MATHEMATICS IN SURVEYING: MOTIVATING AND CHALLENGING STUDENTS THROUGH PRACTICAL APPLICATIONS OF MATHEMATICS

Mary Barnes

Mathematics teachers and surveyors have collaborated to organise "mathematics-in-surveying" excursions for high school students. The aim is to engage them in challenging activities that explore practical applications of mathematics, and to give them some insight into the work of a surveyor. Activities include mapping a garden, measuring the height of a tower, finding the radius of the earth, and setting out a pattern. Students use modern surveying equipment and CAD software as well as simple measuring tapes and compasses. The excursions have met with an enthusiastic response from both the students and their teachers.

Introduction: What do you know about surveying?

Many people notice surveyors working on building sites and highway construction projects, wearing hard hats and reflective vests. This may lead them to think of surveying as a "blue collar" occupation, and they may not realise the wide range of work carried out by professional surveyors, the extent to which they use modern technology and the many ways in which our society depends on their work.

Surveyors play a key role in the planning and construction of all major infrastructure projects—roads, railways, tunnels, airports, buildings, mines, sewerage and drainage systems. Their work is essential to the integrity of our land title system, the planning and management of natural and man-made resources, petroleum and mineral exploration, and the production of topographical maps. Geodetic surveyors use satellites to measure the Earth's surface accurately to help predict earthquakes and monitor sea level changes and continental drift. Hydrographic surveyors map the sea floor, lakes, rivers and ports and help to ensure that shipping channels are kept open. A surveying technique called photogrammetry provides details for maps and for such tasks as structural analysis of bridges and restoring facades of buildings. It is also used in the analysis of crime scenes and medical images, and to combine live action with computer generated imagery in movie post-production, as, for example, in the movie *Fight Club*.

To sum up, surveying involves measurement on and above the earth's surface, underground and under water. Surveyors work in a great variety of situations, in cities, country towns and the remote outback, in an office and out of doors, using computers and highly technical equipment. Surveyors need to have good spatial skills, be able to think and visualize in three dimensions, make measurements with a high degree of accuracy, and pay attention to detail—all qualities developed by good mathematics students. Mathematics is a key component of a surveying degree course. And, an important consideration in today's employment climate, there is a serious shortage of surveyors throughout Australia. Further information about surveying as a career can be found on the website of the Institution of Surveyors, New South Wales (ISNSW) (http://www.isansw.org.au)

Mathematics in Surveying Excursions

In response to the need for surveyors, a surveyor and a mathematics teacher had the idea of organising Mathematics in Surveying excursions for high school students. With the support of ISNSW, they formed a small committee of surveyors and teachers to plan a series of field days. While the surveyors' aim was to raise students' awareness of the possibilities of surveying as a career, the teachers were concerned about decreasing numbers choosing higher level mathematics courses. They hoped to increase students' engagement and interest in mathematics by having them spend a day in a pleasant outdoor location, applying the mathematics they had been learning to a series of practical tasks.

Bicentennial Park (part of Sydney Olympic Park) was selected as the venue, because it has a variety of interesting structures including a tower, a hedged garden, a sundial, a "Peace Monument", and plenty of open space. These provide opportunities for a wide range of challenging mathematical activities.

Teachers and surveyors collaborated to plan the activities and prepare student worksheets. The excursions were free of charge to students—all overheads were met by the ISNSW. Very quickly we had over 200 registrations, as many as we could handle. Students, working in groups of five, rotated through seven activities. Large numbers of volunteers were needed to run these: about 40 surveyors were recruited and they were assisted by trainee teachers and some retired teachers. All specialized equipment required was loaned for the occasion.

Activities

The excursions were designed for Year 10 students but, with a little modification, many of the activities could be used with Year 11 or able Year 9 students. The mathematics involved included measuring distances, using a compass to find bearings, drawing plans to scale, mensuration formulae for areas, angle properties of parallel lines, simple circle geometry, Pythagoras' theorem, similar triangles, and some elementary trigonometry. In some cases the only equipment needed was measuring tapes and magnetic compasses. These activities, which could easily be adapted by teachers for use in or near their own schools, are described in detail below. In other cases more sophisticated surveying equipment was required, but these activities could also be used in a school if a friendly local surveyor could be found and persuaded to visit, bringing a theodolite or other equipment.

Mapping

One of the features of Bicentennial Park is a hedged garden, with lawns and flower beds. There are two lawns, each in the shape of a rectangle, with a piece cut out of one of the long edges, leaving space for a flower bed between them (see Figure 1).

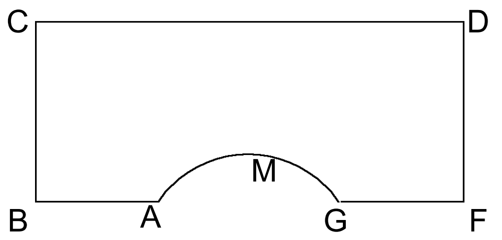

Figure 1. Plan of one of the lawns in the Hedged Garden

Students were asked, first, to measure up the lawn and draw a map of it on grid paper provided. They were instructed first to measure the segments from *A* to *B*, *B* to *C*, and so on round the lawn, ending with the segments from *G* to a point *M* somewhere near the middle of the curved section, and from *M* to *A*; and to use a compass to find the direction of each interval. Then, using this data and their geometry equipment, they were asked to draw an accurate map using a scale of 1:100, and indicating clearly the direction of magnetic North.

The difficulty is that the piece cut out from the rectangle is not a semicircle, so students need to use their knowledge of circle geometry to find the centre of the circle, in order to draw the arc *AMG*. They can do this by drawing the perpendicular bisectors of *AM* and *MG* as shown in Figure 2. Since these are chords of the circle, their intersection gives the centre *O*.

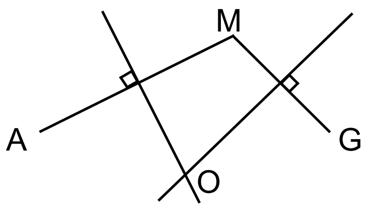

Figure 2.

Having drawn the map, students were then told to imagine that there were two land mines buried underneath the lawn, and asked to mark their positions on their maps. They were given the *bearings* of the first mine from corners *B* and *C* of the lawn, and the *distances* of the second mine from corners *D* and *F*. Since finding the location of the second mine involved intersecting two circles, students had to decide which of the two intersection points gave the correct position. Luckily, one of the points was well outside the lawn area, so the decision was not hard!

A final task was to calculate the area of the lawn. While this could be done by counting squares on their grid paper, a more accurate answer required them to find the area of the segment *AMG* (Figure 1). This again required some use of circle geometry, as well as the areas of a triangle and a sector of a circle. An activity like this could be carried out in any park where there are lawns or flower beds with suitable interesting shapes. Failing that, the outline of a suitable shape could be marked out with coloured tape or ribbon on the school oval.

From the hedged garden, the students were directed to a computer classroom, where they were given a short tutorial on the use of the computer aided design software *miniCAD* (Iredale, 2008). They were shown how to enter the data they had gathered during the mapping exercise into the computer, and use this to produce an accurate map of the area, mark the exact location of the two mines and get the computer to calculate the area. Thus they were able to check the accuracy of their pencil-and-paper work. Finally, they were shown how to superimpose their map onto an aerial view of the garden, and print out the result.

Finding heights using similar triangles

Another activity was designed around Bicentennial Park's "Peace Monument". This monument consists of three structures, all similar to that shown in Figure 3.

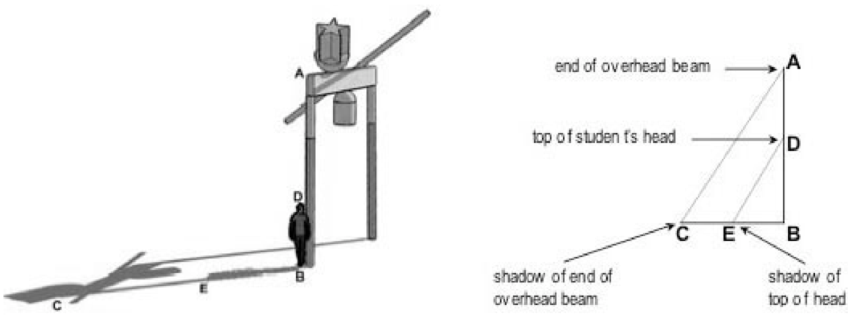

Figure 3. Using shadows to find the height of the Peace Monument

Students were first asked to use shadows to find the height of the top of the horizontal beam above the base of the structure, using no equipment except a measuring tape. The first step was to find the exact height of one member of the group, by having them stand against the upright, as shown in the picture, while the others placed a ruler horizontally across their head, noted where it met the upright (point *D*) and measured from there down to the ground (*DB*). Next, they marked the points where the shadows of the top of the beam and of the student's head fell on the ground (points *C* and *E*), and then measured from the base of the upright to each point (*BC* and *BE*). Finally, they used similar triangles to calculate *AB*, the height of the top of the beam above the ground.

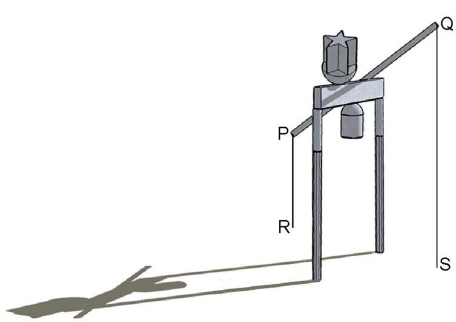

Figure 4. Finding the length of an overhead beam.

A second, more challenging, task was to find the length of the sloping beam *PQ* attached to the top of the monument (see Figure 4). To help in this, a surveyor placed pegs *R* and *S* in the ground in advance, vertically below the endpoints of the beam. These pegs enabled students, using the same method as before, to find *PR* and *QS*, the heights above ground of the two endpoints of the sloping beam. They also measured the distance *RS*. Then, by drawing a diagram in the plane *PQSR*, they could see how to use Pythagoras' theorem to calculate the length of *PQ*.

A problem arose with this activity the first time we tried it. It was cloudy for part of the day, and with no sun, there were no shadows to measure. However, we devised a method for doing the same measurements without shadows. One student was instructed to crouch down, with eyes fairly close to the ground, at some distance from the monument. Another student stood in a straight line between the base of the monument and the crouching student. The crouching student was then instructed them to move forwards or backwards, until the top of their head was aligned with the top of the monument, as shown in Figure 5.

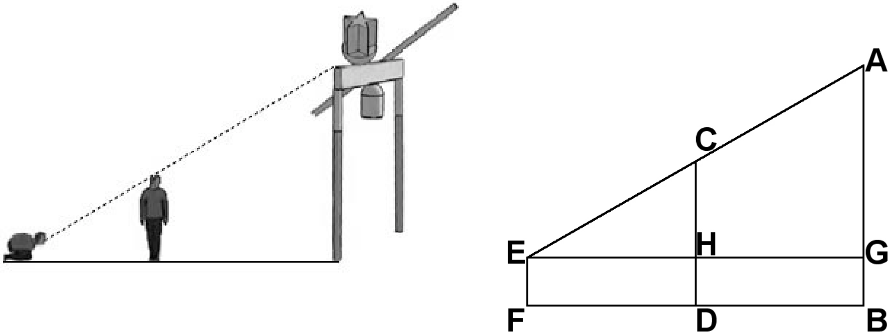

Figure 5. Finding heights if there is no sun

Once the two students were positioned correctly, other members of the group were able to measure the height above the ground of eyes of the crouching student (*EF*), the horizontal distance from the crouching student's eyes to the standing student (*FD*), and the horizontal distance from the crouching student's eyes to the base of the upright (*FB*). These measurements, together with the height of the standing student (*CD*) provided enough information for students to use similar triangles to calculate *AG* and hence find the height of the top of the monument *AB*.

Like the mapping, this activity could easily be carried out in the grounds of a school, or anywhere where there is a suitable structure, such as a building or a flagpole, whose height cannot be measured directly. And it requires no equipment other than a measuring tape.

Finding heights using trigonometry

One of the landmarks in Bicentennial Park is the "Treillage Tower". This tower, with walls made of latticework, sits on top of a stepped base and is approached by a paved pathway with a row of fountains. The students' task was to find the height of the tip of the mast at the top of the tower, using a surveying instrument called a Total Station, a combination of an electronic theodolite for measuring angles, and electronic distance measuring device. A reflector in the form of a prism mounted on a pole was placed at the top of the steps, directly below the tip of the mast, and the Total Station placed some distance away along the path (see Figure 6).

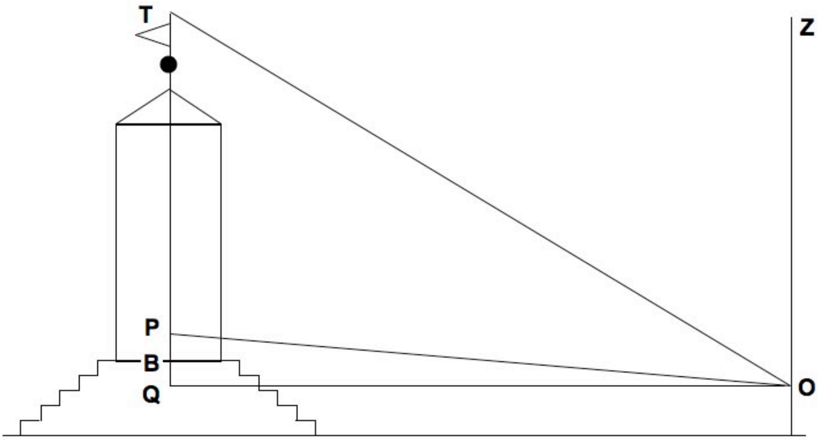

Figure 6. Finding the height of the Treillage Tower

In Figure 6, *O* is the position of the observer's eye (looking through the telescope on the total station), *Z* is the zenith point vertically above *O*, *P* is the prism, *T* is the tip of the mast on the tower, *B* is a point vertically below *T* on the base level of the tower, and *Q* is a point vertically below *T* on the same horizontal level as *O*.

To save time, students were told the height of the prism above the base level of the tower (*PB*). Using the total station, they were able to measure the zenith angle of the top of the tower (angle *ZOT*), the zenith angle of the prism (angle *ZOP*) and the distance from the observer's eye to the prism (*OP*). Since *ZOQ* is a right angle, they were able to find angles *POQ* and *TOQ*. Then, using trigonometry, first in triangle *POQ* and then in triangle *TOQ*, they were able to calculate the lengths of OQ, PQ and TQ. With this information, plus the height of the prism (*PB*), they could find the height of the tower *TB*.

The Sundial

Calculations about shadows are important to surveyors. Applicants for building approvals may ask a surveyor to predict where shadows will fall at different times of year. For example, if a high-rise building was built next door to you, would it cast shade on your backyard swimming pool? The sundial in the park (Figure 7) provided the context for a discussion of shadows, and an explanation of how sundials work and why the time on a sundial is not exactly the same as the time shown on your watch.

Figure 7. The Bicentennial Park Sundial

Finding the radius of the earth

Eratosthenes, a Greek mathematician and astronomer (276 BC – 194 BC) who lived at Alexandria, devised a way of calculating the size of the earth by observing and measuring shadows and using simple geometry and properties of circles.

We adapted Eratosthenes' method for students to use. By measuring shadows on the sundial, they were able to calculate the zenith angle of the sun at midday in Bicentennial Park. They were then told the zenith angle at midday at Rockhampton, (on the same line of longitude as Sydney). Using this, they were able to work out the angle *BOR* in Figure 8. This is the angle subtended between Bicentennial Park and Rockhampton at O (the centre of the earth). Given that the length of the arc *BR* (the distance from Sydney to Rockhampton) is 1160 km, they could find the circumference of the earth and hence its radius.

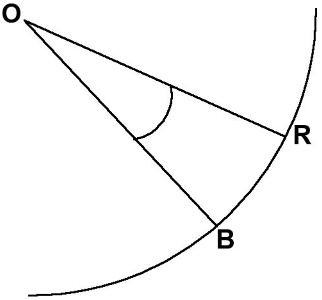

Figure 8

Other activities

Another activity at the excursion involved setting out a pattern, using modern surveying equipment—robotic total stations and GPS units. The coordinates of points in the pattern had been entered into the surveying equipment in advance. Students were able to read out from the onboard computer how far north and how far east they had to go to get to the next point. This proved quite a test of distance estimation skills. When a point was located, they pushed a nail into the ground. At the end, coloured ribbon was wound around all the nails to make the pattern stand out, and everybody climbed to the top of the nearby Treillage tower to see what they had created.

Students also had the opportunity to see demonstrations of photogrammetry and laser scanners, and a display and explanation of historical surveying equipment.

Responses to the excursions

The feedback from students and their teachers about the excursion was very positive. Students described the tasks as interesting and many were clearly excited by the opportunity to use modern surveying equipment and CAD software. They evidently found the tasks engaging, and even when pressed for time were keen to complete a task before moving on to the next activity.

Teachers observed that students were enthused by the activities, and became actively involved. They thought there was a good variety of practical and numerical activities. They were also inspired to see the large number of support staff (surveyors, student teachers and others) who came to help run the day and thought that they effectively communicated their enthusiasm to the students. Perhaps the best indicator of success was the number of schools that expressed interest in coming back to a similar excursion the next year.

The idea for these excursions is catching on, and groups of surveyors and mathematics teachers in other parts of the country are planning similar activities. A surveying day was held recently in Newcastle, and a group of surveyors in Melbourne is interested in planning excursions there, and is looking for interested mathematics teachers to work with them.

References

Institution of Surveyors, New South Wales: http//:www.isansw.org.au

Iredale, I. (2008) miniCAD (Version 5) [Computer software]. Round Corner, NSW: Mapsoft. (http://www.mapsoft.com.au)