UNDERSTANDING FRACTIONS: WHAT HAPPENS BETWEEN KINDERGARTEN AND THE ARMY?

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In this paper I will discuss some issues relating to children's understanding of fractions. These include the key ideas which need to be understood, the principal stages in the development of understanding and the key indicators of quality of understanding which teachers can observe.

Introduction

Learning and mastering fractions is still a major issue for students in both primary and middle schools (Saxe, Taylor, McIntosh and Geahart, 2005) where they show difficulties in understanding and using written notations for fractions. Students bring to their study of fractions a significant amount of informal knowledge related to whole numbers and fractions. One of the most important roles in our job as mathematics teachers is to help students develop a rational number sense, see the connections between fractions symbols; develop verbal knowledge and understand pictorial representations. We also need to take into consideration the kind of intuitive or socially acquired knowledge of numbers which students may bring to classroom. Related to this, we need to take into consideration how different cultural and language backgrounds may influence their learning (Miura & Yamagishi, 2002). Finally, we need to help them understand the meaning of mathematical symbols and procedures (Mack, 1995).

Students' understanding of fractions:

Teaching concepts of fractions, ratios and proportions is still one of the main challenges in teaching mathematics at school. One reason for that is how to engage students in activities that involve such concepts in their everyday life. They construct knowledge about fractions and rational numbers from experiences at home, sports games etc. students then come to school with this "constructed" knowledge which then interacts with the "instructed" knowledge offered by the mathematics curriculum and teachers (Smith, 2002). When students are exposed to fundamental fraction concepts early in the elementary grades, their understanding of fractions develops and changes (Saxe et al, 2005). It is our job as teachers to help students to make the connection between their constructed knowledge and their instructed knowledge as well as the new powerful ideas that we expect them to learn and master.

For example, we need to help them make sense of mathematical expressions such as: " $\frac{4}{7}$ ", " $\frac{1}{2} = \frac{3}{6}$ " and " $\frac{x}{3} = \frac{8}{12}$ " (Smith, 2002).

We also need to consider the changing patterns in students' use of fraction notation and the role of instruction in developmental change (Saxe et al. 2005). At early stages of schooling, students need to develop their intuitive approach to fractional numbers involving explorations with continuous and discrete models. Groups of objects, number lines, and regions are three models that can be used to develop an understanding of fractional numbers. A basic understanding of fraction concepts beginning in elementary grades not only meets the mathematical needs of students, but it also gives them a proper perspective of number structure. By using visual strategies, students are also able to develop number sense for fractions, understand fraction size and estimate reasonable answers for fraction addition problems (Cramer & Henry, 2002). Cramer and Henry also believe that it is more beneficial for students if teachers invest more time in elementary school building meaning for fractions using concrete models and emphasizing concepts, informal ordering and estimation. Much of the symbolic manipulation of fractions can then be adequately addressed later in middle school. However, researchers show that teaching fraction concepts requires linguistic support and an early introduction, which may support the development of rational-number sense and later the development of rational-number understanding (Miura & Yamagishi, 2002). Others see that "fractions are not just an easy step from whole numbers" (Hart, p81), and believe that extra effort has to be made to minimise problems caused by inclusion of fractions in the mathematics curriculum and to ensure that the teaching and learning of fractions is a continuous process from elementary to middle school levels (Hart, 1981).

Finally, difficulties in developing and understanding of fraction concepts and in learning to compute with fractions have been noticed and analysed. Novillis (1976) believes that students are not exposed to a sufficient variety of fraction concepts in elementary schools making it difficult for them to generalize these concepts. More models (e.g. geometric set of objects and unit segment of a number line) need to be presented to them in order to overcome these difficulties (Novillis, 1976).

Issues in computation and operating with fractional numbers:

The traditional teaching approach for fractions has been heavily symbolic and procedural. The rush to show students how to perform procedures can prevent them from establishing a proper understanding of the concept and a solid foundation of operation sense (Huinker, 2002). It would be better to allow students more time to understand problems involving fractions and then help them to write a formal algorithm of the solution or a workable strategy that can be used in other situations. Students need support in refining and extending their interconnections, and consider the necessary mathematical issues to generalize invented strategies into more powerful and symbolically driven procedures (Empson, 2001).

For example, teacher may start a fraction instruction by a problem such as:

Four students in the class need to share three chocolate bars equally. How much can each student have?

Now change the numbers in this problem and let students work out solutions. Such an activity can produce a variety of fractional solutions. It also creates a good environment to introduce and discuss fractional concepts such as equivalent fractions, comparing and operating with fractions. Students show good skills in solving equal-sharing problems using partitioning or transforming shares into biggest possible pieces.

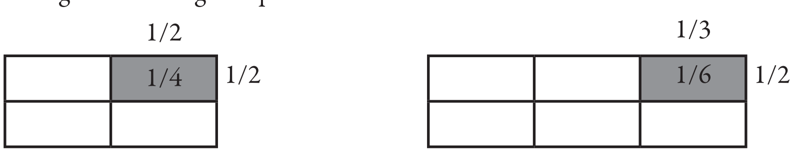
However, it seems that their initial strategies are not readily related to the symbolic procedure for renaming fractions. The following problem could help students and teachers to discuss and discover various dimensions of equivalent fractions: *24 children at a birthday party need to share 8 pizzas equally. How much pizza can each kid have?*

Students may use equivalence-strategies to solve this situation. Teachers should examine these strategies and draw attention to equivalent relationships in order to understand that different partitions result in an equivalent amount when all the sharing material is used. Also, they should examine whether these strategies make sense mathematically and whether they can be generalized (Empson, 2001).

Another issue in operating with fractions is the misconception of multiplication and division. Many students believe that multiplication makes numbers bigger and division makes them smaller.

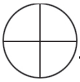
But students will encounter serious problems using this concept, especially when they have to solve multiplication and division involving rational problems (Graeber & Campbell, 1993).

They know from their previous experience that multiplication is a repeated-addition: $2 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$ and $2 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$, but it is very difficult for them to translate and visualise $\frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{2} \times \frac{1}{3}$ using the same strategy. They need to learn different strategies such using area-parts:



The shaded cells represent areas of rectangles (L_W). This strategy helps students to make sense of the fact that multiplication can make numbers smaller. However, students need to be familiar in using area models to interpret multiplication of whole number before being encouraged to use this model to interpret multiplication of fractions (Graeber & Campbell, 1993). In addition, students call on their informal knowledge of whole-number operations when dealing with fractions. They reconceptualise and partition units, when solving problems involving multiplication of fractions, in ways that make sense to them. They establish mental and workable algorithms focusing on the number parts and the equal-sharing strategy. This process has never been simple and straightforward for all students. However, during the learning process, the use of informal knowledge and concepts provides them with a foundation to build a bridge to connect complex ideas as their understanding of the multiplication of fraction, in particular, develops (Mack, 2001).

Division with fractions is another problem area for students. They still call on their experiences with whole numbers when solving division problems which involve fractions but they don't feel comfortable when they have to divide by a fraction e.g. (3 ÷ _). They need to understand the concept of dividing with fractions before they learn how to use a mechanical method to solve the problem. They also need to learn and practice how to make connections with other mathematical operations and concepts; as the learning of fractions division cannot be isolated from the learning of other topics in mathematics, where they can develop their *Proportional* and *Algebraic thinking* (Flores, 2002). They can use measurement and area parts to interpret the division with fractions, where they will be able to analyse results and build up formal algorithms for division. The following simple examples are very useful here:

1. How many halves in one half? The answer is 1 for sure $(\frac{1}{2} \div \frac{1}{2} = 1)$. .

2. How many quarters in one whole? The answer is 4 for sure $(1 \div \frac{1}{4} = 4)$.

3. The doctor told you to take "half" a tablet of your medicine each day until the packet is finished. How many days do you need to take the medicine, if the packet contains 6 tablets? $(6 \div \frac{1}{2} = ?)$

4. My son can swim 20 metres in two and one-half minutes. How many metres does he swim per minute? A possible solution:

20 metres in 2 _

40 metres in 5 minutes

8 metres in 1 minute (40 ÷ 5 = 8). Fraction interpretation for the solution:

$$\frac{20}{2\frac{1}{2}} = \frac{20}{\frac{5}{2}} = \frac{20 \times 2}{\frac{5}{2} \times 2} = \frac{40}{5} = \frac{8}{1} = 8$$

When solving a fraction division problems, students call on their skills and knowledge of whole number division, where "Problem situations can be categorized as measurement division (determine the number of groups); partitive division (determine the size of each group); or the inverse of a Cartesian product (determine a dimension of a rectangular array)" (Sinicrope, Mick & Kolb, p153, 2002). In fraction division, these interpretations, plus the "determination of a unit rate" and the use of division as "the inverse of multiplication", are very helpful for students to understand fractional problem division and then establish workable algorithms which can be used in other similar situations (Sinicrope et al, 2002).

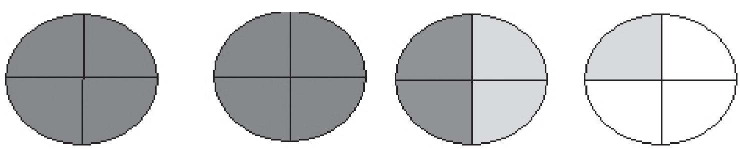
Issues in learning fractions:

Young children begin to show an understanding of fraction concepts after dealing with everyday situations but show a misunderstanding of these concepts later in their middle school years or when they enter the workforce. Researchers try to find "what happens between kindergarten and the army" (Riddle & Rodzwell, p202). Students in elementary schools visualise situations and present solutions by using pictures and diagrams; they use their understanding of fractions to develop procedures that make sense to them. While students naturally think adding to create improper fraction and then dividing to find wholes e.g. (Traditional solution for 2 _ + _): of completing wholes when solving fractional problems; teachers focus on finding common denominators,

$$2\frac{1}{2}+\frac{3}{4}=2\frac{1\times 2}{2\times 2}+\frac{3}{4}=2\frac{2}{4}+\frac{3}{4}=2\frac{5}{4}=2+1\frac{1}{4}=3\frac{1}{4}$$

, few students will use this method.

The most common solution uses the fill up wholes:



$$1+1+\frac{1}{2}+\frac{3}{4}=3\frac{1}{4}$$

Students get confused and start to learn by rote the approach being taught. Moreover, in middle school and older, students memorize rules and procedures for solving computations involving fractions. It seems that the traditional methods of teaching mathematics also don't help students in solving problems involving fractions, but rather can make them more confused. Teachers should use rules to enhance students' understanding of fractions, but they must try to make it as helpful and meaningful as possible (Ploger & Rooney, 2005). After all, in teaching fractions, we must match our instructions to the way in which students think about parts and wholes (Riddle & Rodzwell, 2000).

Conclusion

Students experience an enormous amount of difficulty in developing flexible skills to translate between concrete representations of fractions and mathematical symbols of fractions. In order to overcome these difficulties, teachers need to examine the ways in which students understand mathematical concepts and need to teach them to understand concepts prior to teaching them to perform procedures. Teachers should also encourage students to draw on their informal knowledge rather than using mechanical procedures.

However, teachers need also to rethink the way they teach fractions. They should give students a chance to present their thinking and understanding, because it may be possible for them to relate fraction symbols to informal knowledge in meaningful ways. We as teachers can learn from our students, who may be able to provide us with blueprints for instructions. We can be mathematicians together, each with a role to play in the educational process.

References

Cramer, K. & Henry, A. (2002). Using manipulative models to build number sense for addition of fractions, NCTM 2002 Yearbook: Making sense of fractions, ratios and proportions, (pp 41-48).

Empson, S.E. (2001). Equal sharing and the roots of fraction equivalence, Teaching Children mathematics; V7, No 7, (pp421-425).

Flores, A. (2002). Profound understanding of division of fractions. NCTM 2002 Yearbook: Making sense of fractions, ratios and proportions, (pp237-246).

Graeber, A.O and Campbell, P.F. (1993). Misconceptions about multiplication and division. The Arithmetic Teacher; March 1993: 40, 7 Academic Research Library, (pp408-411).

Hart, K. (1981). Fractions, Children understanding of mathematics: 11-16. CSMS 1981, (pp 66-81).

Huinker, DeAnn (2002). Examining dimensions of fraction operation sense. NCTM 2002 Yearbook: Making sense of fractions, ratios and proportions, (pp72-78).

Mack, N.K. (1995). Confounding whole-number and fraction concepts when building on informal knowledge. Journal for Research in Mathematics education, V 26, No 5, (pp 422-441).

Mack, N.K. (2001). Building on informal knowledge through instructing in complex content domain: Partitioning, units and understanding multiplication of fractions, Journal for Research in Mathematics education, V32, No3, (pp 267-295).

Miura, I.T. & Yamagishi, J.M. (2002). The development of rational number sense. NCTM 2002 Yearbook: Making sense of fractions, ratios and proportions, (pp206-212).

Novillis, C.F. (1976). An analysis of the fraction concept into a hierarchy of selected sub-concepts and the testing of the hierarchical dependencies.

Journal for Research in Mathematics Education Vol. 7, No 3, (pp131-144).

Ploger, D. & Rooney, M. (2005). Teaching Fractions: Rules and Reason,

Teaching Children mathematics; V12, No 4; (pp12-17).

Riddle, M. & Rodzwell, B. (2000). Fractions: what happens between kindergarten and the army? Teaching Children mathematics V7, No 4; (pp202-206).

Saxe, G.B., Taylor, E.V, McIntosh, C., and Geahart, M. (2005). Representing Fractions with standard notations: A development analysis.

Journal for Research in Mathematics Education, Vol. 36, No 2, (pp137-157).

Sinicrope, R., Mick, H.W. & Kolb, J.R. (2002). Interpretations of fraction division. NCTM 2002 Yearbook: Making sense of fractions, ratios and proportions, (pp153-161).

Smith III, J.P. (2002). The development of students' knowledge of fractions and ratios, NCTM 2002 Yearbook: Making sense of fractions, ratios and proportions, (pp3-17).