USING MATHEMATICS EXTENSION MATERIALS IN THE UPPER PRIMARY/LOWER SECONDARY SCHOOL

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According to education research approximately 10% of our students can be classified as being gifted and talented and if another 15% can fit into a high achieving class then about a quarter of the students in our classes are in need of enrichment materials in the mathematics curriculum. These students need challenge above and beyond the normal mathematics curriculum – they need to be presented with materials and experiences to develop their higher order thinking skills. A range of materials that have been trialed in some primary and secondary schools will be presented.

Thinking and working mathematically

In a rapidly changing world it will be the ability of people to think broadly, clearly and creatively to solve problems in a variety of situations that will be of most value to future generations. With the development of ever increasingly powerful technological tools in our society a crucial element of the education system will be the ability of students to properly comprehend the task, understand what mathematical procedures and approaches are appropriate for the solution of those problems and have the competency to complete

the procedure. The role of teachers working within the framework of an appropriate mathematics curriculum needs to be that of guiding and showing students how to think in a creative manner.

The need for problem solving in the mathematics curriculum

The purpose of incorporating a problem-solving approach into the mathematics curriculum is to provide students with a broader experience where the structure of the problem is not restricted to simple, single routine processes. The tasks need to give students the opportunity to think and perform on levels appropriate to their ability using skills from a number of areas to investigate and solve each task. Using a classroom discussion and a series of examples, students can be helped to develop an understanding of what is required in each task and then decide which skills to apply to solve it.

Good mathematics students can understand the problem, apply appropriate solution processes to find solution/s, reflect on the worth of those solutions, pose more questions that result from their investigations and explore and pursue further possibilities for their own interest.

Good mathematical problem-solving tasks are easy to understand, interesting, foster curiosity, involve the application of a number of skills, allow further questions to be posed, challenge students and involve a variety of themes. The solving of these problems requires each student to seek information outside the instruction delivered in the classroom framework, to modify their thinking and be prepared to use and think of new approaches in unfamiliar situations.

Rather than teach students how to think in a strategic-sense the problems presented to students need to encourage the application of general thinking skills as being a natural part of the process of being a problem solver.

From a classroom teaching point of view general problem solving is inclusive and provides the whole group with a better chance of experiencing success with tasks requiring a general approach, linking and combining skills from a number of areas rather than being restricted to one skill. The emphasis of this approach is to empower students to chart their own solution pathway rather than apply a set of algorithms which have been constructed by an unknown "expert" who has contrived a question, setting up a solution pathway.

The finding of a correct answer is not the most useful outcome or skill which can be achieved in a problem solving experience. Students should be encouraged to restate the

problem in their own words to clarify the task, to write a reflective statement as to what they learnt in completing the task and to write what it was that they found confusing and how they were able to move through, around or over any obstacles encountered.

The education system needs to provide opportunities for students to work with tasks that require the application of various problem solving strategies. The best way to improve students problem solving skills is by application rather than by examining them in isolation.

Whilst working with tasks of this type students need to be encouraged

- to think creatively
- to be flexible in the use of new ideas and approaches
- to try a number of ways of looking at the task
- to elaborate on similar ideas and to be original in thinking of new approaches or new ways of seeing problems
- to question all assumptions and initial premises
- to be perseverant and keep trying when things are not obvious
- to find a range of solutions; discounting the incorrect ones and realising why they don't work
- to reflect on the way that they went about finding a solution
- to reflect on the things that they learnt by completing the task

The need for extension tasks in the mathematics curriculum

Students in our classes have a range of academic abilities. Catering to the needs of the higher achieving, gifted and talented section of the group is as difficult as it is to the lower end of the group. It is crucial that high achieving students not be forgotten in the mathematics classroom. In the classroom, teachers have a responsibility to provide these students with the opportunity of realising their potential via the use of suitable approaches and materials. Often these students think in non-routine ways which is difficult for them to describe and difficult for teachers to understand how these students reach their conclusions let alone the thinking pathway that they followed. The communication of the thinking processes employed by students in the solving of tasks in many ways is more important than the conclusions that they reach.

Finding good materials can be a major difficulty. Good, rich and appropriate tasks enable teachers to extend and challenge capable and higher achieving students in their pursuit of either single or multiple solutions to complex tasks requiring mathematics. A

major hurdle in the process is lack of suitable materials which are pitched at appropriate levels. The references section gives a lead to the tasks recommended by the author.

Structure of problem solving tasks

Tasks need to be written and presented in a way that makes it easy for students to understand what it is that they need to solve. It is crucial that students are able to connect quickly with the nature of each task and have a clear initial understanding of what needs to be done. A most concerning problem arises when students who are capable of working through a task are not able to understand what needs to be done through poor literacy skills or other factors, are unable to start and so give up without being able to apply their skills and enjoy the process.

I propose a three-level tiered structure to provide students experience in the problem-solving process.

In section 1 it is the role of the teacher to lead a group discussion and present examples so that the nature of the task can be understood before presenting subsequent application and extension activities. In this way the context of each task is defined and so understanding the nature of the task is quickly covered allowing students to move on to employing suitable thinking and solution skills. From a teaching sense this allows all students access to the task where misunderstandings and other hurdles to making a start are removed.

The tasks in section 2 are open-ended in structure where students are encouraged to find multiple solutions to each task. These tasks can then be approached in a more confident manner where the skills applied to the problems in section 1 can be applied to the tasks in this section. Students need to work in a more independent manner than was applied to the tasks in section 1. The teacher's role with these tasks is more that of a mentor – to suggest possible approaches and clarify aspects leading to the discovery of multiple solutions. Each task has a hint section which can be given to students to help them access the task when appropriate. Solutions are included which can be used by the teacher to quickly see where the task can be taken as well as allowing students the opportunity of checking their work.

The tasks in section 3 involve the skills of reading/comprehension applied to a numeracy context. Reading/ comprehension plays a key role in the English curriculum and the tasks in section 3 require the application of literacy skills to solve problems requiring an application of student's mathematics skills.

Three sections of the problem solving structure

Section 1

These tasks are initially teacher lead to get the whole group of students on track with the spirit or essence of the activity. Each task provides three levels of instruction.

Introductory Lesson – teaching suggestions, notes and examples

An introductory task is given for teachers to use as a problem solving class lesson. Some notes are given as well as some key questions to stimulate discussion. A series of questions are provided that can be given to the class by writing them on the board or given out as photocopies to the class – solutions are provided. This section can be used to explore the context and concepts required in the task.

Refer to the sample in the Appendix

Application Activity

An application activity which builds on the concepts and ideas started in the Introduction lesson can then be used by the majority of students in the class to consolidate the ideas covered in the introductory lesson. This takes the form of a task card. Solutions are provided.

Refer to the sample in the Appendix

Extension Task 1

An extension task is given which presents students with some challenging questions connected to the ideas presented in the Introductory Task. These tasks can be given immediately to

- students who have been identified as being gifted, talented and higher achieving and so, can work with the Extension Activity straight after the Introductory lesson. These tasks require a deeper more sophisticated degree of thought.
- students who have completed the Introductory lesson and, Application Task and want to, or are capable of now working on more complex tasks can now be given the Extension Task.

Refer to the sample in the Appendix

Extension Task 2

Students who have completed Extension Task 1 and would like some more challenge can be given the second extension task. In this way all students of the class have plenty of materials to work through at appropriate levels.

This approach gives teachers a flexible method of use. The whole class can be given the introductory lesson and then the students in the class can be offered two pathways – the majority of students can undertake work with the application task whilst the more capable students work through the extension task. Further tasks can be completed where students who have worked through the application task can work on the extension task whilst the students who worked on the extension task can be given an extra extension task.

This is an attempt to cater for individual differences of students in the classroom by giving a number of pathways for the students to pursue. Also the extension task is not just available for the high achievers but available in time to all students. The high achieving students can work through the extension tasks immediately while other students who will take more time to develop skills appropriate to the task can attempt the extension task after working through the application task.

Refer to the sample in the Appendix

Figure 1.

*Flow Chart showing the structure of a problem solving lesson using tasks from section 1.*
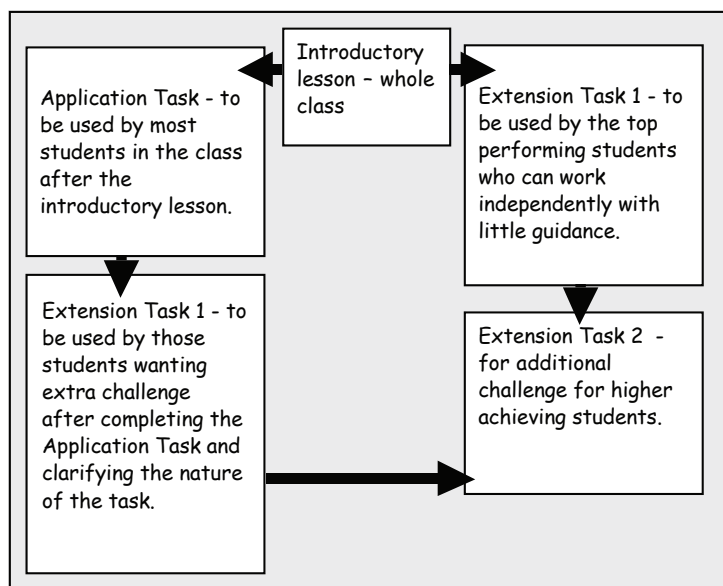
Section 2

The tasks in this section require students to make the first move in understanding the context of the question. An open-ended problem is one that has more than one answer. This for students and teachers means that the pursuit of THE single answer to a question does not take the whole focus of the investigation. Rather than applying a funneled search for a solution to a problem, students need to be encouraged to have a broad-minded approach to finding solutions to mathematics problems.

Figure 2.

Diagram showing the difference between a closed and an open ended problem.
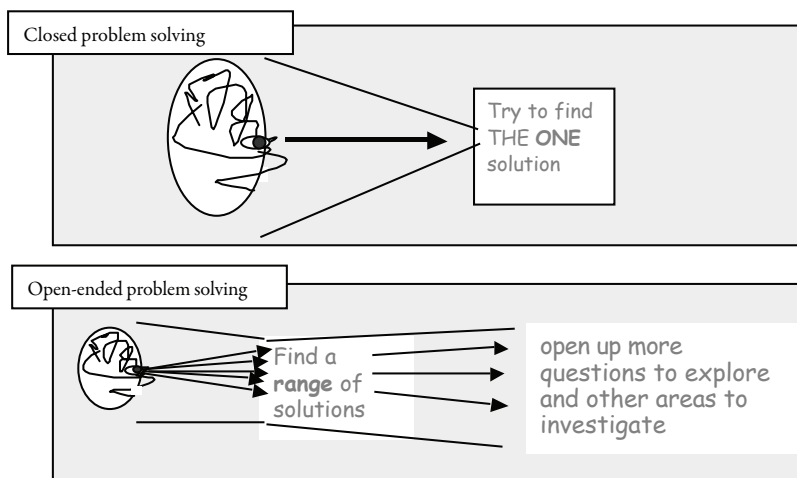
Students need to alter their frame of thinking and need to look at the question in its entirety to identify multiple stages and multiple solutions from the start. Successful strategies such as making lists of possible outcomes, working backwards from the solution, drawing a diagram or picture to show the possibilities, making a model or looking for patterns may all be appropriate. Students need to understand the task, find an initial solution, examine the nature of that solution and look for other related situations that lead to find more solutions. In adopting this approach students can be encouraged to make up their own investigations and design their own tasks which can be shared with the group.

Refer to the sample in the Appendix

Section 3

The questions in this task don't need to be given in sequence with sections 1 or 2 – in fact they can be used singularly or in total at any point within it. As they are different in structure to tasks from the other sections, they provide challenge in a different form.

A command of language, written or verbal is a fundamental component of everything that we do. Before a problem can be solved it needs to be understood on a holistic level leading to the uncovering of smaller details of the problem which need to be specifically understood. The quicker the problem-type can be recognised through language the quicker it can be categorised to branch/branches of mathematics allowing the solution process in turn to begin. Comprehending a problem, using a logical approach to analyse it and then applying mathematics to work through it and expressing the solutions in a clear and coherent manner shows why literacy is a key factor in the solving of all problems. The questions in this section are similar to reading-comprehension problems undertaken in other subject disciplines, with each requiring a mathematical component to complete the task. The questions range from astronomical facts to the discipline of geography and other biological topics, such as hair growth.

A hint section for each sheet is given which makes it quick and easy for teachers to become acquainted with the task without having to work through the whole sheet. Teachers can either copy the hint sections or deliver them verbally to get students back on track with the task.

Refer to the sample in the Appendix

Conclusion: results of trialing in primary and secondary schools

The materials outlined in this paper have been extensively trialed in a variety of schools, both primary and secondary in a number of states across Australia. The success of an educational approach can only be ascertained and judged after students and teachers have had an opportunity to use them. In the light of this the materials described in this paper were trialed in a range of both primary schools and secondary schools predominately in Victoria but also in Western Australia, New South Wales and Queensland. The findings reported by all parties were extremely positive where students responded in a very positive way to the use of the non-subject specific approach. This was seen as providing all students with appropriate challenge but more importantly providing the higher achieving band an avenue to apply their higher order thinking skills. The classroom lead problem solving

section was cited as being pivotal in the process of introducing the idea of problem solving to students, allowing the use of the open-ended tasks in a less teacher focused manner. The literacy-mathematics tasks were used in a variety of ways with most students finding them interesting. The one-off nature of these tasks allowed these tasks to be used when the time permitted.

References

Bull, I. (2009) - Mathematics tasks for gifted, talented and high achieving students – Upper Primary / Lower Secondary. South Melbourne, Vic: Teaching solutions.

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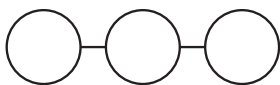Bull. I. Mathematics Revision and Practice. Putney, NSW: Phoenix Education Pty Ltd.
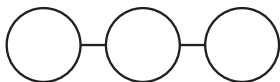
Appendix

Section 1

Introductory Lesson

1. Choose from the numbers {1, 2, 3, 4, 5, 6}. (Use each number only once each time).
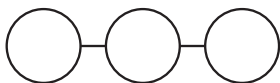
(a) Place the numbers into the circles to make the numbers add to 9.

(b) Place the numbers into the circles to make the numbers add to 10.

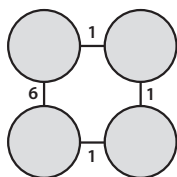(c) Place the numbers into the circles to make the numbers add to 12.

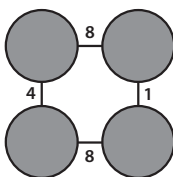2. Jodie needs to put numbers on houses which go from 100 to 108. How many of each digit does she need?

Application Activity

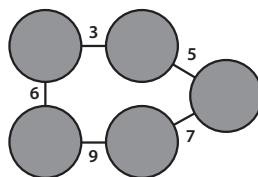1. Write a number in each circle to give the total for each side.
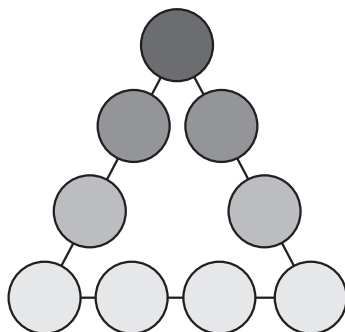
(a) **(b)** **(c)**

2. Place the numbers {1, 2, 3, 4, 5, 6, 7, 8, 9} into the circles so that each line adds up to (a) 17 (b) 20 (c) 23

3. Jordan needs to put numbers on houses which go from 1 to 40. How many of each type of digit does he need?
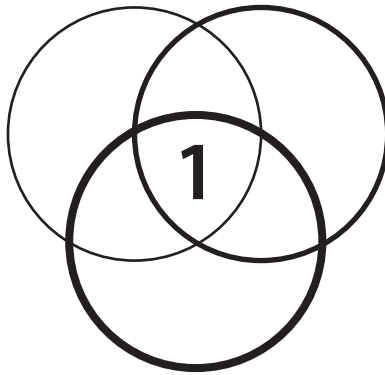
Extension Task 1

1. Louisa wants to place numbers in the puzzle so that the numbers in each circle have the same sum.

The number 1 is a part of each circle.

Place the numbers into the puzzle so that the sum of the numbers in each circle is the same.

(a) {2, 3, 4, 5, 6, 7}
(b) {4, 5, 6, 8, 9, 10}

2. Find the number of the digits from 0 to 9 that are needed for house numbers from 1 to 100. Explain how you found your answer without listing the numbers.

Extension Task 2

1. Palindrome numbers read the same way forwards and back such as 1441, 2003002 or 555.
(a) How many palindrome numbers are there between 1000 and 10 000?
(b) How many of these palindrome numbers are

(i) even   (ii) odd
(iii) divisible by 3   (iv) divisible by 6

2. How many palindrome numbers are there with
(a) 5 digits   (b) 6 digits

Section 2

1. In the opening of a milkbar every 6th customer receives a free drink and every 9th customer gets a free lolly.
(a) If 200 customers come into the shop on the first morning how many drinks and lollies were given out? Explain how you got your answer.
(b) How many customers received both a drink and a lolly? Explain how you got your answer.

2. During the afternoon 20 customers received both a drink and a lolly.

(a) How many customers could have visited the shop in the afternoon?

(b) How many drinks could have been given out.

(c) How many lollies could have been given out?

Hint

Write a list of multiples of 6 and 9 and look at the types of numbers which are both multiples of 6 and 9 at the same time – these get both a drink and a lolly.

1. (a) As every 6th customer receives a free drink then the 6th, 12th, 18th .... customers will receive a free drink and the 9th, 18th, 27th....... will receive a free lolly.

(b) The first customer to receive a free drink and lolly is the 18thcustomer.

2. These customers both share the multiples of 6 and 9 to get both the lolly and drink. More customers will probably come in after the 20th customer got the free drink and lolly just before the 21st customer to get both a drink and a lolly.

Section 3

The Amazon river in South America flows largely through Brazil and is the largest river in the world according to the number of tributaries and volume of water carried. It is 6400 kilometres or 4000 miles long and the only river longer is the Nile which is 4160 miles in length. The Amazon drains an area of about 7 million square kilometres. The Amazon discharges between 34 million and 121 million litres of water per second and deposits 3 million tons of sediment at its mouth per day. The volume of water is one-fifth of the fresh water that flows into the world's oceans.

1. Using the length of the Amazon river how many kilometres are there in a mile and find the length of the Nile in kilometres.

2. List the lengths and widths of three rectangles which have the same area as the area covered by the Amazon river.

3. Find the weight of sediment which is deposited at the mouth of the Amazon each year.

4. Find the minimum and maximum volume of water that the Amazon carries

(a) each minute (b) each hour (c) each day.

5. Estimate the volume that flows from the Amazon river each year and so estimate the volume of water that flows into the world's oceans in a year.