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The importance of
developing
children's counting
skills: the move
from rote to
rational counting

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Counting



In this session the focus will be on:

- Counting Principles
- Framework of Mathematical Learning: Counting
- The Mathematics Online Interview: Counting tasks
- Subitising
- Strategies for developing rational counting
- Victorian Curriculum: Mathematics

Learning to count



- Children start by rote counting. They learn the “song” and sometimes they say words in the wrong order.
- Children then recite numeral names in order.
- As the child gets older s/he attaches a numeral name to a number of things.
- Children learn that each number symbol represents an amount, gradually putting recognition of numerals together with counting.
- Expert counting requires the co-ordination of several different ideas(see the five principles described by Gelman and Gallistel (1978)

Possible Counting Mistakes



- Fail to correspond their pointing to individual objects.
- Fail to correspond the sound with the pointing action.
- Miss an object or miss a number name
- Count an object more than once or apply the same name twice
- Confusing the order of names.
- Lose track of what has been counted & what remains to be counted.
- Don't stop the verbal sequence at the last object, keep on because of the rhythm.
- Don't realise the last number is cardinal.
- Choose to miss some objects because they don't think they should be included in the count because of their colour, shape, position etc.

From reciting sequence of number names to meaningful counting



- Many children can recite numbers in sequence without being able to count.
- Meaningful counting relies on:
 - one to one principle
 - stable order principle
 - cardinal principle
 - abstraction principle
 - order-irrelevance principle(see research by Gelman & Gallistel, 1978)

Rational counting

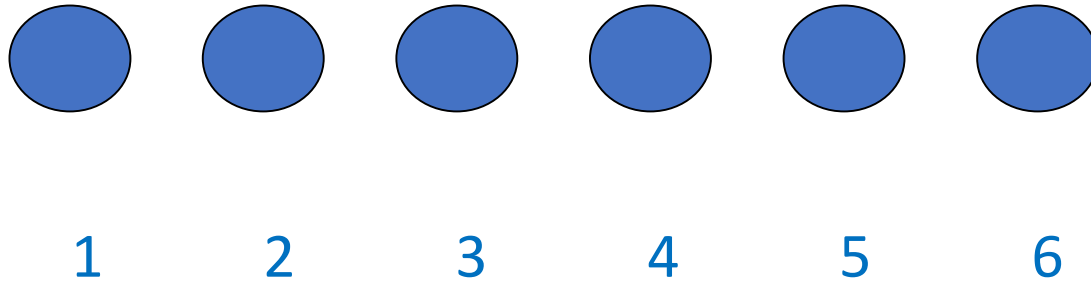


- In rational counting (meaningful counting) the child gives a correct number name as objects are counted in succession.
- In rational counting the child not only uses one-to-one correspondence but also is able to answer the question about the number of objects being counted.
- Rational counters exhibit all the counting principles.

One-to-one principle



Every object to be counted is associated with EXACTLY one of the number names — not more than one and not less than one.



There is one number tag for each object

One-to-one Principle



-
- ▶ The difficulty for most students is to coordinate the recitation of the verbal sequence with checking off the objects.
 - ▶ The coordination of two actions becomes more difficult as the number of objects in the group gets larger, or when the child has to think about what number name comes next.
 - ▶ Students struggle with counting steps because they have trouble co-ordinating stepping from one step to another while saying the number sequence.

Stable-order principle



- The number names are in a fixed order and must always be used in this fixed order (1, 2, 3, 4, 5, 6 ...)

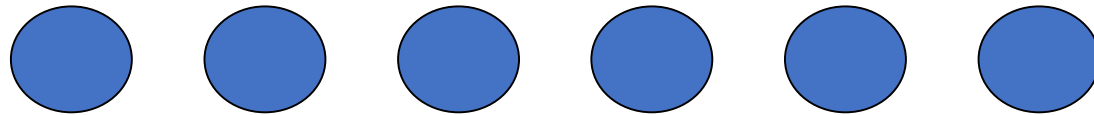
- Must use the number names in the same order every time.

Cardinal Principle



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- If you are counting a set of objects, then the number of objects present is the last number that has been said.



For example “There are ... 1, 2 ,3 , 4, 5, 6 dots. Yes, there are 6 dots altogether.”

- The purpose of counting is to find “how many” and this is given by the last number stated.
- Fully grasping the cardinal principle depends on understanding the previous two principles (one-to-one & stable order)

Cardinal Principle



There are three phases in the development of cardinality (Fuson & Hall, 1983).

1. Reciting the last number with no clear idea that it relates to quantity, but because they realise it is the response the adult expects.
2. Understanding that the last number of the count relates to the quantity.
3. Understanding the progressive nature of cardinality i.e. if they are stopped in the middle of a count they can say how many they have counted so far, then carry on.

Cardinal Principle



It is necessary for the child to grasp the cardinal principle before they can:

- Understand that the next number in the sequence represents a larger quantity.
- Use the strategy of “counting on”.
- Use counting to determine and compare the equivalence of sets.

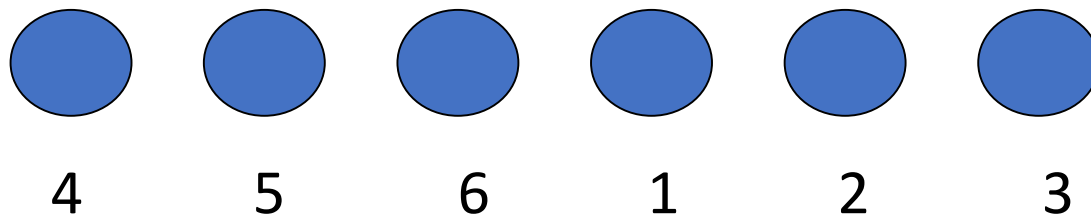
Abstraction Principle



- We can count any specified set of "objects", real or imagined, similar or disparate (different).
- Different types of collections can be counted - not just collections of the same type.
- The arrangement of the objects to be counted does not affect how many there are.
- *Some children will only count red counters because red is their favourite colour.*

Order-irrelevance Principle

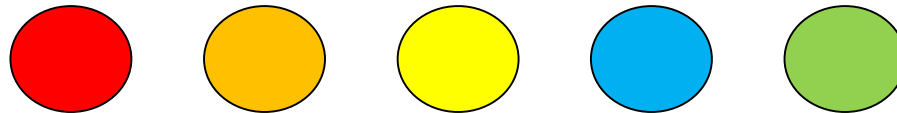
- The order in which the objects in a set are counted does not affect the number in the set (e.g. counting a string of objects can start at the beginning, although it may be unwise for other reasons)
- It doesn't matter in which order the objects in a collection are counted. Can start in the middle of a row – doesn't always have to be counted left to right.



Order- irrelevance Principle

Students who can count quite well will have difficulty with this until counting is well established.

For example, put 5 objects of different colours in a row, with the yellow one in the middle.



- Ask children to count all the objects, starting with the yellow object as '1'.
- For a greater challenge, ask the children to count all the objects, making the yellow one '5'.

Order-irrelevance Principle

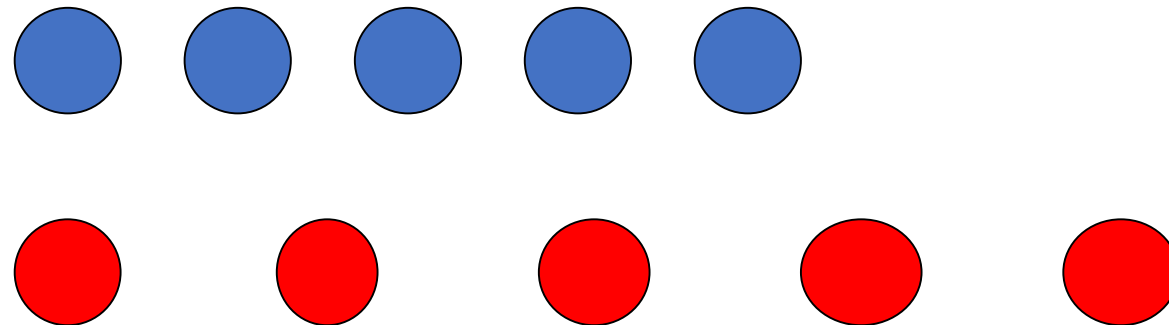


This principle requires knowledge of the previous four principles and entails understanding that:

- ▶ Each counted item is still a "thing" not a "one" or "two" etc.
- ▶ The name tags were temporarily given and do not necessarily adhere to the objects once the counting is finished.
- ▶ Whatever order the objects are counted in the same cardinal result occurs.
- ▶ *Need to grasp principle of abstraction in order to be able to generalise the use of counting as a tool*

Order-irrelevance Principle

- helps us confirm the consistency of the quantity of a set, and it is confidence in that consistency that enables us to be sure about making comparisons.
- this confidence helps us to override perception when spatial changes make things appear bigger, and it may therefore underlie our ability to recognise the conservation of number.



Framework of Mathematical Learning: Counting



<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/enrp/enrplaf.htm>

0. Not apparent.

Not yet able to state the sequence of number names to 20.

1. Rote counting

Rote counts the number sequence to at least 20, but is not yet able to reliably count a collection of that size.

2. Counting collections

Confidently counts a collection of around 20 objects.

3. Counting by 1s (forward/backward, including variable starting points; before/after)

Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.

Framework of Mathematical Learning: Counting



4. Counting from 0 by 2s, 5s, and 10s

Can count from 0 by 2s, 5s, and 10s to a given target.

5. Counting from x (where $x > 0$) by 2s, 5s, and 10s

Given a non-zero starting point, can count by 2s, 5s, and 10s to a given target.

6. Extending and applying counting skills

Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical task

Mathematics Online Interview Tasks



<https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/onlineinterviewbklet.pdf>

1 Teddy Task

Show the child the teddies and get the cup. Cup must be able to hold more than 20.

Please take a big scoop of teddies. . . . Please put a few more teddies in to fill up the cup (at least 20).

- a) Hold them in front of you. . . . Tell me how many teddies you think are in the cup.
- b) Please check to find out. 😊 → Q2

If the child is in the first year of school or had difficulty counting the teddies, move to the detour on page 24 now, otherwise continue on.

Mathematics Online Interview Tasks



<https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/onlineinterviewbklet.pdf>

2 Counting Forwards, Backwards, and Breaking the Sequence

Please count for me by ones without the teddies. Start counting from ... I'll tell you when to stop.

- a) 1 ... 32 😊 → part (e)
- b) 53 ... 62 😊 → part (d)
- c) 84 ... 113 (*remember to note first difficulty*)
- d) Count backwards from 24. I'll tell you when to stop (24, 23, ..., 15).
(*If child hesitates, say 'like 24, [pause] 23, ...'*) 😊 → Q3 😊 → part (e)
- e) Count backwards from 10. I'll tell you when to stop. → Section B
(*If child hesitates, say 'like 10, [pause] 9, ...'*)



<https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/onlineinterviewbklet.pdf>

3 Before and After Tasks

- When you are counting forwards, tell me the number that comes after 56.
- Tell me the number that comes before 56.

4 Counting from 0 by 10s, 5s, and 2s

Count by 10s for me (then 5s, 2s) as far as you can go. [at any stage] 😊 → B
(Stop child after 110, 45 and 30, respectively.)

5 Counting from x by 10s and 5s

- Start at 23 and count by 10s. (Stop the child after 103.) 😊 → B
- Start at 24 and count by 5s. (Stop the child after 44.) 😊 → B

<https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/onlineinterviewbklet.pdf>

6 Counting from x by a single digit number

- a) Start at 11 and count by 3s. (Stop the child after 35.) 😐 → B
- b) Start at 20 and count by 7s. (Stop the child after 55.) 😐 → B

7 Counting Money

Tip out the collection of money onto the table.

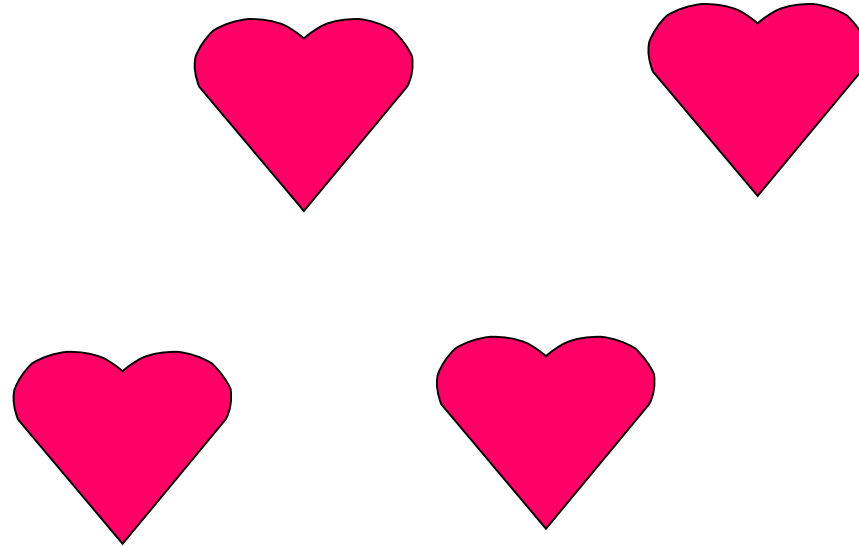
- a) Please count the money for me. (No recording allowed for this task.)
- b) What did you find? 😐 → B
- c) How much more money would you need to have \$5? (Any counting method yielding the correct answer is acceptable.)

Subitising



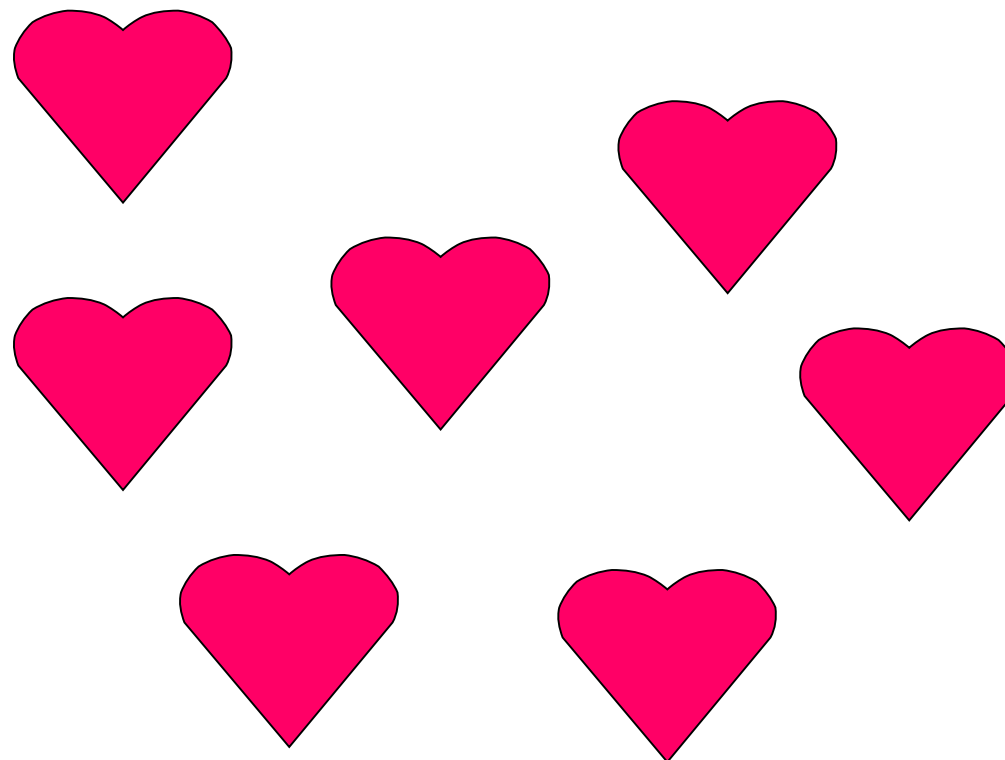
- The process of immediately recognising how many items are in a small group
- This name comes from the Italian word subito, which means "immediately" or "right now".
- Subitising contributes to early forms of grouping by "chunking" information,.
- The process of subitising can also be used with seeing parts in the whole.
- When playing a game with dice we normally recognise the number of dots immediately. **Is this subitising?**

Conceptual knowledge and subitising

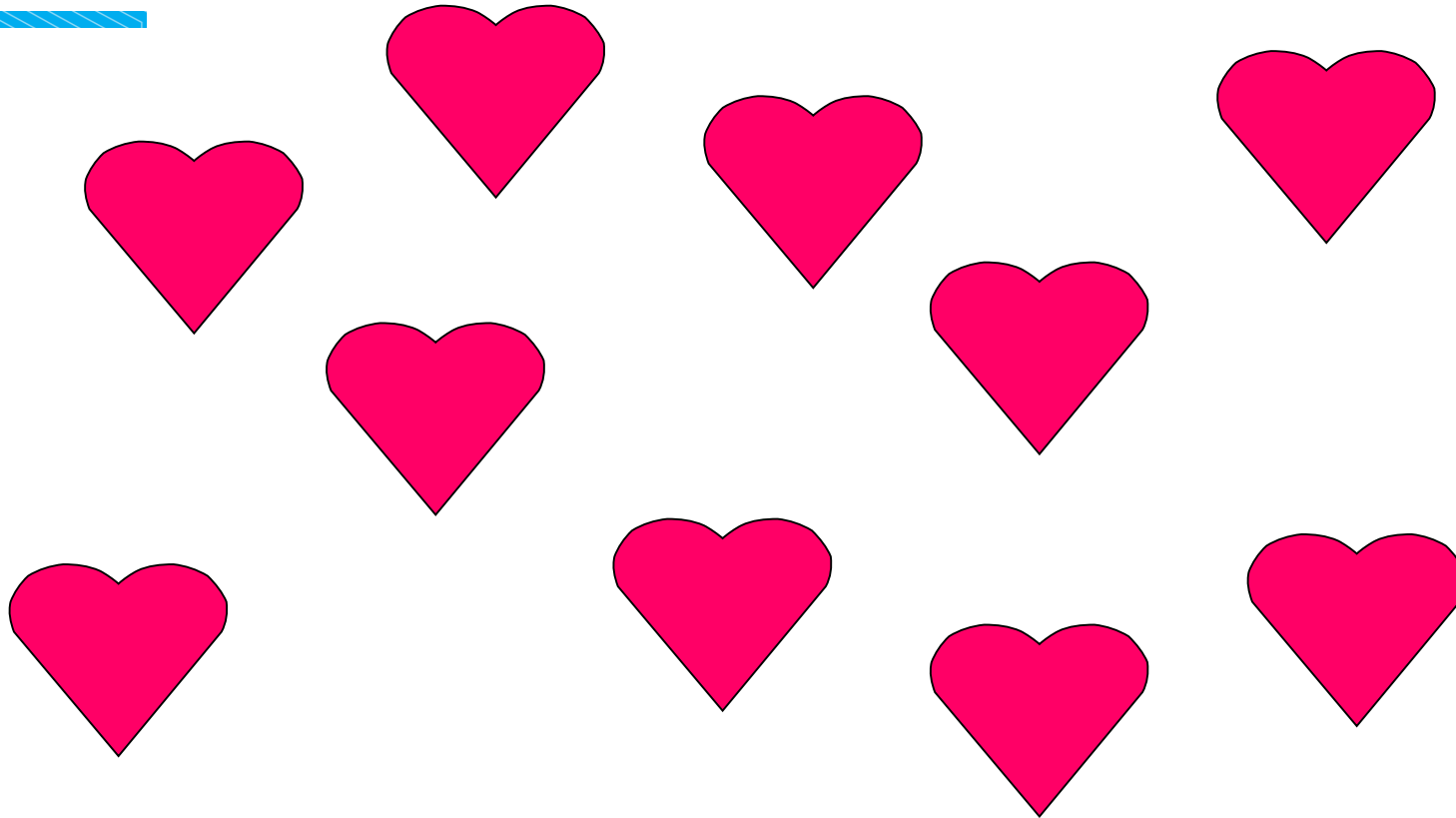


- How many?
- Did you think “1, 2, 3, 4”?

At the limits of subitising



Beyond subitising



What strategy did you use?

Subitising



Common Misunderstandings

Level 1.1 Subitising Tool

<http://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/assessment/lvl1cardset.pdf>

Unbreakable chain to breakable chain to bi-directional chain

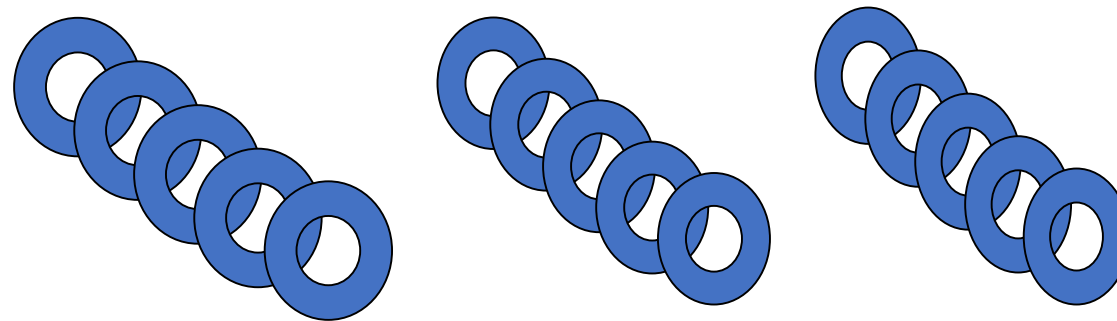
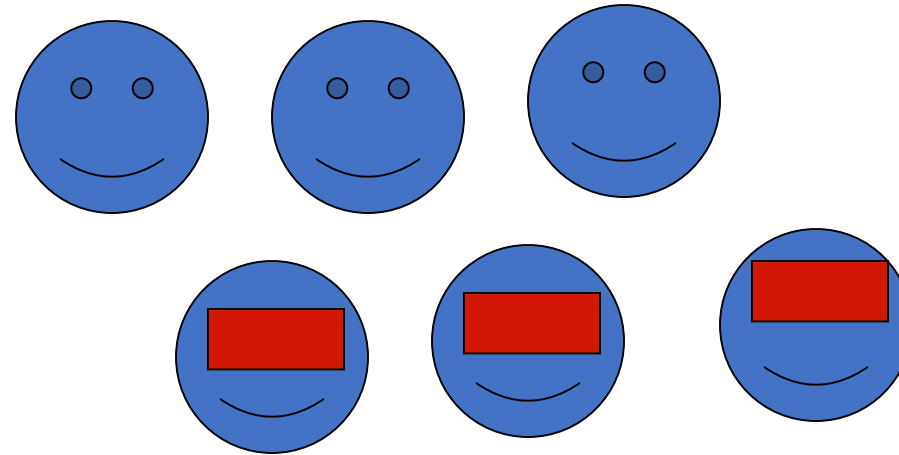


- Rote Counting – from zero or given number (unbreakable)
- Counting from a given number to another given number – from 4 to 18 (breakable)
- Counting from 12 to 24 and back to 20 – by 1s, 2s (bidirectional)

Counting on and back are essential skills for adding and subtracting and eventually multiplication and division.

Perceptual to imagined to composite units

- perceptual units
- imagined units
- composite units



(essential for multiplication)

Counting by ones to skip counting



- skip counting occurs frequently in everyday life e.g. time in minutes, sport scores, money
- facility with skip counting is necessary for multiplication, as well as “clever counting”
- students develop skills with skip counting by 10s, then 5s, then 2s, then others
- develop skip counting through “whisper counting”
 - 1 2 3 4 5 6 7 8 9 10 11 12 13 14
- Use games such as “Buzz”, “Fizz Buzz” etc into middle years.

Strategies for developing counting



Reality

- ▶ Give a purpose to the counting. Involve children in needing to count. e. g. Do we have enough pencils for everyone?

Modelling

- ▶ Teacher says "watch me" and does a count, naming each object with a correct numeral, and using the last tag as the final number for the count. Talk out loud. Do it at every opportunity.

Making small groups

- ▶ Give a group of counters to children, ask them to make groups of two. Then push them all back together and ask them to make twos again. Repeat until this becomes quicker. Then introduce threes and fours.

Strategies for developing counting (cont.)



Move to less concrete

Hide the counting objects under a card. Show the objects, count, add one more and cover. Ask the child to count again.

Different arrangements

Arrange groups of counters differently. See if children know where they started counting so they can decide where to stop.

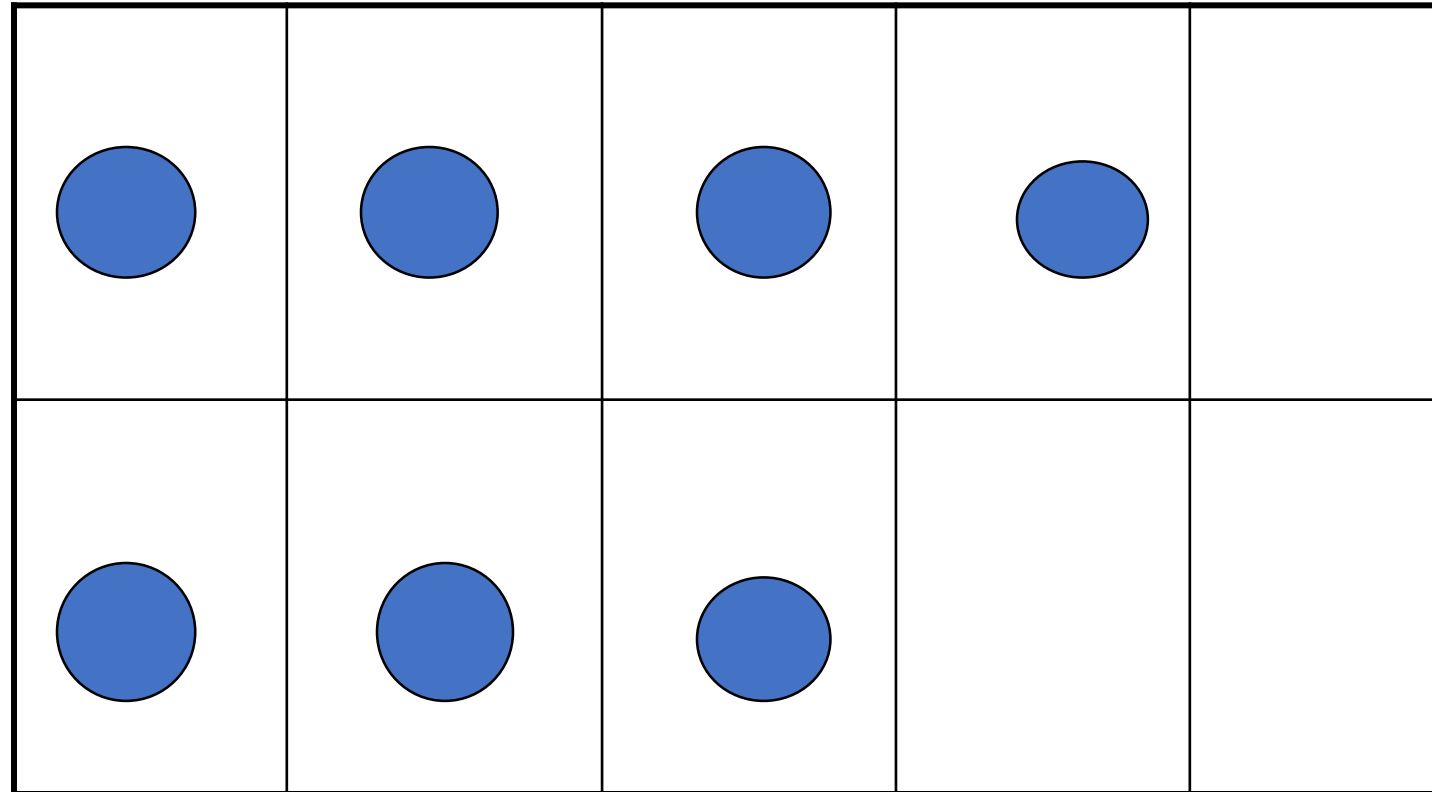
- a line
- a circle
- randomly

Part - whole



- How many different ways can the child show you 5 sticks in two hands?
 - one and four
 - two and three
 - three and two
 - four and one
 - five and none
- Tens Frames can be used to explore part whole concepts and provides a strong conceptual base for addition and subtraction strategies.
 - use an egg carton with two of the hollows cut off
 - use counters, sultanas or Smarties
 - individual Ten Frame
 - vary the size of the Tens Frames
 - Play games with Tens Frames

Using Tens Frames



Tens frames can be used as subitising cards as well as being made with counters on paper

Activities that encourage development of counting include:



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- singing counting songs
- listening to counting books
- setting the table
- keeping score
- playing board games
- manipulating numerals
- placing numbers in order
- playing Number Bingo, card games
- matching numerals
- making sets of objects

Victorian Curriculum: Mathematics



Foundation

- Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point ([VCMNA069](#))
- Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond ([VCMNA070](#))
- Subitise small collections of objects ([VCMNA071](#))
- Compare, order and make correspondences between collections, initially to 20, and explain reasoning ([VCMNA07](#))

Victorian Curriculum: Mathematics



Year 1

- Develop confidence with number sequences to and from 100 by ones from any starting point. Skip count by twos, fives and tens starting from zero ([VCMNA086](#))
- Recognise, model, read, write and order numbers to at least 100. Locate these numbers on a number line ([VCMNA087](#))
- Count collections to 100 by partitioning numbers using place value ([VCMNA088](#))

Year 2

- Investigate number sequences, initially those increasing and decreasing by twos, threes, fives and ten from any starting point, then moving to other sequences ([VCMNA103](#))
- Recognise, model, represent and order numbers to at least 1000 ([VCMNA104](#))
- Group, partition and rearrange collections up to 1000 in hundreds, tens and ones to facilitate more efficient counting ([VCMNA105](#))

Victorian Curriculum: Mathematics



Year 3

- Investigate the conditions required for a number to be odd or even and identify odd and even numbers ([VCMNA129](#))
- Recognise, model, represent and order numbers to at least 10 000 ([VCMNA130](#))
- Represent money values in multiple ways and count the change required for simple transactions to the nearest five cents ([VCMNA137](#))

Year 4

- Investigate and use the properties of odd and even numbers ([VCMNA151](#))
- Recognise, represent and order numbers to at least tens of thousands ([VCMNA152](#))
- Investigate number sequences involving multiples of 3, 4, 6, 7, 8, and 9 ([VCMNA154](#))

Reflection



Look at the two NAPLAN questions:

- What is the purpose of each question?
- Why have they chosen each of the distractors?
- Do you think students at these level would be successful?
- What would be the greatest difficulty for unsuccessful students?
- Do you think these are 'fair' questions?

NAPLAN tasks

26 Dana started at 10 and made this number pattern.


10, 11, 13, 16, 20, 25, ?

What is the next number in the pattern?

- 26 30 31 35

Year 3

35 This grid shows the numbers from 1 to 24.
Mike has crossed off the number 1.

Write your answer
in the box. 

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| X | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | | | | | | |

Year 5

He crosses off all the multiples of 2, then of 3 and then of 5.

How many numbers will still be showing on the grid?

Implications for primary mathematics classrooms?

Mathematics Developmental Continuum



One to one correspondence

<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/continuum/Pages/onetoone.aspx>

Counting up to 20 objects

<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/continuum/Pages/countingobj10.aspx>

Counting with Two Digit Numbers

<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/continuum/Pages/countingtwo125.aspx>

Skip Counting

<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/continuum/Pages/skipcount20.aspx>

Advanced Skip Counting

<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/continuum/Pages/advskipcount25.aspx>