MENTAL COMPUTATION IN THE PRIMARY CLASSROOM

Angela Rogers

St. Monica's Primary School, Moonee Ponds

The importance of teaching the skills and strategies associated with mental computation has become particularly apparent over the past 15 years. Considerable research has been conducted into this area and educators and governments alike now acknowledge the important place it holds in the Australian Mathematics curriculum. In order for students to be properly prepared for the mathematical demands of life, school and work they must be assisted to be proficient users of mental computation. Primary Schools must take a whole school approach to introducing and developing these skills so mental computation becomes not just a 'school skill' but a 'life skill'.

Mental Computation vs Mental Arithmetic

For many years mental computation has been confused with mental arithmetic. Yet educationally there are distinct differences. Mental computation is based upon the constructivist approach. It develops children's understanding and promotes metacognition. According to Caney (2004) "experiences that encourage discussion and learning are far removed from the emphasis on activities that focus on testing that have dominated mental computation in primary and secondary classrooms for so long" (p.10) Mental arithmetic is based on the quick and accurate recall of number facts, and relies mainly upon the skill of a child's memory. Timed mental arithmetic sessions can take the emphasis off children comprehending how they are finding their answers and may not provide them with the skills to further develop their understandings. These sessions can simply make those children with a strong memory feel good about themselves and leave the rest of the class feeling disappointed. This is supported by McIntosh (HREF2, 2004b) who states "these daily speed and accuracy tests did not make the children noticeably more competent, but it did make them slightly more neurotic about numbers" (p.1).

Interestingly Van den Heuvel-Panhuizen (1992) in Beishuizen and Anghileri (1998) suggests that there is a component of mental computation which relies on the commitment of number facts to memory, mental arithmetic style. They state "automatising basic number bonds like complements in 10 is an important prerequisite for flexible mental arithmetic" (p. 521). Therefore it is important for children to develop instant recall of certain basic number facts so they can apply these in more complicated mental computation calculations. As such, it seems mental arithmetic sessions do have a place in the modern classroom if teachers and students are aware of the important link they have to mental computation .

One of the major aims of the school mathematics curriculum is to "strive to assist each child to gain a level of numeracy essential for successful participation in schooling, in work and in everyday life" (HREF1, 2004b, p.xii). A major part of achieving this goal is through providing children with mental computation skills. Mental computation is arguably one of the most used mathematical skills that children take with them into their lives. McIntosh's (HREF2, 2004b) research suggests that "adults use mental computation for over three quarters of all their calculations... and written calculation and calculator use are each involved in less than 15% of all calculations"(p. 2). This leads one to question the significant amount of teaching time that is directed towards the introduction and practice of algorithms. It makes sense that if schools are preparing children to tackle life, their focus should be on mental computation.

Written vs Mental Computation

Mathematical computation involves both written and mental computation. Both forms of computation have their place in the mathematics curriculum and the skills associated with mental and written computation can compliment each other. For example, a solid understanding of mental computation allows children to check the reasonableness of their written computations and written computation can allow for children to develop further their mental computation strategies. Reys (1984) suggests mental computation by definition "produces an exact answer, and the procedure is performed mentally, without using external devices such as pencil and paper" (p.548). These skills can be developed well before the written algorithm and often are quite natural to children. Bebout (1990) in Mardjetko and McPherson (2007) reports that "very young children have effective strategies for mental computation".

While computing mentally comes naturally, teachers rarely value this and often rush children into using the written algorithm. This can cause children to stop their intuitive thinking strategies and blindly follow the prescriptive steps of the algorithm. Kamii's (1994) research suggests an early emphasis on learning algorithms is a hazard that inhibits children's own number thinking, retarding development of number sense and adding to children's confusion with place value. This is supported by Heirdsfield (2004) who states "vertical algorithms dictate a rigid procedure, and do not lend themselves to encouraging students to manipulate numbers flexibly" (p.8).

Clearly the introduction of the written algorithm can have dire consequences for children's learning, therefore the timing of this needs to be carefully considered by schools. It seems that the later the algorithm is introduced the better this is for children. This is supported by McIntosh (2005) who suggests that "delaying the introduction of written algorithms is beneficial to students" (p. 4). Clearly the deeper children's understanding of mental computation and informal methods, the easier it is for them to understand the purpose and procedure of the written algorithm.

Westwood (2000) summarises this by stating "children should have no problem mastering these procedures [algorithms] if they are linked as closely as possible with the more informal methods of adding...that are typically used by children... difficulties arise if the processes are taught without reference to children's prior learning or way of recording." (p. 47).

Mental Models

It is clear there needs to be focused instruction given to children to develop their mental computation skills and assist them to make a flawless transition between informal strategies and the formal algorithm. This transition is best supported by the use of appropriate 'mental models'. Mental models "assist students to both construct and co-construct specific mathematical concepts such as number and operation, numeration and number facts- these concepts are essential for mental computation" (Heirdsfield and Lamb, 2007, p.4).

Dutch researchers have successfully introduced a mental model called the 'empty number line (ENL)' (See Figure 1) to assist with developing children's informal computation skills (Beishuizen and Anghileri, 1998). The ENL allows children to use mental computation strategies whilst moving toward understanding the purpose of a standard written algorithm. Gravemeeijer (1994) suggests that the ENL is well suited to the development of computation as it reflects informal methods that children develop.

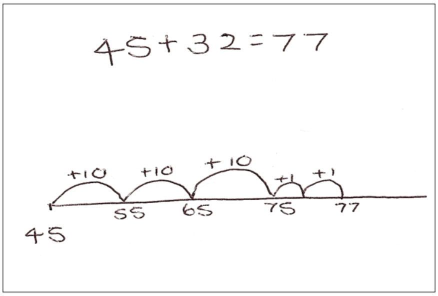

Figure 1: Example of a child using an Empty Number Line (ENL) to complete a problem.

The ENL has several advantages for both the teaching and learning of mental computation. It is a very transparent mental model that allows teachers to immediately understand the method and thinking that children are using to arrive at their answer. It is also very user-friendly. Children can easily understanding the concepts behind it, allowing them to feel in control of the computation. The ENL is also a very flexible method, which can be applied to addition and subtraction of any magnitude. Finally and most importantly the ENL is a clear visual representation of the actual procedure that takes place in addition and subtraction, a capacity that the written algorithm does not offer.

As with every mental model there are some downsides to using the ENL that teachers must be aware of. The ENL is best suited to sequential strategies with valuable strategies such as splitting ten not able to be simply represented e.g: 13+18=10+10+8+3. Also an overuse or overreliance on the ENL may limit the development of more sophisticated strategies by some children. This may lead children to simply blindly follow a procedure when using the ENL, rather than applying the most appropriate strategy to a particular problem. Finally, the ENL does not easily lead to an informal written method which is likely to be an adequate long term method, and as such children must be encouraged to move onto more appropriate and efficient written methods once they have mastered the ENL. As Beishuizen and Anghileri (1998) suggests the success of a mental model such as the ENL can only be "gauged by students' engaging in mental computation without these models"(p.526).

Whole School Approach

One of the most effective ways to introduce mental computation to children is through a whole school approach. According to Mardjetko and MacPherson (2007) "Research has shown that a targeted program can result in a rapid improvement in the development of strategies for mental computation" (p.6). These "targeted programs" encourage teachers and students to see the importance of mental computation throughout Primary school, from Prep to Year Six. Schools which place an emphasis on introducing mental computation strategies for the four operations as well as fractions and decimals, provide their students with the chance to develop these skills over a number of years. Callingham's (2005) research into Jacaranda Public School's implementation of a whole school approach showed positive results in children's scores on standardized tests of mental computation from K-6. This research along with McIntosh's (2004a) work in several schools in Tasmania and Canberra suggests that a whole school approach is a very effective way to consolidate children's mental computation skills.

One of the most important outcomes of a whole school approach is a significant increase in teacher's knowledge and understanding of the explicit teaching of mental computation. Callingham (2005) noted that "all teachers…did indicate that they had changed their teaching to address mental computation more explicitly, with less emphasis on drill and practice of written algorithms" (p. 206). Heirdsfield and Lamb's (2005) research also supported the whole school approach by stating:

> this study provides further evidence for the need for continuing professional development, as well as more focused teacher education programs, to improve teacher content knowledge on mental computation along with Pedagogy specifically focused on the importance of this new challenging addition to the Number strand. (p.425)

Clearly in order to improve student outcomes in mental computation the development of teachers' Pedagogical Content Knowledge (PCK) needs to be a focus.

Schools also need to have a strategic approach to implementing a mental computation program into their classrooms. Beishuizen and Anghileri (1998) note that "International work does suggest that…[mental computation] strategies can be taught and should not be left to chance" (p.520). Therefore teachers must use and develop programs that ensure this does not occur.

Wigley, (1996) suggests that "Leaving pupils to find their own methods will deprive many of more advanced strategies- a better approach recognises that there are a few big ideas which should be taught to everyone (Wigley, 1996 in Beishuizen and Anghileri, 1998). Supporting this assertion, Alistair McIntosh developed a strategy based program which scaffolds the development of the 'big ideas' Wigley, (1996) refers to.

Mental Computation Strategies

"Mental Computation: a strategies approach" (McIntosh, 2004a), takes teachers through a step by step guide to introducing the strategies that most assist students to become 'flexible' users of mental computation. Heirdsfield and Lamb (2007) describe flexibility in mental computation as "employment of a variety of efficient mental strategies, taking into account the number combinations to inform the mental strategy choice."(p.2). McIntosh's program introduces a 'variety of efficient mental strategies' including Count on, Doubles, Near Doubles, Spin Arounds, Bridging Ten, Tens Facts and using place value to add ten. These are seen as the building blocks of developing the 'flexibility' that is considered by Heirdsfield and Lamb (2007) as the key to assisting children to become proficient users of mental computation. As children come to school with natural mental computation strategies, it is important to begin mental computation discussions as early as Prep.

Once these basic strategies have been introduced and mastered it is important for children to develop more sophisticated strategies. McIntosh's (HREF2, 2004b) research highlights the importance of developing the scope of mental computation strategies further to include strategies for double digit computations (p.9) This is supported by Beishuizen and Anghileri (1998) who suggest that "In the past research into addition and subtraction strategies concentrated almost entirely on smaller numbers up to 20. Today however strategies with larger two digit numbers up to 100 are gaining more attention" (p. 524).

These two-digit strategies include:

- Aggregation (eg: 28+35: 28+5=33,33+30=63),
- Wholistic (eg: 28+35: 30+35=65, 65-2=63)
- Separation right to left (eg: 28+35: 8+5=13, 20+30=50, 13+50=63)
- Separation left to right (eg: 28+35: 20+30=50, 8+5=13, 50+13=63)

(Heirdsfield and Lamb, 2003, p.1)

As with the basic strategies, the two-digit strategies need to be explicitly taught to children. Aggregation and wholistic computation are considered by Thompson and Smith (1999) as the most sophisticated for children to use. This is further supported by Beishuizen's (1993) research which explained that "weaker students tended to use less efficient separation strategies" (p. 298)

Assessment

In order to assess their student's capabilities and to identify inflexible and/or inaccurate mental computers in their classrooms, teachers need to regularly assess mental computation. Some teachers consider the assessment of mental computation to be a challenge because 'it happens in the children's heads'. This does not need to be the case. In fact, children who have solid mental computation skills should have the knowledge to provide much more detailed answers than 'I just knew it'. If regular discussion of strategies is supported and valued in classrooms, children will develop the skills to clearly articulate the methods they use to solve a problem. As McIntosh (HREF2, 2004b) states "mental computation can be simply and effectively assessed by a written class test" (p.9). He explains that if children are asked to solve a problem and write a detailed explanation of the procedure and strategies they used to solve it, teachers can readily assess the depth of their mental computation knowledge.

Mental computation has been shown to have strong links to the development of number sense in children. Number sense is "a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements" (McIntosh, Reys & Reys, 1992 in McIntosh & Dole, 2000, p. 401). Mental computation promotes number sense through a development of understanding of how numbers work and relate to each other.

Mental Computation Errors

In his three year study of students in Tasmania and the Australian Capital Territory, McIntosh discovered patterns in children's mental computation errors (HREF2, 2004b). McIntosh makes a clear distinction between children's procedural and conceptual errors. McIntosh explains "a conceptual error is one made because the student does not understand sufficiently the nature of the numbers or operation involved" (HREF2, 2004b, p.8), whereas a "procedural error is one in which the student although having an overall strategic understanding of what to do, makes either a careless error or other error in carrying out the strategy" (HREF2, 2004b, p.8)

An interesting pattern that McIntosh (HREF2, 2004b) noted in children's errors was that most of them were incorrect by one. McIntosh (HREF2, 2004b) suggests that "it appears very probable that in many cases the children's strategy was to count up or down by ones" (p.8). Clearly this is a most inefficient strategy to use as it relies on the children having to keep track of large numbers in their mind. Disturbingly, McIntosh (2004a) states that up to 20% of upper primary students continue to use counting by ones for two digit addition and subtraction questions.

When analysing errors teachers must check that these errors are not procedural, such as "when adding 6 and 3, counting 6,7,8" (HREF2, 2004b, p.9). Clearly this incorrect procedure must be quickly rectified to avoid problems in the future. McIntosh (HREF2, 2004b) also suggests that children must also be "weaned off" the count on or count back strategy and introduced to more efficient derived strategies such as "doubles, near doubles, bridging ten, adding tens, using compatible numbers, using related known facts" (p.9).

Other errors have been noted when children attempt to imagine and complete the written algorithm in their mind. This often leads to errors, particularly where carrying is concerned as this is a step that is difficult to track mentally. As such, this is not a mental computation strategy that is recommended to be taught to children.

Professional Development

In order to effectively develop children's mental computation skills, schools must take a systematic whole school approach to its teaching and learning. This includes the development of teacher's mental computation PCK through regular and quality Professional Development (PD). This PD should include providing teachers with the knowledge to identify common mental computation errors and how to remedy these. This will also alleviate the tension that teachers sometimes feel when a child suggests a mental computation strategy they are unfamiliar with. Teachers PCK will allow them to evaluate the children's methods and decide if that particular approach can be successfully applied to other problems they may encounter. Teachers should also be provided with a solid understanding of assessment, mental models and strategies which can assist their students to develop into flexible mental computers. In turn, each teacher's theoretical knowledge of Mental Computation must be supported in practical terms through the implementation of a quality mental computation framework such as that developed by McIntosh (2004a).

In Summary

Mental computation is a skill that has applications in almost every calculation children will attempt throughout their school life and beyond. As such, formal instruction in this area should permeate every aspect of numeracy teaching. The importance of mental computation in a child's life cannot be underestimated, from working out how many minutes of school are left, to calculating how much they can afford to spend at the supermarket, children and adults alike must be competent, proficient and flexible mental computers to ensure they can function in society successfully. In conclusion, as McIntosh (HREF2, 2004b) suggests:

> If mental computation is to take the place in schools that both society and the pronouncements of curriculum developers at system level encourage, then it needs to be given attention in terms of teaching time, exploration of efficient teaching strategies, and resourced equivalent to those that hitherto have been given to the teaching of the formal written algorithms (p.10).

References

Beishuizen, M. (1993) Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second grades. *Journal for Research in Mathematics Education,* 24*(4),* 294-323.

Beishuizen, M., & Angileri, J. (1998). Which Mental Strategies in the Early Number Curriculum? A comparison of British ideas and Dutch Views. *British Educational Research Journal*, 24*(5),* 519-538.

Callingham, R. (2005). A whole-school approach to developing mental computation strategies. In H. L. Chick & J. L. Vincent, (Eds), *Proceedings of the 29th Annual Conference of the International Group for the Psychology of Mathematics Education*, (*2*), pp. 201-208. Melbourne: PME.

Caney, A. (2004) Numbers+Magic=Answer. Students explaining: Make the most of mental computation. *Australian Primary Mathematics Classroom,* 9*(3),* 10-14.

HREF1: **http://www.coag.gov.au/reports/docs/national_numeracy_review.doc**, Council of Australian Government. (2008) National Numeracy Review Report.

Gravemeijer, K. P. E. (1994) Developing Realistic Mathematics Education. Utrecht, The Netherlands: Freudenthal Institute, University of Utrecht.

Heirdsfield, A.M. (2004). Enhancing mental computation teaching and learning in Year 3. In *Proceedings Australian Association for Research in Education*, Melbourne.

Heirdsfield, A. & Lamb, J. (2005) Mental Computation: The Benefits of Informed Teacher Instruction. In: MERGA 28 - 2005 *Building connections: Theory, research and practice*, July, 2005, Melbourne.

Heirdsfield, A. & Lamb, J. (2007) Year 2 inaccurate but flexible mental computers: teacher actions supporting growth. In *Proceedings Australian Association for Research in Education*, Adelaide, Australia.

Kamii, C. (1994). Young Children Continue to reinvent Arithmetic-3rd Grade: *Implications of Piaget's Theory*. New York: Teachers College Press.

Mardjetko, E & Macpherson, J. (2007). Is your classroom mental? *Australian Primary Mathematics Classroom*, 12*(2)* 4-9.

McIntosh, A (2004a). Mental Computation a Strategies Approach. Hobart: Department of Education.

HREF2: **http://www.mai.liu.se/SMDF/madif5/papers/McIntosh.pdf**, McIntosh, A. (2004b) Mental Computation of School-Aged Students: Assessment, Performance Levels and Common Errors.

McIntosh, A. (2005). Developing computation. Hobart: Department of Education, Tasmania.

McIntosh, A., & Dole, S. (2000). Mental Computation, number sense and general mathematics ability: Are they linked? In J. Bana & A. Chapman (Eds.)., *Mathematics Education Beyond 2000* (pp. 401-408). Perth: MERGA.

Thompson, I., & Smith, F. (1999) Mental calculation strategies for the addition and subtraction of 2-digit numbers. Final Report. University of Newcastle, University upon Tyne.

Reys, R. E. (1984). Mental Computation and estimation: Past, present and future. *Elementary School Journal*, 84, 546-557.

Westwood, P. (2000). *Numeracy and Learning Difficulties: Approaches to teaching and assessment.* Camberwell: ACER.