REASONING AND COMMUNICTION IN THE MATHEMATICS CLASSROOM – SOME 'WHAT' STRATEGIES

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What is reasoning?

Reasoning is the ability to think, understand, and form opinions or judgments that are based on facts (Longman, 1987). It is the process of making inferences from a body of information. For example, given the information that Lala is a spider, it is reasonable to conclude that Lala has eight legs. Likewise, given the information that Sally's room is a rectangle 4 m wide and 6 m long, it is reasonable to conclude that the area of the room is 24 sq m.

Reasoning may also be deemed to be the alliance of problem solving and communication, as shown by the item in Figure 1. This item from TIMSS 1999 (Kelly, Mullis & Martin, 2000) has three parts, A, B and C. The cognitive domain or performance expectation of Parts A and B are *investigating and solving problems* while that of Part C is *communicating and reasoning.*

The figures show four sets consisting of circles.

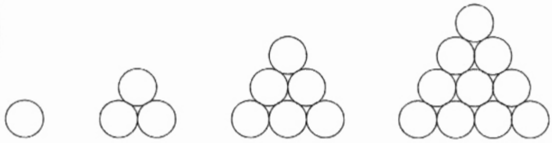

Figure 1 Figure 2 Figure 3 Figure 4

a) Complete the table below. First, fill in how many circles make up Figure 4. Then, find the number of circles that would be needed for the 5th figure if the sequence of figures is extended.

Figure	Number of circles
1	1
2	3
3	6
4	
5	

b) The sequence of figures is extended to the 7th figure. How many circles would be needed for Figure 7?

Answer: ________________

c) The 50th figure in the sequence contains 1275 circles. Determine the number of circles in the 51st figure. Without drawing the 51st figure, explain or show how you arrived at your answer.

Figure 1

Can mathematical reasoning be taught? What has research to say?

According to Steen (1999) research does support a few general conclusions related to the nurturing of mathematical reasoning amongst school students. They are as follows:

- *Firstly,* successful learners are mathematically active (Anderson, Reader & Simon, 1997 quoted in Steen 1999). Passive strategies (memorization, drill, templates) are much less likely than active tasks (discussion, projects, team-work) to produce either lasting skills or deep understanding.
- *Secondly,* successful mathematics learners are more likely to engage in reflective (or metacognitive) activity (Resnick, 1987). Students who think about what they are doing and why they are doing it are more successful than those who just follow rules they have been taught.

- *Thirdly,* students differ: no single strategy works for all students, nor even for the same student in all circumstances. Howard Gardner's theory of multiple intelligences (Gardner 1983, 1995) supports the practice of experienced teachers who create multiple means for students to approach different topics. Diverse experiences provide implicit contexts in which mathematical reasoning may emerge. But we cannot be sure that it will emerge for all.

Some "What ..." strategies to help develop reasoning amongst middle school students

In this section, four strategies that may be used to facilitate the development of reasoning amongst middle school students are presented. The four strategies are what number makes sense, what's wrong, what if, and what's the question if you know the answer? These strategies are based on the assumption that mathematical tasks that can be solved through mathematical proficiency and obvious arithmetic or procedural operations hinder mathematical reasoning whereas those that emphasize sense making and interpretative skills enhance it.

Strategy 1: What number makes sense?

In "What number makes sense?" students are presented with a mathematics version of a cloze passage that many would be familiar with in their English Language lessons. Students are presented with problem situations which has numerical data missing. A set of numbers is provided and students determine where to place each number in the passage so that the situation makes sense. The steps given as part of the task sheet help to focus the students on the steps they need to take and also explain their thinking. The task is meant for group work. The teacher must ensure that group interaction is followed by class discussion so that students have the opportunity to explain their thinking and also learn of ways of solving problems that differ from their own. As students work through tasks of this nature, they practice computation and increase their repertoire of problem-solving skills. Reasoning skills are improved by being exposed to a variety of ways to solve a problem (Krulik and Rudnick, 2001). Such a task can be easily crafted from a typical textbook question. An example follows. A typical textbook question found in many Singapore textbooks, shown as example 1 is transformed into a version of what number makes sense type of task, example 1A (See Figure 2).

Example 1

A typical textbook question (Teh & Looi, 2003, pg 119)

Q14a A locomotive is 10 m long and weighs 72 tonnes.
A similar model, made of the same material is 40 cm long, find the mass of the model.

Q14b Suppose the tank of the model locomotive contains 0.85 litres of water when full, find the capacity of the tank of the locomotive.

Example 1A

What number makes sense?

Read the problem. Look at the numbers in the box.
Put the numbers in the blanks where you think they fit best.
Read the problem again, do the numbers make sense?

A Toy Locomotive

A toy locomotive, made of the same material and density, is an exact model of a real one. The locomotive is _____ long and weighs _____.
The toy locomotive is _____ long and weighs ______. The capacity of the locomotive's oil tank is _____ and this is ______ times the capacity of the toy locomotive's oil tank.

4.8 kg	10 m	40 cm	75 tonnes	3125 litres	25^3

Figure 2

Strategy 2: What's wrong?

In, "What's wrong?" the students are provided with an opportunity to use their critical thinking skills. They are presented with a problem and its solution. However the solution contains an error, either conceptual or computational. The student's task is to recognize the error, correct it and then explain what was wrong, why it was wrong and what was done to correct the error (Krulik and Rudnick, 1999). Students may be asked to complete the task in small groups or individually. The teacher must ensure that students are engaged in class discussion after completing the task so that they get the opportunity to see ways of solving problems that differ from their own. Furthermore, these discussions often lead to deeper mathematical understanding (Krulik and Rudnick, 2001). Such tasks are not difficult for teachers to craft as they are constantly exposed to such errors students make in class and in their written assignments. Example 2, shown in Figure 3 is an example of one such task.

Example 2

Washing dinner plates

During the school camp, Sally was in charge of Kitchen duty. After each meal, she had to get a few students to wash the dinner plates. The students were very cooperative and they all washed at the same rate. On the first day, she asked 8 students to wash the plates and they took 2 hours. On the second day, she asked 12 students to do the washing and told the camp commander that the students will complete the washing in 3 hours.

Sally's solution:-

8 students	take	2 hours
1 student	takes	2 ÷ 8 = 0.25 hours
2 students	take	0.25 x 12 = 3 hours

There is something wrong with Sally's solution.
Show how you would solve the problem.
Explain the error in Sally's solution.

Figure 3

Strategy 3: What if ?

In "What if ?" kind of tasks two kinds of demand are made on the students' cognition. The first is when the given information is changed. This modification permits students to re-examine the task and see what effect these changes have on the solution process as well as the answer. In this way students are reinforcing their critical thinking as they analyze what is taking place (Krulik and Rudnick, 1999). The second is the generation of "what if" questions after they have solved some given "what if" questions. This draws on their creative thinking skills and engages them in problem posing (Brown and Walter, 1985). Problem posing is the generation of new problems and the reformulation of given ones

(Silver, 1994). Whole class discussion must precede individuals working on such tasks because students need to share the "what if" tasks they created with others and also make their thinking visible. Example 3, shown in Figure 4 is an example of one such task.

Example 3

Cylindrical Tank

An open cylindrical tank with diameter 28 cm and height 50 cm contains water to a depth of 20 cm. Find

i) the volume of the water inside the tank, giving your answer in litres;
ii) the total surface area of the tank that is not in contact with the water.

What if the cylindrical tank is closed?
What if the dimensions of the cylindrical tank are doubled?
What if the dimensions of the cylindrical tank are halved?
What if the depth of the water is reduced by 10 cm?
What if the orientation of the cylindrical tank is changed such that it is lying on its curved side?

Generate another 3 "What if" tasks and answer them.
Look out for any interesting observations / patterns.

Figure 4

Strategy 4: What's the question if you know the answer?

In "What's the question if you know the answer?" kind of tasks students are presented with the context and data but with the question/s missing. Students are asked to write a question that matches a given answer. Such tasks provide students with opportunities to engage in critical thinking. Whole class discussion must precede individuals working on such tasks as it is important for students to learn that several questions may have the same answer, but certainly different solutions.

Example 4, shown in Figure 5 is an example of one such task.

Example 4

Just one card

Eleven cards numbered 11, 12, 13, 14, ……, 21 are placed in a box.
A card is removed at random from the box.

1. What's the question if the answer is $\frac{5}{11}$?
2. What's the question if the answer is $\frac{4}{11}$?
3. What's the question if the answer is $\frac{9}{11}$?
4. What's the question if the answer is $\frac{6}{11}$?
5. What's the question if the answer is $\frac{3}{11}$?

Figure 5

Some pedagogical musts and suggestions

The four strategies described in this paper place much emphasize on engaging students in class discussion after a task has been completed. It is important for students to explain and justify their solutions because in so doing they are also clarifying their own thinking and often self-correct their errors if there are any. According to Gunningham (2003) when students hear their classmates' reasoning/thinking and solutions they get an opportunity to:

- learn to value thinking as well as answers,
- add to their bank of problem-solving strategies,
- realize that there are many paths to finding a solution,
- develop an appreciation of creativity in solving problems,
- compare the efficiency of different strategies for solving different problems, and
- build more complex connections between different mathematical concepts.

It is important that teachers support students in their efforts to explain and justify their answers by:

- providing adequate time for the explanation to be completed,
- resisting the temptation to immediately highlight flaws in reasoning / thinking,
- praising creative approaches to problem-solving,
- making explicit the links between different concepts,
- questioning students to draw out key ideas, and
- summarizing the strategies presented and discussing the merits of using some strategies compared to the others.

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