom Research at the University of Melbourne. In particular, my argument here draws on analyses of 95 lessons recorded in nineteen classrooms located in Australia (Melbourne), China (Hong Kong and Shanghai), Germany (Berlin), Japan (Tokyo), Korea (Seoul), and the USA (San Diego). The complete research design has been detailed elsewhere (Clarke, 2006). Three video records were generated for each lesson (teacher camera, student camera, and whole class camera), and it was possible to transcribe three different types of oral interactions: (i) whole class interactions, involving utterances for which the audience was all or most of the class, including the teacher; (ii) teacher-student interactions, involving utterances exchanged between the teacher and any student or student group, not intended to be audible to the whole class; and (iii) student-student interactions, involving utterances between students, not intended to be audible to the whole class. All three types of oral interactions were transcribed, although type (iii) interactions could only be documented for two selected focus students in each lesson. Where necessary, all transcripts were then translated into English.

A five-stage analysis focused on the significance of the situated use of spoken mathematical language in these classrooms. The results of the first and second analytical stages focused on public oral interactivity (frequency of public utterance) and public mathematical orality (spoken use of key mathematical terms) (Clarke & Xu, 2008). We distinguish *private* student-student interactions from whole class or teacher-student interactions, both of which we consider to be *public* from the point of view of the student. Our major concern in the first four stages of the analysis was to document the opportunity provided to students in the mathematics classroom to use the relatively sophisticated mathematical terms that formed the conceptual content of the lesson in both public and private classroom talk. The fifth stage of our analysis examined student use of mathematical terms in post-lesson video-stimulated interviews.

Public Mathematical Orality: Who gets to speak publicly and do they talk mathematics?

The ‘Asian’ data set analysed included three sequences of five lessons from three mathematics classrooms in Shanghai, three similar sequences from Hong Kong, three sequences from Tokyo, and three sequences from Seoul. ‘Western’ classroom practice was represented in this analysis by three sequences of five lessons from Melbourne, two sequences from Berlin and two sequences from San Diego. The data from San Diego 3 and from Berlin 3 were excluded because of difficulties in applying the definition of “public” in both classrooms.

In our first analytical pass, we counted the number of utterances made by anyone participating in a whole class or teacher-student interaction (a “public utterance” from the student perspective). Figure 1 shows the average number of utterances per lesson occurring in whole class and teacher-student interactions in each of the classrooms studied in Shanghai, Hong Kong, Seoul, Tokyo, Berlin, Melbourne and San Diego.

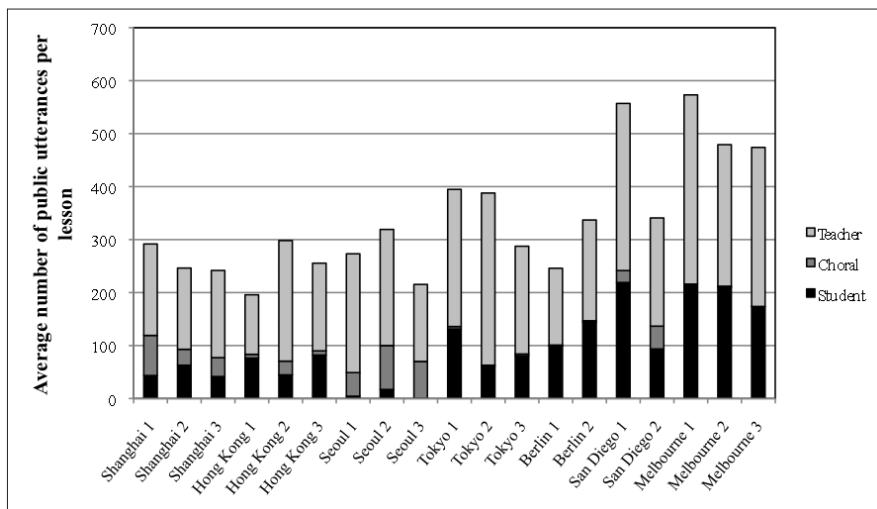


Figure 1. Average Number of Public Utterances per lesson in Whole Class and Teacher-Student Interactions (Public Oral Interactivity)

An utterance is a single, continuous oral communication of any length by an individual or a group (choral). The average number of public utterances per lesson provides an indication of the spoken public interaction occurring in a particular classroom. Figure 1 distinguishes utterances by the teacher (light grey), individual students (black) and choral responses by the class (e.g. in Seoul) or a group of students (e.g. in San Diego) (dark grey). Any teacher-elicited, public utterance spoken simultaneously by a group of students (most commonly by a majority of the class) was designated a “choral response.” Lesson length varied between 40 and 45 minutes and the number of utterances has been standardized to 45 minutes.

The classrooms studied can be also distinguished by how frequently the public spoken interactions made use of mathematical terms. This included the use made of the choral recitation of mathematical terms or phrases by the class. This recitation included both choral response to a teacher question and the reading aloud of text presented on the board or in the textbook.

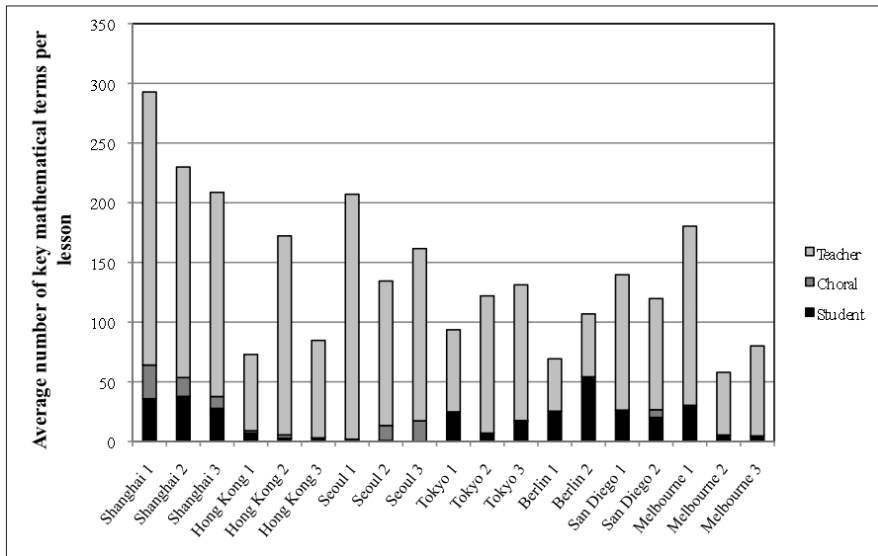


Figure 2. Frequency of Occurrence of Key Mathematical Terms in Public Utterances (Mathematical Orality)

Figure 2 shows how the frequency of occurrence of key mathematical terms varied among the classrooms studied. In classifying the occurrence of spoken mathematical terms, we focused on those terms that were central to the lesson content (e.g. terms such as “equation” or “co-ordinate”). This meant that our analysis did not include utterances that constituted no more than agreement with a teacher’s mathematical statement or utterances that only contained numbers or basic operations that were not the main focus of the lesson. In the case of the Korean lessons, in particular in Seoul 1, the choral responses by students frequently took the form of agreement with a mathematical proposition stated by the teacher. For example, the teacher would use expressions such as, “When we draw the two equations, they meet at just one point, right? Yes or no?” And the class would give the choral response, “Yes.” Such student statements did not contain a mathematical term or phrase and were not included in the coding displayed in Figure 2.

Similarly, a student utterance that consisted of no more than a number was not coded as use of a key mathematical term. It can be argued that responding “Three” to a question such as “Can anyone tell me the coefficient of x ?” represented a significant mathematical utterance, but, as has already been stated, our concern in this analysis was to document the opportunity provided to students for the oral articulation of the relatively sophisticated mathematical terms that formed the conceptual content of the lesson. Frequencies were again adjusted for the slight variation in lesson length.

From the results displayed in Figures 1 and 2, we suggest that the instructional practices of the teachers in the various classrooms assigned spoken mathematics a very different function in public classroom discourse. One of the most obvious inferences from comparison of Figures 1 and 2 is that the most talkative classrooms were not necessarily the same as the classrooms where spoken mathematical terms were most frequently used.

The Significance of Student-Student Interactions

The private conversations recorded in any one lesson were only those of the two focus students and their immediate neighbours. Two different focus students were recorded in each lesson. In this paper, I report the frequency of utterances (uninterrupted oral communications) and key mathematical terms (defined below) in both public and private arenas with respect to the two focus students. In the third-pass analysis, all utterances made by the two focus students were classified according to whether the utterance was targeted at a public audience or a private audience. Public utterances were those made to the teacher (either in one-on-one interaction or in the form of a choral response) or to another student, but audible to the whole class. Private utterances included statements made to a student peer in private or to oneself.

In Figures 3 and 4, the results quoted for both public and private Oral Interactivity and Mathematical Orality are per focus student per lesson and have been averaged over the spoken contributions of around 10 students per classroom. This should minimize the effect of individual student timidity or extroversion, although awareness of being recorded was a common characteristic of all focus students (and of their teachers). The number of utterances and key mathematical terms was standardized to a lesson length of 45 minutes.

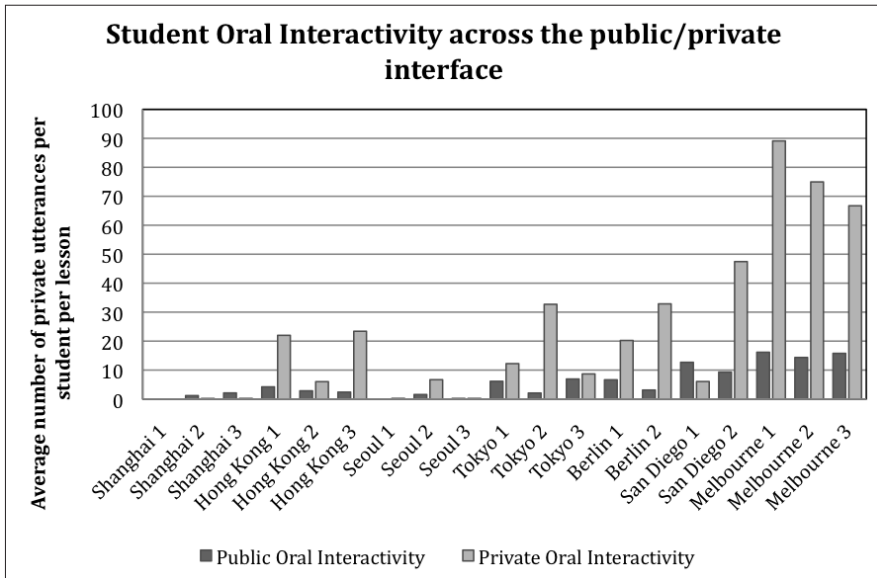


Figure 3. Public and Private Oral Interactivity: Frequency of utterance per student per lesson (each bar represents the average of two students for each of five lessons – ie. ten students)

Five classrooms stand out in Figure 3 because of their extremely low frequency of student-student interaction: those in Shanghai and Seoul (except Seoul 2). In these five classrooms, student-student conversation can be discounted as an instructional strategy (or as a subversive practice by students). For example, in Seoul classroom 1, there were no instances of student private talk in the first four recorded lessons and only two private utterances from one of the focus students in lesson five, an average of 0.2 utterances per student per lesson. The first utterance was “That’s yours” and the second was “No.” Obviously, neither involved any technical mathematical terms.

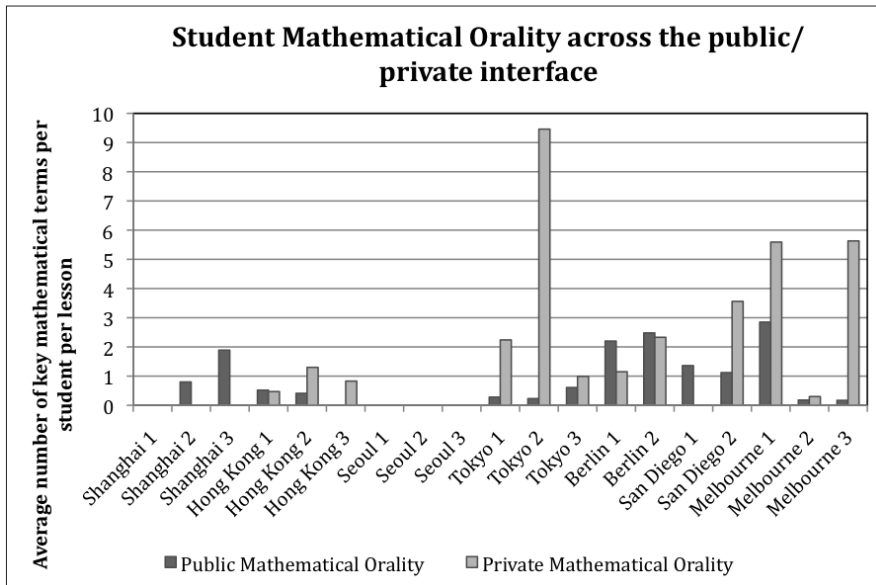


Figure 4. Public and Private Mathematical Orality: Frequency of use of technical terms (each bar represents the average of two students for each of five lessons – ie. ten students)

Figures 1 and 2 show relatively high levels of whole class public mathematical orality in the Shanghai classrooms, but this is not evident in Figures 3 and 4 because the typical public contribution of an individual Shanghai student occurs within a class of fifty students (at least ten more than the average for classes in any of the other cities) and a specific individual's contributions will consequently be less frequent than in smaller classes, unless the teacher makes frequent use of choral responses. Rather than characterising aggregated whole class behaviours, Figures 3 and 4 express their findings in terms of the individual student.

Spoken Mathematics in the Classroom: Key Points Summary

International classroom research projects such as the Learner's Perspective Study (Clarke, Keitel & Shimizu, 2006) or the TIMSS-R Video Study (Hiebert et al., 2003) provide the opportunity to interrogate the capacity of our theories to accommodate classroom practice in cultural settings other than those in which the theories themselves were developed. The possible primacy of language in knowledge construction can then be examined without the distorting prejudice of a context in which particular types of oral performance are already privileged.

The prevalence of spoken mathematics in the nineteen classrooms studied differed in the following respects:

- the frequency of public utterance
- the relative prominence of the teacher or the students' voices in public discourse
- the frequency of public use of spoken technical terms, most particularly by students
- differences in the extent to which student use of spoken mathematics was strategically facilitated by teachers
- the extreme differences in the occurrence of student-student (private) use of spoken mathematics

It is highly instructive to consider the results displayed in Figures 1, 2, 3 and 4 with respect to different individual classes and to ask the question, "How might we describe the role of spoken mathematics in this classroom?" Any Australian mathematics teacher should then ask the further question, "And which of these classrooms resembles mine?"

Differences in classroom practice may reflect different pedagogical traditions, but may also reflect different theories of learning on which the observed instructional practices are based. In some classrooms, student-student spoken mathematics was an essential component of the dominant pedagogy. In other classrooms, it was entirely absent. These extreme differences allow us to ask the question: "With what consequences?"

Spoken Mathematical Fluency as a Valued Learning Outcome

Do we want our students to be able to "talk mathematics"? Research conducted in Western classrooms suggests that this is essential (Walshaw & Anthony, 2008; Silverman & Thompson, 2008). Is talking mathematics an essential instructional approach or an important capability that we want to develop in our students?

If student facility with technical mathematical vocabulary is a valued outcome, then the analysis of the post-lesson interviews (see Figure 5) suggests that the public scaffolding of student technical fluency can be as effective as the encouragement of student-student spoken mathematics.

However, where the classroom provides students with no opportunity for spoken mathematics (Seoul), there appears to be little inclination (and possibly capacity) to do so, even in interview situations where the invitation to use spoken mathematics was explicit ("Tell me what the lesson was about"). Student inclination to employ other mathematical terms ('other terms') in addition to those specific to the lesson could reflect more interconnected knowing. Student descriptions of lesson content and learning provide

a different type of mathematical performance. It appears that the spoken articulation of mathematical understanding is being unevenly achieved even where it is explicitly valued. Mathematical speech seems to require scaffolding, whether overt (Shanghai One) or covert (San Diego Two).

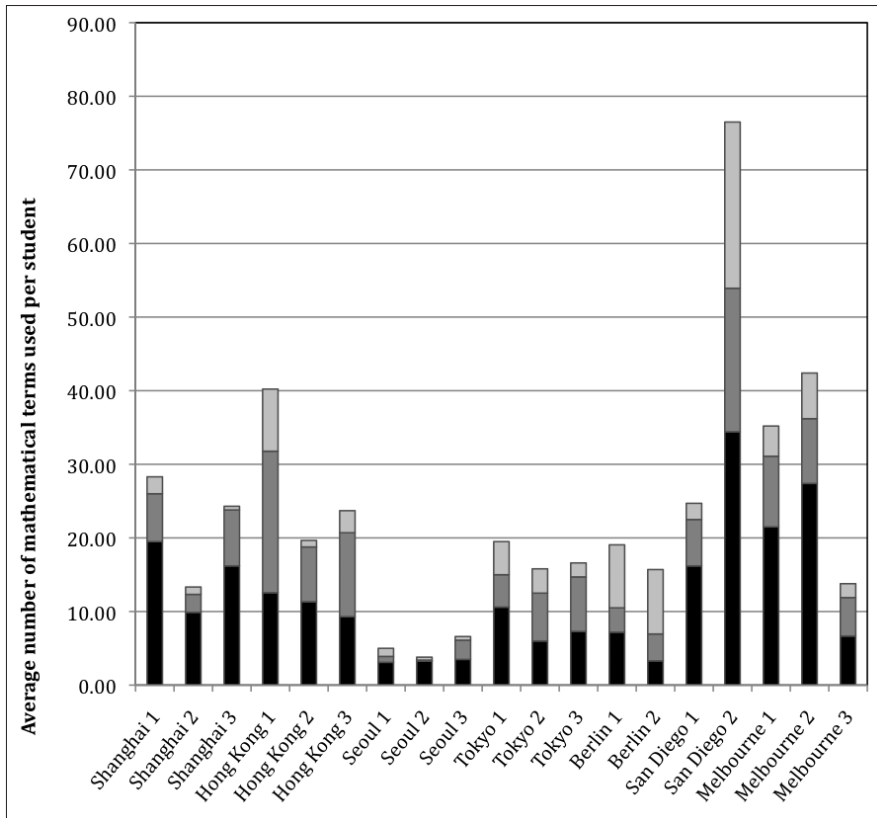


Figure 5. Frequency of use of technical terms in post-lesson interviews (each bar represents an average per student over ten student interviews) (black = key terms, dark grey = related terms, light grey = other terms)

The Asian classrooms in this study varied from no spoken mathematics by students (Seoul) through only public spoken mathematics by students (Shanghai) to spoken mathematics by students in both public and private classroom settings (Tokyo). Differences in outcome may reflect differences in aspiration (rather than simply differences in

success) – different cultures valuing different types of mathematical performance. What performances do Australian mathematics classrooms seek to promote? And what is the role and significance of spoken mathematics?

Talking about the mathematics classroom

I want to shift attention now to the language that we use when talking *about* the mathematics classroom. My involvement in international classroom research has made me very aware of the rich vocabulary available to some of my colleagues from non-English speaking countries. This was most evident when a Chinese colleague, Professor Cao Yiming, was working at the ICCR in Melbourne, coding the activities in Chinese mathematics classrooms. Professor Cao was coding events for which I had no names! Together, we were able to develop descriptions of the classroom events he was identifying. Some of these could be expressed very succinctly. For example, “jianping” referred to “the public evaluation of a student’s solution.” Some were more difficult to describe. “Pudian” seemed to be the teacher’s attempt to construct through discussion a connection between content previously covered, or the students’ existing mathematical knowledge, or other student experiences and the mathematics to be addressed in that day’s lesson. The term “pudian” seemed to encompass both the act of establishing these connections in discussion with the class and the teacher’s planning as to what connections might be established and how this should be done. We used “bridging” as an English shorthand for “pudian” but that was only possible because we had developed a more elaborate English description.

I was already aware of the extensive vocabulary available to mathematics teachers and researchers in Japan to describe classroom events. In particular, “Kikan-Shido” (Between-desks-instruction) had already become a familiar term to everyone in the Learner’s Perspective Study because the action of moving around the classroom monitoring and guiding student work was so familiar to everyone (O’Keefe, Xu, & Clarke, 2006). For example, the French use the phrase “passe dans les rangs” (walks between rows) to signify the same event, but they have not developed a set of principles to optimise the teacher’s use of this practice. By contrast, the Japanese even distinguish “Kikan Shido” from “Kikan Junshi” (between desks patrolling).

Some time earlier, I had discovered that the work of educational theorists such as Vygotsky (1962) was profoundly misrepresented by English translations that mistook the Russian term “obuchenie” for “learning,” when it actually refers to an activity that we might call “teaching/learning” in which the teacher and the students are collaboratively

engaged (Clarke, 2001). Since then, I have found that other languages, such as Dutch and Japanese, name classroom activity in the same way. It became increasingly apparent that the way that we see classrooms, participate in classrooms, research classrooms, and theorise about classrooms was highly dependent on the names we could give to the events of the classroom. Lack of the name for an event has the effect of making the event almost invisible. And differences in meaning could have profound consequences for teaching.

What's in a name?

Once we have a name for a classroom activity, we can ask at least two questions: “Did it occur?” and “How well was it done?” Without the name, we may not even recognise that the event occurred. One of my current research activities is to work with colleagues from non-English speaking countries to develop an international lexicon of terms for the activities of the mathematics classroom, together with an illustrative library of video material.

A preliminary survey of classroom-related terms has been conducted in the following languages: Japanese, French, Chinese, Czech, Korean, Finnish, Swedish, Portuguese and Spanish. The results are fascinating. It is clear that the dominance of English as the lingua franca of the international education community is restricting access to a vast number of sophisticated terms that other cultures have developed to talk about the significant actions and events of the mathematics classroom. These actions and events are significant precisely because someone has thought them to be of sufficient importance to deserve a specific name. The terms being collected not only make new aspects of classroom practice visible, they also shed light on what elements of classroom practice are valued in different cultures. It is not possible to share all the richness of the collected terms here – but the following two sections will hopefully illustrate some of the ways in which Australian mathematics teachers might benefit from an awareness of how their colleagues overseas describe their practice.

Some Japanese terms

Japanese mathematics teachers, teacher educators, and researchers have an extensive vocabulary with which they can discuss the mathematics classroom. Here are some examples of Japanese terms:

- *Honji no Nerai* – “the purpose of today’s lesson”
- *Jiriki Kaiketsu* – “solving the problem on one’s own”
- *Neriage* – “the kneading, mixing together of the students’ ideas” or “polishing students’ ideas through whole-class discussion”
- *Matome* – “summing up”

- *Kyoshi Shudo* or *Seito Shutai* – “teacher leads the lesson” or “students lead the lesson”
- *Kikan Shido* – “between desks instruction”
- *Yamaba* – “the climactic moment of the lesson”
- *Koteiteki Hyoka* – “positive evaluation”
- *Shido* – “instruction (more teacher directed)”
- *Shien* – “support (less teacher directed)”
- *Hansei* – “self-critical reflection”

“Matome,” for example, refers to an event in which the teacher talks to the whole class to highlight and to summarize the main point of the lesson. What the students have discussed in the lesson is reviewed briefly and what they have learned during the lesson is highlighted and summarized by the teacher in the whole-class setting. In reporting his research of Matome as a classroom phenomenon, Professor Yoshinori Shimizu stated:

For the Japanese teachers, the event Matome appeared to have the following principal functions: (i) highlighting and summarising the main points of the lesson, (ii) promoting students’ reflection on what they have done, (iii) setting the context for introducing a new mathematical concept or term based on previous experiences, and (iv) making connections between the current topic and previous one (Shimizu, 2006, p. 141).

But Shimizu also draws connections between pedagogical terms such as Yamaba and Matome and corresponding structural elements from Japanese literature or drama (Shimizu, 2006, p. 143). The essential point is that Japanese mathematics teachers have access to a sophisticated and coherent vocabulary that allows them to discuss the components of the mathematics lesson, reflect on their teaching, offer and receive advice, and which provides a powerful tool for pre-service and in-service teacher education.

Multiple Perspectives on Lesson Events

The other benefit offered by considering non-English pedagogical terms is that new perspectives are offered on familiar moments in the lesson. In the Learner’s Perspective Study, these familiar moments were called “Lesson Events” (Clarke et al, 2008). Consider the beginning of the lesson. There is a high level of consistency in the use by American teachers of an activity they call “warm-up” (see Mesiti & Clarke, 2006). This term is so widely used that an American teacher can open the lesson by saying, “Okay guys, let’s go ahead and get started on today’s warm-up” (Mesiti & Clarke, 2006, p. 56). A warm-up activity typically consists of a set of short answer questions written on the board and corrected immediately. The questions may or may not be connected to the subsequent content of the lesson. The purpose of warm-up has been described as “focusing” and as “review.” It also appears to have

classroom management characteristics – establishing a working atmosphere. French teachers will frequently commence their lessons with “*rappel de cours*” (review of content). The purpose of *rappel de cours* is to establish a common knowledge foundation among the class on which the lesson’s mathematical content can be constructed. This should be compared with the Chinese use of “*pudian*,” which was discussed above. Australian mathematics teachers planning their lessons might usefully consider which of these perspectives best match their intentions: warm-up, *rappel de cours*, or *pudian*.

Consider “whole class discussion” - there are many situations in the lesson when a mathematics teacher might employ whole class discussion. In French mathematics classrooms, “*mise en commun*” is seen as highly important. *Mise en commun* is a whole class discussion in which the teacher guides the students in the construction of the lesson’s main points by synthesising different student solutions. In this description of *mise en commun* we can see similarities with *Matome*. There is a similar emphasis on bringing together the main conceptual threads of the lesson. One difference could be the role of the students, whose contributions are central to the conduct of *mise en commun*.

Kikan-Shido has already been discussed. The analysis conducted by O’Keefe et al. (2006) identified over a dozen distinct purposes that teachers in different countries pursued during *Kikan-Shido*. In Portuguese, a teacher engaging in *Kikan-Shido* is commonly described as “*A professora esclarece dúvidas*” (the teacher removes doubts). In Japan, one of the key teaching activities during *Kikan-Shido* is the unobtrusive selection of interesting student solutions that will provide the catalyst for discussion in the *Matome* phase of the lesson. The student’s method is noted by the teacher, while moving around the classroom, and then used because it provides a particularly clear example of the correct method or even because it illustrates a common difficulty. The teachers studied in Hong Kong were much more likely to intervene immediately: frequently stopping the whole class for brief instruction on a misunderstood procedure or admonishing a particular student for not employing the correct method.

The essential benefit of considering such different approaches to familiar classroom events is that it makes us question our assumptions and consider alternatives where we might otherwise have been guided only by habit or tradition.

Talking in and about mathematics classrooms

This paper has been about differences in the ways that mathematics educators and mathematics students use language to describe their practice. I believe that our capacity to improve our practice requires a language through which to discuss and reflect upon it – whether we are teachers or students.

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