DEVELOPING MATHEMATICAL LANGUAGE – KEY TO CONCEPTUAL DEVELOPMENT

Brian Tweed

Massey University College of Education, Palmerston North, NewZealand

The New Zealand Numeracy Development Project (in both English and Māori languages), has highlighted many issues connected with how students learn Mathematics. One issue is the role that language plays in student achievement. Some interconnections between Mathematical language development, pedagogy and conceptual development will be described along with implications for how mathematics courses could be planned, implemented and assessed. An emerging framework for assisting the development of language in mathematics activities is presented.

Background

The New Zealand Numeracy Development Project (NZNDP) has been implemented in New Zealand Primary and Secondary Schools in both English and Māori medium contexts. One feature of the project is the requirement for students to select and explain the strategies they use when they are solving problems and to develop new concepts through co-operative working in small groups. In this context students need to develop both concepts and language simultaneously.

The idea that Mathematics is in fact a specialised form of the language function has been explored in some depth (Lakoff & Nuñez, 2000; Devlin, 2000) and supports the idea that interweaving rich language and rich mathematics is not only a good idea, it is an essential ingredient in any successful mathematics learning.

Much of the focus of the NZNDP so far has been on developing students' strategies for solving number problems through the use of a 'teaching model' involving material representations and mental imaging with students eventually arriving at a point where only number properties need to be referred to. This is based on the Pirie-Keiren model of understanding growth in mathematical understanding. (Pirie & Keiren, 1994; 1992). A simplified version as used in the NZNDP is described in New Zealand Numeracy Development Project Book 3 (2008). This is within a strategy and a knowledge framework (described in NZNDP Book 1, 2008) which guides teachers in the planning of next steps of learning for students. There is also an associated classroom management system based on grouping students according to their numeracy stage to allow tailoring of activities and resources to the needs of each group.

Within this structure the teacher works in very interactive ways with small groups of students and engages in a learning conversation with students to support the learning of new strategies. This can also provide a very rich flow of information for teachers to diagnose problems and assess progress. An enhanced opportunity is also provided in this situation for students to develop their use of language along with their conceptual development.

At heart, the NZNDP is based on constructivist principles. The real business of learning takes place by students interacting with their environment and with other people. The current developmental stage (i.e. their fund of prior experience and knowledge) of the student plays a large role in how well the learner learns. Language plays a key role in new learning as the medium through which new experiences are communicated, analysed, interpreted and fitted into existing schema. This aligns with Vygotsky's ideas about how students acquire new learning. (Vygotsky, 1986; 1994)

It could be considered that the NZNDP has reached a natural end of 'phase 1' of its implementation. The question now is what next? What will 'phase 2' look like? The suggestion here is to incorporate the work done so far into a broader framework integrating language development and richer problem solving activities and contexts.

A Broader Learning Framework

The following framework is intended to build on the constructivist approach seen here as already embedded in the NZNDP and is based on the following ideas:

- Mathematical language is unique to each student, and is to be developed by them as a response to the demands made during involvement in mathematical tasks.

- Mathematical language is developed through interaction between students and teachers as they engage in mathematical activity.
- The construction of an effective representation using materials or diagrams is also an effective platform for developing language appropriate to the concepts being represented especially when students themselves do the construction.
- Student engagement in mathematics learning will be enhanced when they work on problems that have direct connections to their own experience and interests.
- Student engagement in mathematics learning will be enhanced when they work on problems that offer challenge and growth with clear connections to further progress and opportunities to implement solutions in realistic situations
- Teachers play a vital role in both concept and language development as people who are located securely in the culture of mathematics and are able to model and explain effective practices and language as they and their students engage interactively in mathematical activities. Teachers not only create the mathematical experiences for students and support the learning, they also provide 'meta information' - they inform students about the nature of mathematics, dispositions of mathematicians and thinking skills.
- Use of language by students when doing mathematics will be enhanced when there are clear demands made on students to develop or create language that helps them solve problems and communicate the mathematics they learn and their solutions to problems.
- The pedagogy in use in a classroom reflects this interactive approach in which students work in small groups and engage in problems appropriate to their developmental stage i.e. their NZNDP stage.

The broader framework outlined in this section is intended to be most appropriately used in a setting where students are routinely engaged in problem solving/investigations. Referring back to the numeracy project context described earlier, small groups of students would be presented with authentic problems at the appropriate numeracy stage and supported in developing solutions for them through the use of this framework.

The components of the framework in Table 1 are essentially an attempt to describe what a student has to do in order to produce an effective solution for a problem from the first engagement to the final 'publication' of a solution. Interwoven throughout the framework are ideas from a literacy perspective. (Kenney, 2005; Hyde, 2006; Keene, 2007)

Table 1: A possible framework for developing language, conceptual and problem solving capabilities.

	Teacher	**Student**
Task Selection Aim: to match the task to a student group for both numeracy and language requirements.	Assess task for numeracy requirements and match with target student group.	Students can be involved in task selection as they gain confidence and experience.
	Ensure that the type of problem allows for generalised solution descriptions. i.e. it is a rich problem.	
	Gather resources, check language requirements.	
	Gain familiarity with possible solutions and connections with major mathematical ideas, next steps, generalisations.	
Initial Engagement Aim: to have a clear understanding about the context and requirements of the problem.	Deal with any coding issues, unfamiliar terms and phrases, inappropriate prior understandings.	Interact with resources, read descriptions of problems, diagrams, graphs, tables etc.
	Support students through use of comprehension strategies.	Draw on previous experiences, problems already encountered to add meaning to current task.
	Model own thinking when interpreting information about a problem.	Use a comprehension strategy to process information.

	Teacher	**Student**
Cognition Aim: To construct an effective representation of the mathematics involved in the problem.	Work interactively with students to support conceptual understanding. (NZ Numeracy Project Teaching Model)	Use materials to represent the problem.
		Develop diagrams to represent the problem.
Solution: Aim: To arrive at a solution of the problem in an algebraic or 'algebra like' form.		Try other parameters.
		Develop mental imaging techniques.
		Describe the solution in general/algebraic terms.
		Give examples of solutions for different parameters.
Conference Aim: to consult with 'colleagues' about the solution, seek advice and feedback and modify solutions if necessary.	Teacher involved as participant. Ask questions, listen, assess, make suggestions.	Discuss with other groups working on the same problem.
		Check that all members of the group fully understand all the work that has been done.
		Consult 'experts'.
Checking/Publishing Aim: To test out solutions in actual situations and publish the work for others to comment on and learn from.	Assist with checking. Read the publication and offer comments.	If the solution can be tested directly, test it. eg. by constructing a model, or performing actions.
		Produce a ''publication' suitable for a wider audience and publish it.

The kind of problem chosen for a group of students is of great importance. The Numeracy Project has provided one essential component - the accurate matching of the numeracy demands made within a problem to the numeracy stage of the students. In addition, it is important that the problems chosen have a measure of reality sufficient to allow students to connect their own experiences. Ideally, problems stem from actual and recent experience which may have been deliberately orchestrated by the teacher. The problems must have genuine mathematical scope with requirements for generalisation, extension and connection to other problems and concepts. These are 'rich' in mathematical experience but it is also required that they offer rich language experience also.

Various skills are woven into this framework and come to prominence at different stages within it. To begin with it is largely reading and interpreting the information given about the problem and processing this to bring out the mathematics involved. The 'standard' NZNDP model i.e. the process of moving from material representation to mental imaging to number properties, is placed as a tool for supporting the solution of problems in the middle phase of the framework. It is here that additional concept learning as required in the solution of the problem could be targeted. The later stages are intended to support students in developing, checking and justifying their solutions with others (students, teachers, family members) and will usually involve the 'publication' of their solutions for others to see. An important feature is that students are involved in linking their solutions back to the problems in order to provide a 'natural consequence' to their mathematical work. This could involve the testing out of solutions in the realistic context of the problem.

Essentially the framework intends to support students to work through a natural and complete process of mathematical learning in which realistic contexts, language and communication and the production of real solutions with practical applications are given status.

The Framework also describes some aspects of the teachers' role. This is important because the teacher works interactively with students at every stage and is a vital participant in the process.

At the start of the process the teacher needs to put in the effort to select and gain familiarity with appropriate problems for the groups in the class. As progress is made the responsibilities shift more onto the students themselves with the teacher adopting a supporting and mentoring role. Towards the end of the process it could be completely student driven.

Much of the work is informal early in the process and becomes more formal as it progresses. Language development and conceptual development happen simultaneously through interacting with material and diagrammatic representations, being asked to explain and justify, and listening to the language of the teacher and others. The intention is for the final publication to be presented as a 'good copy' with as much use of formal symbols and language as is appropriate for the developmental stage of the student.

Concluding Comments

If the framework is adopted as a model for how mathematics usually happens in a classroom then there are implications for course design, resources and assessment practices. However this is a common challenge that constructivist approaches make to 'traditional' teacher and content oriented methodologies. (Cobb, Perlwitz, & Underwood-Gregg, 1998; Draper, 2002; Stevens, 2000; Richardson, 1997)

One suggestion is to view the mathematics course as a network of inter-related tasks and problems that groups of students work through. Learning pathways can be mapped out from problem to problem in the network. As students gain experience they are able to continue on to problems that progressively develop concepts and language.

In terms of assessment, there may need to be a period of formal assessment at the beginning of a course (or data may be carried over from previous years) but after that the day to day monitoring and noticing of student work could provide a large part of the necessary data to allow for further planning and possibly other assessment requirements.

In terms of resourcing, the creation of a collection of inter-related problems as indicated above is a major challenge in itself and the subject of further debate, research and trialing.

If the idea that developing language is a key to developing conceptual understanding is accepted, then the conventional forms of mathematical text and resources usually seen in mathematics classrooms may be inappropriate. The exact nature of the resources needed to effectively support this idea is also awaiting further development.

It is interesting to note that the open nature of the framework and the focus on authentic contexts for learning can be applied to many situations such as developing a social or political awareness in students or for improving responsiveness to students from differing cultural backgrounds (Gutstein, 2006). Of particular importance in the New Zealand context is how this framework could support the further development of the use of Māori language (the indigenous language of New Zealand) and its inherent cultural underpinnings in Mathematics learning. (Christensen, 2004; Trinnick & Stevenson, 2007; 2007)

References

Christensen I. (2004). Te Reo Pāngarau: Learning and teaching Mathematics in Māori. *Language Acquisition Research. Papers presented at Ministry of Education Forum 2003*, 117-127. New Zealand Ministry of Education.

Cobb, P., Perlwitz, M., & Underwood-Gregg, D. (1998). Individual construction, mathematical acculturation, and the classroom community. In M. Larochelle, N. Bednarz, & J. Garrison (Eds.), Constructivism and education (pp.63–80). New York: Cambridge University Press.

Devlin K. (2000). *The Math Gene: How Mathematical Thinking Evolved And Why Numbers Are Like Gossip.* Basic Books

Draper, R..J. (2002). School mathematics reform, constructivism, and literacy: A case for literacy instruction in the reform-oriented math classroom. *Journal of Adolescent & Adult Literacy 45:6 March 2002, 520-529*

Gutstein, E. (2006). *Reading and Writing the World with Mathematics: Towards a Pedagogy of Social Justice.* New York, NY: Routledge, Taylor & Francis

Hyde, A. (2006). *Comprehending Math - Adapting Reading Strategies to Teach Math, K-6.* Portsmouth, NH: Heinemann.

Kenney, J.M., Hancewicz, E., Heuer, L., Metsisto, D., Tuttle, C.L. (2005). *Literacy Strategies for improving Mathematics Instruction*. Alexandria VA: ASCD.

Keene, E.O. (2007). *Mosaic of thought : the power of comprehension strategy instruction.* Portsmouth, NH : Heinemann

Lakoff, G, Nuñez, R.E. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being.* New York: Basic Books

NZNDP New Zealand Numeracy Development Project (2008). *Book 3: Getting Started.* New Zealand Ministry of Education

(Also available on-line: **www.nzmaths.co.nz/Numeracy/2008numPDFs/pdfs.aspx**)

NZNDP New Zealand Numeracy Development Project (2008). *Book 1:The Number Frameworks.* New Zealand Ministry of Education

(Also available on-line: **www.nzmaths.co.nz/Numeracy/2008numPDFs/pdfs.aspx**)

Pirie S.B., Keiren T. (1994). Growth in Mathematical Understanding: How Can We Characterise It and How Can We Represent It? Educational Studies in Mathematics, *Vol. 26, No. 2/3, Learning Mathematics: Constructivist and Interactionist Theories of Mathematical Development (Mar., 1994), pp. 165-190*

Pirie S.B. and Keiren T (1992). Creating Constructivist Environments and Constructing Creative Mathematics. *Educational Studies in Mathematics, Vol. 23, No. 5, Constructivist Teaching: Methods and Results (Oct., 1992), pp. 505-528*

Richardson, V. (1997). *Constructivist teaching and teacher education: Theory and practice*. In V. Richardson (Ed.), Constructivist teacher education: Building new understandings (pp. 3–14). Bristol, PA: Falmer.

Stevens, R. (2000). Who Counts What as Maths? Emergent and assigned Mathematics Problems in a Project Based Classroom. In Boaler J. (Ed.), *Multiple Perspectives on Mathematics Teaching and Learning* (pp. 105-145) Westport, CT: Ablex Publishing

Trinnick T. and Stevenson B. (2007). Te Poutama Tau: Trends and Patterns. *Findings from The New Zealand Numeracy development Projects 2006*, 44-54. New Zealand Ministry of Education.

Trinnick T. and Stevenson B. (2007). Te Ara Poutama: An Evaluation of Te Poutama Tau. *Findings from The New Zealand Numeracy Development Projects 2007*, 29-38. New Zealand Ministry of Education.

Vygotsky, L.S. (1986). *Thought and Language*, A. Kozulin, (ed. and trans.), MIT Press, Cambridge, Mass.

Vygotsky, L.S. (1994). *The Vygotsky Reader*, R. van der Veer and J. Valsiner (eds. and trans.), Blackwell Publishers, Oxford.