THE MATHEMATICAL ASSOCIATION OF VICTORIA

MATHS TREATS BY LUCIANA THE POSSUM

MATHEMATICS BEHIND CALCULATORS

Mathematics underpins modern technology. Everyday mathematics is usually written using the symbols of the Hindu-Arabic numeral system. We refer to this as a base 10 numeral system or 'decimal system' (based on the Latin prefix *deci*, from *decimus*, meaning tenth). Decimal numbers use ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. In contrast, the binary numeral system (base 2) underpins most electronic devices such as your calculator and computer. The two digits of a base 2 system are usually 0 and 1. Physically, the 0 represents 'off' or low voltage and 1 represents 'on' or high voltage electricity.

BINARY NUMBERS

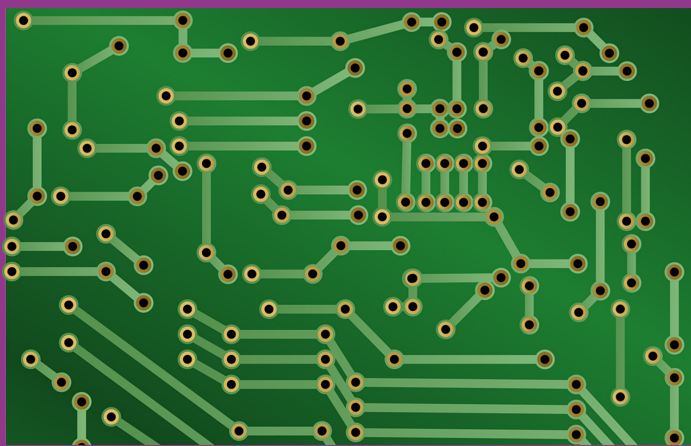
Gottfried Leibniz created the modern binary numeral system in 1689. The base 2 numeral system is a positional numeral system just like the decimal system, but in this case the place value of each column is a power of 2.

In decimals, 613 (base 10) = 600 + 10 + 3 = $(6 \times 10^2) + (1 \times 10^1) + (3 \times 10^0)$. So, in binary, 101 (base 2) = $(1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 4 + 0 + 1 = 5$ (base 10).

ACTIVITY

Write the first twenty decimal numerals in binary notation. What pattern can you see? How would you convert from decimal notation to binary notation? Explain your process in writing. How would you add or subtract numbers in binary notation? How would you multiply or divide numbers in binary notation? In what ways is the process the same or different from operations with decimal numbers? What would a non-integer base numeral system (e.g. base 2½) look like?

BOOLEAN ALGEBRA



George Boole first introduced Boolean algebra in 1847 to describe logical operations where the variables are statements which are true (1) or false (0). The main operations in Boolean algebra are AND (two statements are both true), OR (either or both statements are true) and NOT (something is not true). These basic operations work the same way as Venn diagrams. Boolean algebra is used in set theory and statistics and is key to computer circuits and computer programming.

ACTIVITY

Draw Venn diagrams to illustrate the following: (p AND q), (p OR q), (NOT p). Draw up a 'truth table' to find the values of (p AND q) and (p OR q) for different combinations of values of p and q (true=1, false=0). What happens when you add NOT into the mix? Draw Venn diagrams to illustrate NOT(c AND d) and NOT(c OR d). How do the results relate to the circle representing c and the circle representing d? Can you think of some weird and wonderful composite operations that combine several of these basic operations? Draw up a truth table of resultant values for each composite operation.

REFERENCES AND FURTHER READING

https://en.wikipedia.org/wiki/Positional_notation

https://en.wikipedia.org/wiki/Binary_number

https://en.wikipedia.org/wiki/Non-integer_base_of_numeration

https://en.wikipedia.org/wiki/Electronic_circuit

https://en.wikipedia.org/wiki/Boolean_algebra

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